

# Cooperative Transmission Scheme with Two Relay Stations and Phase Feedback Channels

Bruno Fontana da Silva, Renato Machado, and Andrei P. Legg

**Resumo**—Neste artigo, propõe-se um novo esquema de comunicação cooperativa com duas estações *relays* e canal de realimentação de fase entre as estações *relay* e o nó destino. O uso de um pré-processador nas estações *relay* possibilita que o esquema proposto alcance um grau de diversidade completa e também um ganho de arranjo no nó destino. Um estudo do efeito da realimentação de fase quantizada para o esquema proposto é apresentado, assumindo-se que os canais sofrem um desvanecimento do tipo Rayleigh, plano e quase-estático. Uma análise do projeto de alocação espacial das *relays* é realizada também.

**Palavras-Chave**—Canal *relay*; diversidade de transmissão; realimentação de fase; sistemas cooperativos.

**Abstract**—In this paper, we propose a new transmission scheme with two relay stations and phase feedback channels between the relays and the destination node. The preprocessor use in the relay stations is managed to achieve a full diversity order and also an array gain at the destination node. A phase feedback investigation for the proposed scheme is presented assuming quasi-static flat Rayleigh fading channels. An analysis of the relay spatial allocation design is also performed.

**Keywords**—Cooperative systems; phase feedback; relay channel; transmit diversity.

## I. INTRODUCTION

The well-established knowledge over multiple-input multiple-output (MIMO) wireless communication systems due to its feature of improving capacity and reducing fading sensitivity is already deeply exploited in the scientific literature. Recently, Kaiser et al. [1] have published an overview on the challenges regarding the ultra-wide band (UWB) systems and its potential combining it with MIMO schemes. The authors' statement reveals that although UWB systems provide an extensive bandwidth (i.e., significant diversity in the time domain), higher data rates can be achieved combining UWB and MIMO schemes.

A simple and effective combination of both spatial and time transmit diversity was proposed by Alamouti [2], and his idea was further extended in [3] giving rise to the well known class of orthogonal space-time block codes (OSTBC). MIMO wireless communication systems can obtain significant performance improvements if the use of a quantized feedback channel is considered. In [4], Choi et al. proposed a phase-feedback-assisted scheme with four transmit antennas based on

the Alamouti code in a typical MIMO communication system considering the use of a phase feedback channel in order to achieve full diversity and to increase the array gain.

For network systems, where multiple antennas cannot be supported, improved signal coverage and wireless system performance can be accomplished by using relay based strategies [5]. Relays are typically a low cost solution, linked to a specific source (base station or a node), which could be applied in initial stages of a network in order to provide coverage to a large area at lower cost rather than having more base stations.

Considering the use of space-time diversity techniques in a relay scenario, a cooperative diversity scheme using two relay stations and linear coherent detection was proposed in [6]. It consists of a transmit node, two relay stations and a destination node. It is shown that full diversity can be achieved if the feedback channel is properly used.

Led by this background, this work proposes a cooperative transmit scheme using two relay stations and a phase feedback channel between the destination node and the relay stations. This contribution addresses the following issues: first, a spatial diversity technique is exploited aiming the improvement of the overall signal-to-noise ratio (SNR) performance at the destination node. For this, we consider a phase feedback strategy that was motivated by [4]. Last but not least, a relay spatial allocation design is proposed. Using a log distance large scale path loss model and a simple geometric evaluation based on the line-of-sight distance between the transmitter and the receiver, it is possible to estimate the best distance for placing the relay stations depending on some design parameters.

The rest of this paper is organized as follows. Section II presents the system model. Section III details the proposed transmission scheme and also derives the relay spatial allocation design equations. In Section IV, simulation results are presented. Finally, Section V outlines the main considerations and final remarks of this paper.

## II. SYSTEM MODEL

We consider a cooperative wireless communication system consisting of one source node (node  $s$ ), two relay stations ( $R_1$  and  $R_2$ ), and one destination node (node  $d$ ). The schematic diagram is shown in Figure 1. Herein we assume that each relay station has  $M_R = 2$  antennas. The wireless communication channels are assumed to be flat quasi-static Rayleigh fading channels.

