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On the Impact of Non-Linear High Power Amplifiers on Coded OFDM

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***Abstract*— The high peak-to-average power ratio (PAPR) is one of the main disadvantages of OFDM systems, which reduces the transmission efficiency with non-linear power amplifiers and causes unwanted distortions and out-of-band radiation. There are many schemes proposed in the literature to mitigate this problem. However, the majority of these contributions only analyse the PAPR reduction and do not consider the system BER performance in the presence of an amplifier. When BER analysis for non-linear amplifiers is found in the literature, it is usually done not considering channel coding, even though OFDM in most real systems is coded. In this contribution we investigate the performance of an OFDM system in terms of total degradation, also considering channel coding, and show that with strong coding the effect of a non-linear amplifier is far less negative than in an uncoded system. We also see that some schemes that effectively reduce PAPR, may in fact bring about very little performance gain when coding is taken into account.**

Keywords*— *OFDM, PAPR, non-linear power amplifier, total degradation, channel coding.

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is being used in many wireless communication standards, both currently deployed and under deployment. We can cite, for example: IEEE 802.11a [1] (WiFi), IEEE 802.16 [2] (WiMAX), 3GPP-LTE [3], among others. The main idea behind OFDM is to split the input data in many subchannels and transmit each one of them in a different subcarrier with a low transmission rate, with subcarriers chosen such that they are mutually orthogonal. Each subchannel experiments a narrow band channel making the receiver structure less complex, as equalisation can be made by a single tap. As a consequence, OFDM systems are considered robust against multipath propagation and permit the use of modulation techniques with high spectral efficiency.

In spite of its many advantages, the waveform of OFDM signals presents a wide amplitude dynamic range. The amplitude, and, consequently, power variation, is usually quantified by the peak-to-average power ratio (PAPR) [4]. The high PAPR of OFDM signals is one of its main disadvantages. Because of this, OFDM systems suffer from non-linear effects common to high-power amplifiers. To transmit an OFDM signal with little distortion using an amplifier, it is necessary to have a high power backoff, resulting in a low power efficiency. On the other hand, if a small backoff is selected for high power efficiency, the signal is clipped due to amplifier saturation level, and the signal suffers from out-of-band radiation and constellation distortions, what degrades the bit error probability [5]–[7].

There are several techniques used in the literature aiming at decreasing the PAPR value of an OFDM signal, making it possible to transmit with smaller backoff values and higher power efficiency with tolerable signal degradation. However, many of these methods require extra signalling, such as the partial transmit sequence [8]–[10] and selective mapping [11], [12] methods; or receiver modifications to identify the OFDM original signal [13]. One of the most promising PAPR reduction schemes is the active constellation extension (ACE) [15], [19], which can be employed at most existing systems without any major modifications to the standards. ACE can bring about a substantial reduction to the PAPR, but at the cost of a higher complexity at the transmitter.

However, the ultimate goal of any transmission scheme is not to reduce the PAPR, but rather to improve the system performance in terms of BER in the presence of a non-linear transmitter, preferably with a low power backoff. Because of this, our analysis also considers a figure of merit called total degradation, which is also used for instance in [14]. Most papers only consider the efficiency of their proposed methods in PAPR reduction, but this is clearly not enough, because a given method can reduce the PAPR, but not improve or even worsen the BER performance, as it is the case of a deliberate clipping of the signal.

In [18] it was argued that the high PAPR of OFDM is an issue, but maybe not as serious as previously considered, when one takes into account transmission in a frequency-selective channel. Another aspect of a transmission system that most publications do not consider when a PAPR reduction method is introduced and analysed is channel coding, even though nearly all existing technologies employ some form of coding. This paper will show that the performance loss with non-linear amplifiers can be quite different when coding is used, when compared with an uncoded system. In particular, we will show that the total degradation in coded OFDM systems is much lower than what is generally obtained in the literature with uncoded OFDM. Hence, not only the presence of frequency selectivity, but also the use of channel coding may reduce substantially the problems caused by a high PAPR.

Furthermore, we will also see that PAPR reduction techniques may not significantly improve performance in a coded system, even though the peak amplitudes are substantially reduced. As an example, we test the ACE scheme [15], which is one of the most investigated methods in the literature, and see that in view of its high complexity and diminishing gains as we increase coding efficiency, its use may not be justifiable.