The path gains  $h_{i,s}^{(j)}$ , associated with the channels from node  $s$  (source node) to the  $j$ -th antenna of the  $i$ -th relay

Bruno Fontana da Silva and Renato Machado are with the Signal Processing and Communications Research Group, Department of Electronics and Computing, Federal University of Santa Maria, Santa Maria, RS, 97105-900 – Brazil. Andrei P. Legg is with the Space Science Laboratory of Santa Maria, Federal University of Santa Maria, Santa Maria, RS, 97105-900 – Brazil. Email: fontanads@gmail.com, renatomachado@ufsm.br, andrei.legg@gmail.com

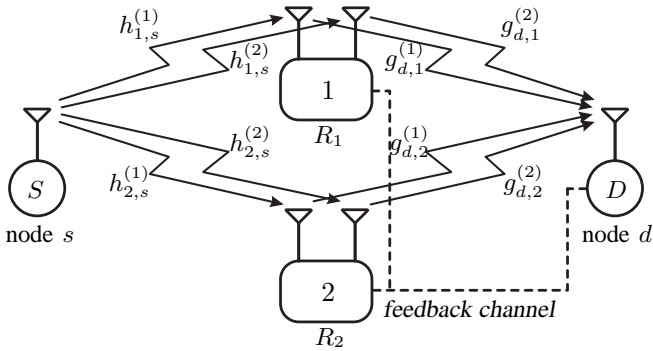


Fig. 1. Schematic diagram of the system model.

station, and  $g_{d,i}^{(j)}$ , associated with the channels from the  $j$ -th antenna of the  $i$ -th relay station to node  $d$ , where  $i, j = 1, 2$ , are modeled as independent zero mean circularly symmetric complex Gaussian random variables with variance  $\sigma = 0.5$  per dimension.

The total average signal energy per time slot is  $E_s$ . The transmitted signals are also corrupted by an additive white Gaussian noise (AWGN), with zero mean and variance  $\frac{N_0}{2}$ , at the receivers.  $\eta_i^{(j)}$  and  $\eta_d$  denotes de AWGN terms at  $j$ -th antenna of the  $i$ -th relay station and the destination node, respectively.

### III. COOPERATIVE TRANSMISSION SCHEME WITH TWO RELAYS AND PHASE FEEDBACK CHANNEL

Assume that the channel fading coefficients are known at the receiver, and that an error- and delay-free feedback channel is available by in which  $b$  bits can be broadcasted from the destination node to the relay stations.

In the proposed scheme, node  $s$  sends an information symbol,  $s_1$ , to the relay stations at the first time slot. The relays perform the detection. At the second time slot, based on the instantaneous channel state information (CSI), node  $d$  determines which antennas (two antennas), and which phase  $\theta$  from a set of  $2^b$  phases yields the maximum instantaneous SNR. It then sends  $b + N$  bits through the feedback channel indicating the best phase/antenna combination to be used by the relays to code the information detected on the previous time slot. After this preprocessing, the relays forward the coded symbols to the destination node.

#### A. Transmission Scheme

Let  $s_1$  be the transmitted data symbol, drawn from a signal constellation with unit average energy (real or complex), at the first time slot. As mentioned earlier, in this contribution, we assume perfect CSI at the receivers. Thus, the output baseband received signals processed by a maximum ratio combining technique at the relay stations  $R_1$  and  $R_2$  are respectively given by

$$\tilde{s}_{1,1} = s_1 \left( |h_{1,s}^{(1)}|^2 + |h_{1,s}^{(2)}|^2 \right) + \eta_1 \quad (1a)$$

$$\tilde{s}_{1,2} = s_1 \left( |h_{2,s}^{(1)}|^2 + |h_{2,s}^{(2)}|^2 \right) + \eta_2 \quad (1b)$$

where  $\eta_i = \sum_{j=1}^2 h_{i,s}^{(j)*} \eta_i^{(j)}$ , and  $\eta_i^{(j)}$  is the AWGN noise at  $j$ -th antenna of the  $i$ -th relay station, and  $(\cdot)^*$  denotes the complex conjugate operation.

Equations (1a) and (1b) produce the desired inputs to the maximum likelihood decoder. Since  $h_{i,s}^{(j)}$  is Gaussian,  $\tilde{s}_{1,i}$  is a chi-squared random variable with  $2M_R$  degrees of freedom. A well-known result is that the diversity order, i.e., the slope (in a log-log scale) of the average error probability curve of a digital modulation over a channel with SNR modeled as a chi-squared random variable with  $2d$  degrees of freedom is equal to  $d$  [7]. It is clear that each one of the relay stations offers a diversity order of  $M_R = 2$ .

Proceeding to the time slot two, both the relay stations code the detected data symbol based on the instantaneous CSI that come from the destination node. For the proposed scheme, we assume that only two antennas are allowed to transmit during the second time slot. The channels are assumed to be uncorrelated. The selected antennas transmit the coded symbols. In this propose, there are two different preprocessing functions multiplying the data symbol, one per relay station.

Thus, the received signal at the destination node  $d$  is given by

$$y_d = s_1 \left( f_1(\theta)g_{d,1}^{(b)} + f_2(\theta)g_{d,2}^{(b)} \right) + \eta_d \quad (2)$$

where  $g_{d,1}^{(b)}$  and  $g_{d,2}^{(b)}$  are the best fading gains (in order to provide the highest signal-to-noise ratio at the destination node) in a particular frame,  $f_1(\theta)$  and  $f_2(\theta)$  are the preprocessing functions of  $\theta$ , and  $\eta_d$  represents a zero-mean complex Gaussian noise with variance 0.5 per dimension.

Assuming that perfect channel state information is available at the receiver, maximum likelihood decoding can be obtained by computing the decision metric

$$\hat{s}_1 = y_d G^* \\ \hat{s}_1 = s_1 |G|^2 + \eta'_d$$

where

$$G = \left( f_1(\theta)g_{d,1}^{(b)} + f_2(\theta)g_{d,2}^{(b)} \right) \\ \eta'_d = \eta_d (f_1(\theta)g_{d,1}^{(b)} + f_2(\theta)g_{d,2}^{(b)})^* \\ |G|^2 = |g_{d,1}^{(b)}|^2 f_1^2(\theta) + |g_{d,2}^{(b)}|^2 f_2^2(\theta) + \\ 2f_1(\theta)f_2(\theta)\Re\{g_{d,1}^{(b)*}g_{d,2}^{(b)}\}$$

By choosing similar preprocessing functions as shown in [4], we define  $f_1(\theta) = \sin(\theta)$  and  $f_2(\theta) = \cos(\theta)$ . Due to the orthogonal relation between this functions, this choice also assists the weighed power allocation of the transmit antennas. The resulting equation for the detected signal at the destination node is

$$\hat{s}_1 = s_1 \left( |g_{d,1}^{(b)}|^2 \sin^2(\theta) + |g_{d,2}^{(b)}|^2 \cos^2(\theta) + \sin(2\theta)\Re\{g_{d,1}^{(b)*}g_{d,2}^{(b)}\} \right) + \eta'_D \quad (5)$$

The SNR of the received data symbol in a particular frame is

$$\gamma = \frac{|G|^2}{2} \gamma_0 \quad (6)$$

where  $\gamma_0 = E_s/N_0$ . Since the antennas are selected from a set of four antennas, and this selection is made in order to provide

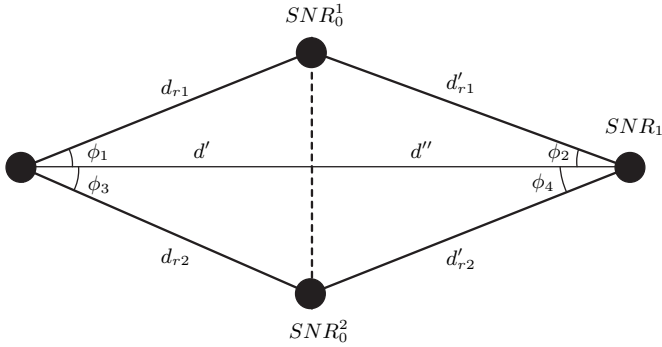


Fig. 2. Spatial configuration of the proposed scheme.

the maximum instantaneous SNR, the proposed scheme offers a diversity degree equal to four [8].