The paper is organized as follows. In Section II the OFDM system model, the amplifier model and the PAPR definition are presented. In Section III, the active constellation extension method is briefly described. Simulation results are shown and the argument for the different behaviour of a coded system in the presence of a non linear amplifier is introduced in Section IV. Finally, some concluding remarks about the paper are made in Section V.

II. System Model

In an OFDM transmission, the input bits are divided into K subchannels corresponding to the number of available orthogonal subcarriers and to the word size designated by the modulation technique used in each subcarrier. Therefore, an OFDM symbol in time domain is the sum of K independent symbols mapped in K subcarriers. It can be represented as

$$s(t) = \sum_{n=-\infty}^{\infty} \sum_{k=0}^{K-1} x_{k,n} e^{j2\pi f_k t} g(t-nT), \qquad (1)$$

where $x_{k,n}$ is the symbol mapped at the n-th OFDM symbol at the k-th subcarrier with frequency f_k, with $f_k = k/T_S$. T_S is the useful OFDM symbol interval, and $T = T_S + T_G$ is the total OFDM symbol interval including the guard interval T_G, which is filled up by a cyclic prefix. $g(t)$ is the shaping pulse, which is usually a rectangular pulse with width T.

As it is well known, a critically sampled version of this signal can be easily implemented by the inverse fast Fourier transform (IFFT).

Then, the signal in (1) is submitted to a power amplifier. We can represent the input base band signal in an amplifier using a polar notation:

$$s(t) = A(t)e^{j\Psi(t)}, \qquad (2)$$

where $A(t)$ is the signal amplitude and $\Psi(t)$ its phase.

The output base band signal can be represented by

$$y(t) = G[A(t)]e^{j\{\Psi(t)+\Phi[A(t)]\}}, \qquad (3)$$

where $G[A(t)]$ and $\Phi[A(t)]$ represent the AM/AM and AM/PM conversion, respectively, caused by the non-linear amplifier.

The amplifier model used in this work is the Rapp model [16] for a solid state amplifier, in which AM/PM conversion can be neglected. The AM/AM conversion of the Rapp model is described by

$$G(A) = \frac{vA}{\left[1+\left(\frac{vA}{A_0}\right)^{2p}\right]^{1/2p}}, \qquad (4)$$

where A_0 is the maximum amplitude, v is the small signal gain and p is the smoothness factor. Both the maximum amplitude and the small signal gain can be normalized to a unit without loss of generality.

Figure 1 shows the amplitude transfer function of the above cited amplifier, with $p = 2$ and $p = 3$. It can be seen that the higher the p value the smoother the transition to saturation. In this work, $p = 3$ is used in every situation.

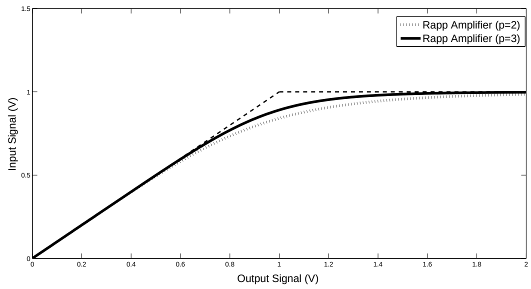

Fig. 1. Transfer Functions of the amplifiers.

Besides, in any analysis that involves amplifiers the definition of input backoff (IBO) and output backoff (OBO) is indispensable:

$$IBO_{dB} = 10log_{10}\left(\frac{P_{sat,in}}{P_{in}}\right) \qquad (5)$$

$$OBO_{dB} = 10log_{10}\left(\frac{P_{sat,out}}{P_{out}}\right), \qquad (6)$$

where P_{in} and P_{out} are the input and the output average power, respectively; and $P_{sat,in}$ and $P_{sat,out}$ are the input and the output saturation power, respectively, obtained from the AM/AM conversion function.

It is important to point out that the closer to saturation region, the greater the efficiency of the power amplifier is. Analysing Figure 1 again, we see that if the signal amplitude is constant, it is possible to work closer to saturation region with high power efficiency.

However, as we see in (1), an OFDM signal corresponds to the sum of many modulated subcarriers, which, depending on the transmitted symbols, may add up constructively or destructively. Hence, an OFDM signal does not have constant amplitude. On the contrary, a large amplitude variation can be observed. In other words, OFDM signals usually have high values of peak-to-average power ratio (PAPR). Because of this, we either choose an operation point well below the saturation power, such that even the highest peaks are amplified linearly, but with low power efficiency; or work close to saturation knowing that part of the signal with high amplitude will be distorted and clipped. Besides causing constellation errors, it is well known that non linear distortions cause out of band radiation.

The PAPR of an analog signal in an OFDM symbol with duration T_S is usually measured for each symbol, and is defined as

$$PAPR(s_n(t)) = \frac{\max\limits_{nT<t\leq(n+1)T} |s(t)|^2}{\frac{1}{T}\int_{nT}^{(n+1)T} |s(t)|^2 \, dt} \quad (7)$$

and is a metric commonly used to gauge the amplitude variability of a signal. Furthermore, if we want to work efficiently with low distortion using non-linear amplifiers it is desirable to reduce the PAPR as much as we can, while maintaining the positive features of an OFDM signal.

III. The Active Constellation Extension Method

One of the most promising PAPR reduction techniques is the active constellation extension method (ACE) [15], which consists in modifying the signal constellation without increasing its error rate. The effect of this modification is to add sinusoids in some frequencies that can lead to the cancellation of some signal peaks.

Figure 2 shows the constellation of a 16-QAM modulated signal after being modified by the method, where it is possible to see that the optimum decision regions are not abandoned. It is clear that the method involves increasing the transmit signal power at some modulation symbols. However, this power increase is not meaningful when compared to the gains provided by the method [15], [19].

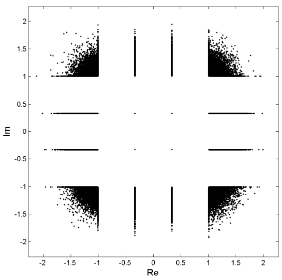

Fig. 2. 16 QAM constellation with the ACE method.

The algorithm must modify only the data subcarriers, letting the pilot and guard subcarriers unchanged. It can be described by the following steps, which are executed for each OFDM symbol:

1) According to input data, designate the K constellation points X_k ;
2) Via IFFT, construct the sampled time domain signal x_l;
3) Compare the magnitude of all samples with $L_{\max}$, the maximum allowed amplitude;
4) For samples that exceeded $L_{\max}$, resize them, i.e, make $\tilde{x}_l = L_{\max}e^{j\theta_n}$, where $x_l = |x_l|e^{j\theta_l}$
5) Regain the frequency domain signal via FFT
6) Restore the pilot and zero subcarriers,as well as the interior constellation points to the original values.
7) Return to the second step until no time sample is resized or the maximum number of iterations is reached

The algorithm complexity depends on the number of iterations, basically requiring one further IFFT for each iteration, but only a few iterations are usually enough to reach a satisfactory result [19]. A particularly positive feature is that its application does not require extra signalling or any change in the receiver. This makes it acceptable for existing wireless technologies that employ OFDM. The only adjustable parameters are the maximum number of iterations and the value of L_{max}, whose impacts are analysed in [19].

IV. Results

In this section, we show some simulation results. We consider an OFDM system with 512 subcarriers, of which 420 transmit data. The DC and the extreme subcarriers are null to maintain the spectral conformation. The subcarriers are separated by 15 Khz and the cyclic prefix corresponds to 1/16 of the useful symbol interval.

Figure 3 shows the PAPR cumulative distribution function (CDF) of an OFDM signal, where it is possible to see that the probability of high peak values in the signal decreases substantially when the ACE method is used. Also, we note that the decrease in PAPR is larger, the lower the value of the amplitude limit $L_{\max}$ is.

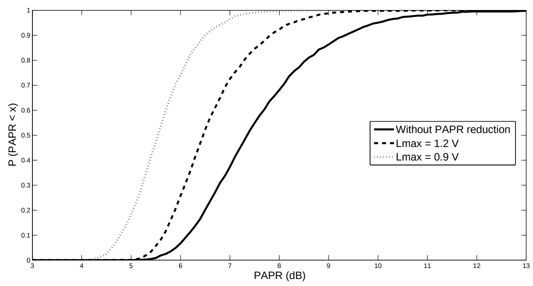

Fig. 3. PAPR CDF of an OFDM signal with and without ACE, varying the value of $L_{\max}$.