The phase  $\theta$  yields the possibility of using the feedback channel to maximize the overall SNR at the destination node. Using similar derivations as presented in [4], we define:

$$\alpha = |g_{d,1}^{(b)}|^2 - |g_{d,2}^{(b)}|^2 \quad (7a)$$

$$\beta = \Re\{g_{d,1}^{(b)*} g_{d,2}^{(b)}\} \quad (7b)$$

$$\dot{f}(\theta) = -\alpha \sin(2\theta) + 2\beta \cos(2\theta) \quad (7c)$$

$$\ddot{f}(\theta) = -2\alpha \cos(2\theta) - 4\beta \sin(2\theta) \quad (7d)$$

what leads us to the following equation for the  $\theta$  that ensures full diversity gain and maximum array gain:

$$\theta_{max} = \tan^{-1} \left( \frac{-\alpha + \sqrt{\alpha^2 + 4\beta^2}}{2\beta} \right) \quad (8)$$

Equation (8) assumes  $\ddot{f}(\theta) < 0$ .

### B. Quantized Feedback Channel

This section shows how to properly use the quantized feedback channel for the proposed scheme. Uniform phase quantization is assumed, as presented in Table I, where  $\theta \in [-\pi/2 \ \pi/2]$ . Table I also states the criteria for choosing a quantized value of  $\theta$  within a limited set given by the number of feedback bits.

 TABLE I  
 QUANTIZATION OF  $\theta$ .

	$\beta > 0$				$\beta < 0$			
	$\alpha < 0$		$\alpha > 0$		$\alpha > 0$		$\alpha < 0$	
1 bit	$\pi/4$				$-\pi/4$			
2 bits	$2\pi/6$		$\pi/6$		$-2\pi/6$		$-\pi/6$	
3 bits	$7\pi/12$	$5\pi/12$	$3\pi/12$	$\pi/12$	$-7\pi/12$	$-5\pi/12$	$-3\pi/12$	$-\pi/12$

### C. Spatial Allocation of the Relays

Into this section we set up a simple solution for estimating the spatial allocation of the relay stations. Figure 2 illustrates a spatial configuration of the proposed scheme.

A simplified log distance large scale path loss model is adapted from [9], from the equation

$$P_L(\text{dB}) = 10n \log_{10} \left( \frac{d}{d_0} \right) - k(\text{dB}) \quad (9)$$

where  $n$  is the environment path loss exponent,  $d_0$  is the close-in distance, and  $d$  is the line-of-sight distance between the source and destination. The  $k$  term is a constant which depends on the channel average path loss and antenna gain. For the sake of simplicity, we assume here  $k = 0\text{dB}$  and the reference distance  $d_0 = 1$  m. Thus, Equation (9) can be written as

$$P_L(\text{dB}) = 10n \log_{10}(d) \quad (10)$$

Aiming a simple analysis and design method, the following assumptions are made

- $d_{r,1} = d_{r,2} = d_r$
- $\phi_1 = \phi_3$  and  $\phi_2 = \phi_4$
- $SNR_0^1 = SNR_0^2 = SNR_0$
- the environment losses exponent is  $n$  for all the signal paths of the system.