However, as mentioned before, reducing the PAPR is not necessarily enough. A good system performance also needs to be guaranteed. Therefore, when a non-linear amplifier is used, the analysis of an OFDM system needs to be done considering some metrics such as the bit error rate (BER), the backoff, the out of band radiation, among others.

When an OFDM signal is transmitted, it is very usual to have a power backoff to reduce the non-linear effects. This procedure must be included in the analysis of an amplified system, because the backoff represents an available power that is being wasted. There is no sense in analysing backoff only, we must also consider the BER performance loss. An intelligent analysis is to determine the Total Degradation (TD) of a system, which is defined by

$$TD_{dB} = OBO_{dB} + \left[EbN0_{(OBO)} - EbN0_{(linear)}\right]_{BER}, \quad (8)$$

where $EbN0_{(OBO)}$ and $EbN0_{(linear)}$ are, respectively, the required bit-energy to noise-spectral-density ratios E_b/N_0

in dB for the non-linear amplifier and for an ideal linear amplifier for a given output backoff OBO_{dB}. In other words, the first term indicates the decrease in power transmission in amplifier, while the second term represents the degradation due to the non-linear distortion caused by the amplifier. This figure of merit is measured for a specific channel and for a specific BER. In the analysis done in this work we considered an AWGN channel and $BER = 10^{-3}$.

An analysis of a total degradation curve leads us to reach an optimum backoff value, which is the value for which we have the least degradation of the system.

In Figure 4, we can verify, in terms of TD, the application of the ACE PAPR reduction method for an uncoded system. It illustrates the situation in which there is no PAPR reduction and the situation in which the method is applied using QPSK, 16-QAM and 64-QAM. In this situation, ACE not only reduces the probability of high signal peaks, as shown in Figure 3, but also lowers the optimum OBO value and the TD of the system.

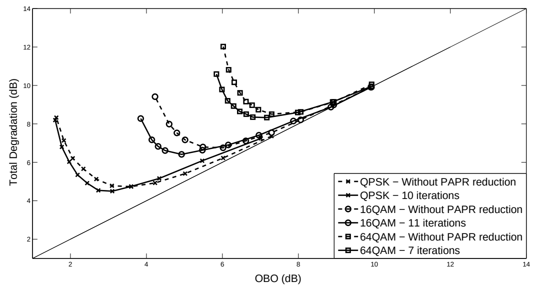

Fig. 4. TD using ACE for QPSK, 16QAM and 64QAM.

However, the situation is very different when the signal is coded. In Figure 5 it is possible to compare the TD performance of a coded system with and without ACE, using a convolutional code with different rates and QPSK modulation. It can be seen that with lower code rates, the performance gain when the ACE method is used gets smaller. In other words, the higher the code redundancy, the smaller the effect of the method is. The method was simulated using 11 as the maximum number of iterations and $L_{\max}$ = 0,9. The code generators and the puncturing pattern for the convolutional codes were extracted from [17].

Another interesting fact that can be noticed assessing Figure 5 is that the optimum values of OBO and TD lower significantly when coding is used and its rate decreases.

Considering the fact that coding lowers the optimum OBO and optimum TD, a system with different code rates and two kind of codifications was simulated, in order to analyse the impact of coding when non-linear amplification is done, as we can see in Figure 6. Besides the convolutional codes, a turbo code with rate $R = 1/3$ was also considered. Comparing the curves we see that without coding the total degradation tends to infinity as the backoff approaches zero, and that in the best situation a total degradation of nearly 5 dB can be achieved. With turbo coding, the situation is somewhat different, even with zero backoff, we still reach a relatively low total degradation, of about 2 dB.

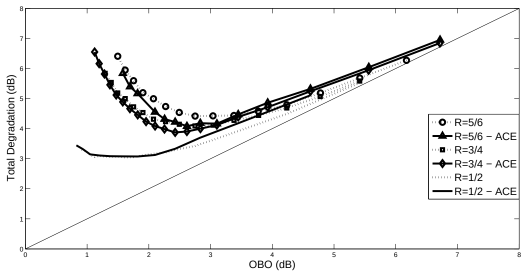

Fig. 5. TD using convolutional coding and ACE method.

By looking at Figures 5 and 6 one can raise the question of why the high PAPR of OFDM signals becomes less of a problem when coding is considered. It can be easily seen that coding per se does not reduce the PAPR, so that the justification must be looked elsewhere.

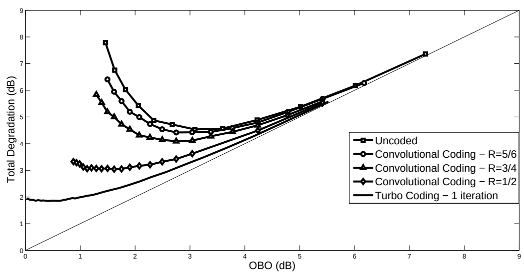

Fig. 6. TD using coding with different coding rates.

Considering an amplified system in an AWGN channel, we can model the received signal as

$$y(t) = x(t) + w(t) + n_{\text{amp}}(t), \tag{9}$$

where $w(t)$ is the Gaussian white noise component with power N_W and n_{amp} a non-Gaussian noise component representing the non-linear amplifier distortion with power N_{amp}. Consequently, the signal-to-noise ratio at the receiver is

$$SNR = \frac{S}{N_W + N_{\text{amp}}}, \tag{10}$$

where S is the signal power.

Figure 7 shows BER plots for an OFDM system in AWGN channel with convolutional codes at different rates. It is possible to see that in a coded system, as we decrease the coding rate it becomes possible to achieve low BER with even lower SNR values. Considering a fixed signal power, at the operation point of coded systems the noise power N_W can be relatively high. As already said, when the signal is amplified, besides the channel noise, the signal receives the addition of an amplification noise N_{amp}. However, this noise becomes less significant as the SNR decreases.

We can also notice that the power spectral density of amplification noise is nearly constant along the signal bandwidth, in other words, it looks like a white noise. This behaviour can be seen in Figure 8, where the PSDs of three signals are plotted: an OFDM signal, an amplified OFDM signal and non linear noise.

In [18], it is said that most papers overestimate the effect of non-linear distortion because they do not take into account the shrinking of the constellation or the fact that non linear distortion is added in transmitter, and not in the receiver. Here we see that if channel coding is not considered, the non-linear effect can also be overestimated. Furthermore, we also show that PAPR reduction methods can be very ineffective in terms of total degradation performance in coded systems, and may introduce an unneeded complexity to the transmitters.

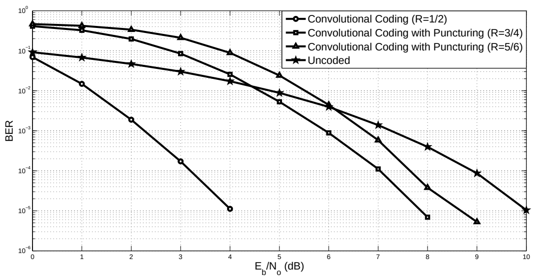

Fig. 7. BER curves for different convolutional code rates.

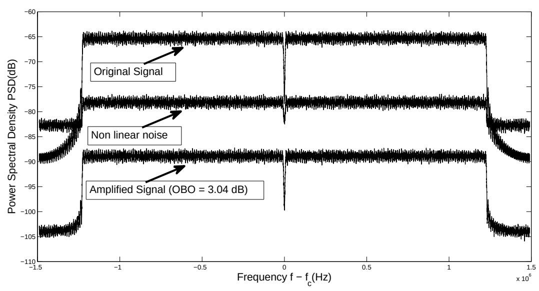

Fig. 8. PSD of an OFDM signal, an amplified signal and non linear noise.

In this paper we haven't directly considered the issue of the out-of-band radiation generated by the non-linear amplifiers. Even though the total degradation and the needed backoff can be significantly lower with channel coding, the out-of-band radiation tends to increase substantially as we reduce the backoff. Nevertheless, out-of-band radiation can be efficiently controlled by clipping and filtering the signal at the transmitter, and can be viewed as a less critical issue than the BER degradation.

V. Conclusions

The high PAPR of OFDM signals is commonly regarded as one of the major issues in their practical implementations. In our contribution we claim, based on simulation results, that this may not be such a severe problem when channel coding is considered, and that it may be not necessary to apply complex PAPR reduction schemes in coded OFDM systems, as their gains can be limited in this case. PAPR remains though a serious problem in signals without coding or with very high coding rates, and efforts to devise efficient schemes to mitigate this problem are still needed.

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