Thus, according to Equation (10) and Figure 2, it is possible to derive the following equations

$$d_T = d' + d'' \quad (11a)$$

$$d_r = \frac{d'}{\cos(\phi_1)} \quad (11b)$$

Proceeding then to the large scale path loss equations

$$P_L(d_r)_{\text{dB}} = 10n \log_{10}(d_r) \quad (12a)$$

$$P_L(d_r)_{\text{dB}} = 10n \log_{10} \left( \frac{d'}{\cos(\phi_1)} \right) \quad (12b)$$

$$P_L(d'_r)_{\text{dB}} = 10n \log_{10} \left( \frac{d''}{\cos(\phi_2)} \right) \quad (12c)$$

$$SNR_0 = \frac{SNR_1}{2} + P_L(d'_r) - P_L(d_r) \quad (12d)$$

Equations (12) show the  $SNR_0$  dependence on the overall  $SNR_1$ , which means that a higher  $SNR_0$  must be provided for the relays during time slot one when compared to  $SNR_1$  at node  $d$  during time slot two, so that the overall system error performance is not harmed. Carrying on with this assumption, the following definitions are taken

$$SNR_0 = \frac{SNR_1}{2} + G_R(\text{dB}) \quad (13a)$$

$$SNR_1 = 2(SNR_0 - G_R(\text{dB})) \quad (13b)$$

$$G_R(\text{dB}) = P_L(d'_r) - P_L(d_r) \quad (13c)$$

Deriving Equation (13c), it is possible to attain some equations that allow the spatial allocation design of the relay stations based on the line of sight distance between transmitter and receiver. The design also depends on the parameters,  $\phi_1$  and  $\phi_2$ , and the SNR gain ( $G_R$ ) desired<sup>1</sup> for the relay stations.

<sup>1</sup>The desired gain ( $G_R$ ) at the relays is the one required to ensure that full spatial diversity order will be obtained at the destination node.

The equations are given by

$$G_R(dB) = 10n \log_{10} \left( \frac{\sin(\phi_1)}{\sin(\phi_2)} \right) \quad (14a)$$

$$\phi_1 = \arccos \left( \sqrt{\frac{d' \left( 10^{\frac{G_R(dB)}{5n}} - 1 \right)}{d_T \left( \frac{d_T}{d'} - 2 \right)}} \right) \quad (14b)$$

$$d' = d_T \cos(\phi_1) \left( -\mu \pm \sqrt{\mu^2 + \frac{\mu}{\cos(\phi_1)}} \right) \quad (14c)$$

where  $\mu = \frac{\cos(\phi_1)}{\left( 10^{\frac{G_R}{5n}} - 1 \right)}$  and  $\phi_2(d', G_R)$  is easily calculated

by simple geometry after the choice of  $\phi_1$  and  $G_R$  in Equation (14a). As we can observe, the choice of two parameters defines the third one.

The last two equations are constrained to real solutions due to the boundary conditions of the geometric spatial model presented earlier. The constraint functions are given by:

$$\sqrt{\frac{d' \left( 10^{\frac{G_R(dB)}{5n}} - 1 \right)}{d_T \left( \frac{d_T}{d'} - 2 \right)}} \leq 1 \quad (15a)$$

$$\phi_1 \leq \arcsin \left( 10^{\frac{G_R}{10n}} \right) \quad (15b)$$

One can see in Figure 3 the  $G_R$  sensitivity in regard to the parameters  $d'$  and  $\phi_1$ . Figure 3 also shows that  $G_R > 0$  only when  $d' < 0.5d_T$ .

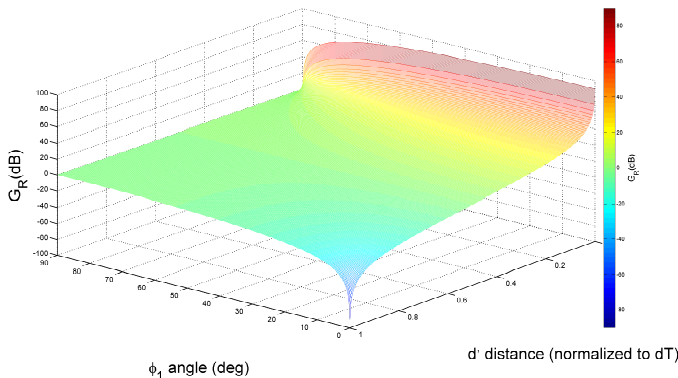


Fig. 3. Relation between spatial design parameters from Equation (14a).

#### IV. SIMULATION RESULTS

In this section, we present the simulation results for the afore-described proposed scheme. The results are obtained through computer simulation and are expressed in terms of bit error rate (BER) versus SNR ( $\gamma_0$ ). All the simulations consider a 16-QAM constellation, two antennas available per relay station and only one antenna at the source and destination nodes. The BER estimation is obtained with a minimum of 500 error bits. The channels are assumed to undergo flat and quasi-static Rayleigh fading, in other words, the path

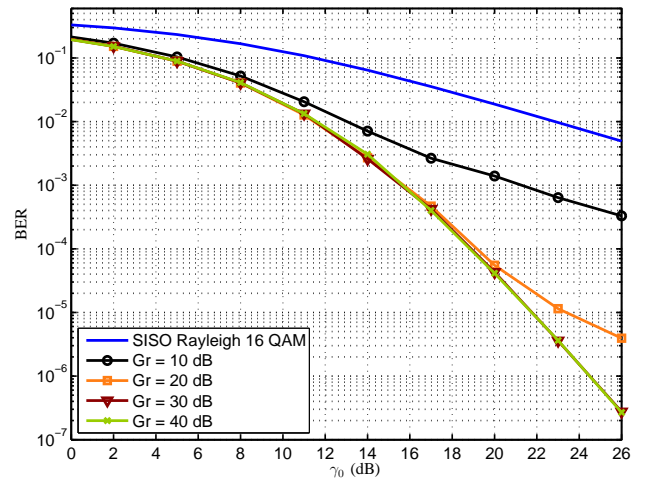


Fig. 4. Performance of the proposed scheme for different distances between the relays and source node.

gains are assumed to be independent samples of a zero-mean complex white Gaussian random process with variance  $1/2$  per dimension, and it is constant for one frame and changes independently from one frame to another. For simplicity, we normalized the noise power  $N_0 = 1$ , so that we have the received power identical to the SNR. In our simulations, we assume that  $d_T = 1$  and the number of feedback bits ( $b_f$ ) is given by  $b_f = b + N$ , where  $b = 0$  (no preprocessing),  $1, 2, 3, \dots, \infty$  ( $\theta_{max}$ ) and  $N = 1, 2, 3$ . Therefore,  $b_f$  bits are used to inform the relays which is the best possible combination transmit antenna/theta to be used for coding and transmitting the data symbol at the second time slot. For purpose of comparison, we also included in the figures the error curves for the no-diversity case (i.e., one antenna in each side of the link).

Figure 4 shows the error performance of the proposed scheme for different  $SNR_0$  conditions and un-quantized feedback channels. In other words, we are verifying which is the lower bound for the proposed scheme if the  $G_R$  gain requirement is not obeyed. We consider four different scenarios. For  $G_R = 10, 20,$  and  $30$  dB the proposed scheme has performance loss (the lower is  $G_R$ , the higher are the losses). For  $G_R = 40$  dB we observe that the BER curve keep the full-diversity slope. Regarding these results, it is clearly noticeable that the relay spatial allocation has primordial importance on the overall performance of the proposed scheme.

From simulation tests, we observe that  $G_R = 30$  dB is good enough to ensure full-diversity gain at the destination node for a BER of about  $10^{-6}$ . Therefore, henceforth, we assume that  $G_R = 30$  dB in order to evaluate the proposed scheme for different levels of phase-feedback quantization. Thus, considering (14a)–(14c) and adopting an arbitrary distance, e.g.,  $d' = 0.06d_T$ , we obtain  $\phi_1 = 20.51$  degrees. This system configuration is adopted in Figures 5 and 6.

In Figure 5 we evaluate the error performance of the proposed scheme for different levels of theta quantization. We

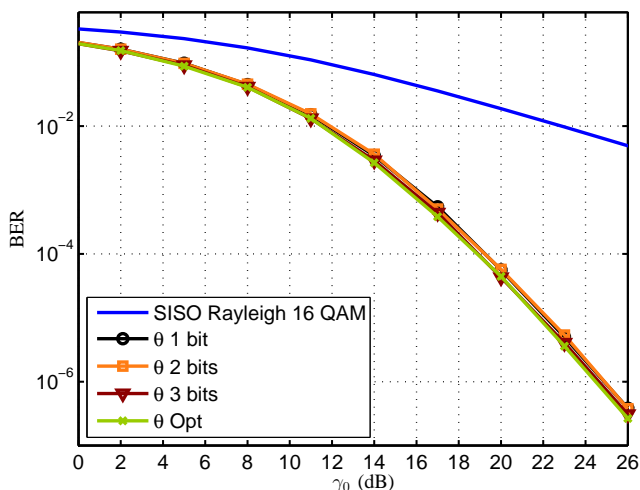


Fig. 5. Performance of the proposed scheme for different levels of theta quantization.

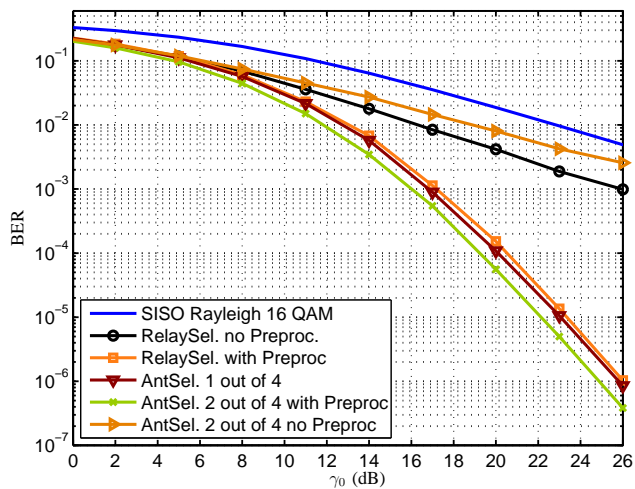


Fig. 6. Evaluation of the different uses of the feedback channel.

can clearly see that with only one feedback bit for theta it is possible to attain a good error performance (very close to the best one) which makes the proposed scheme very attractive. In this figure, the best pair of antennas is chosen from a set of four antennas (three feedback bits is required). “Best”, in this case, means the pair of antennas that maximizes the instantaneous SNR at the destination node. Hence, the proposed scheme needs only four feedback bits to reach a near-optimum error performance.

Figure 6 shows the BER  $\times$  SNR curves for different uses of the feedback channel. For example, if the destination node fed four bits ( $b_f = 4$ ) back to the relays, it is possible to select the best pair of transmit antennas and consider  $\theta \in \{\pi/4, -\pi/4\}$ . Thus, the proposed scheme achieves a near-optimum error performance, as reported in Figure 5. However, if only one feedback bits ( $b_f = 1$ ) is used, then

the best relay is selected, but no smart preprocessing and no antenna selection are possible. As a consequence, we have almost all-performance loss. In the same way, if three feedback bits ( $b_f = 3$ ) is used, then the best pair of antennas is selected, but no smart preprocessing is possible. As a result, we have loss of performance again. On the other hand, if two bits ( $b_f = 2$ ) are used to select the best antenna [8], the error performance obtained is quite good, but it is still 0.9dB lower than the proposed one. Nevertheless, if two feedback bits ( $b_f = 2$ ) are used, then the best relay is selected and a smart preprocessing is possible. As a consequence, a good error performance is achieved, but a little bit lower than the one that select the best transmit antenna with no preprocessing [8]. Therefore, if we use the feedback channel adequately, the proposed scheme becomes a very attractive solution extracting all possible benefits that this scheme can offer.

### V. CONCLUSIONS AND FINAL REMARKS

In this paper, we proposed a new transmission scheme with two relay stations and a quantized feedback channel between the relays and the destination node. Using a few feedback bits, the proposed scheme can obtain a full diversity and also an array gain at the destination node. A phase quantization for the proposed scheme is presented and an analysis of the relay spatial allocation design is also performed. We shown that for the proposed relay scheme presents a good error performance, a higher  $SNR_0$  must be provided for the relays during time slot one when compared to  $SNR_1$  at node  $d$ . Thereby, a suitable spatial allocation of the relay stations is necessary for the proposed scheme to work properly.

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