Capacity of CSMA under Nakagami Fading

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***Abstract*— This paper investigates the throughput performance of carrier sense multiple access (CSMA) in a packet radio network and Nakagami fading environment. The approach considered includes the signal capture model with incoherent addition of interfering signals. The cases of uniform attenuation for all terminals (or perfect power control) as well as unequal average power levels are studied. Analytical and numerical results are presented.**

***Keywords*— Wireless communication, CSMA, Nakagami fading.**

I. Introduction

Wireless local area networks (WLANs) are experiencing rapid development in part stimulated by the deployment of systems compatible to the IEEE 802.11 standards. They offer data communication between terminals within radio range while allowing a certain degree of mobility. In order to serve terminals exhibiting bursty traffic behaviour, WLANs make use of packet radio techniques with random access to a transmission channel shared by multiple users. Specifically, variations of carrier sense multiple access (CSMA) is generally used to access the wireless medium [1]–[4]. The capacity of the channel is then influenced by the probability of packet collision and by the signal degradation due to mutual interference and signal attenuation. In other words, it is influenced by the medium contention resolution algorithm and by the channel characteristics.

Intuitively one might expect that original (wireline) CSMA systems show better system performance than wireless systems because of more hostile channel characteristics found in the latter. However, this is not necessarily the case. For instance, in a channel model that takes into account the effects of fading, competing packets arriving at a common radio receiver antenna will not always destroy each other because they may show different and independent fading and attenuation levels [5], [6]. This leads to expect that wireless systems may actually exhibit successful reception rate higher than that of original (wireline) systems. In fact, Arnbak and Blitterswijk have shown this to happen with slotted Aloha over Rayleigh fading channels [7].

In this paper, we investigate the throughput performance of CSMA in a packet radio network with Nakagami fading environment. The performance of the original (wireline) CSMA is presented in [8] and is here extended to the wireless environment, for which the Nakagami-m fading conditions are assumed. A number of closed form as well as series expressions are found. To the best of the authors' knowledge, these results are novel contributions.

This paper is organized as follows. Section II describes the analytical model. Section III considers the case of incoherent packet addition at the receiver's antenna, and extends the model to include spatial coverage such as in a cell. Comments and conclusions are given in Sections IV and V, respectively.

II. Analytical Model

A. Nonpersistent CSMA

For nonpersisting CSMA, a terminal ready to transmit first senses the channel. If it senses the channel idle, it transmits the packet. Otherwise, it schedules the (re)transmission of the packet to a later time according to some randomly distributed retransmission delay. After the retransmission delay has elapsed, the terminal repeats the procedure described above. In this paper, the channel is considered to be memoryless, i.e., failures to capture the channel and future attempts are uncorrelated. In addition, all packets are assumed to have fixed length and to require p seconds to transmit. Finally, each packet is assumed to have a single destination.

B. Probability of Capture

Given the transmission of an arbitrary test packet over a wireline channel, it is generally assumed that a successful reception can only occur if no other transmission attempt is made during the test packet reception, i.e., if there is no signal overlap at the receiver. However, in wireless systems the radio receiver may be able to be captured by a test packet, even in the presence of n interfering packets, provided that the power ratio between the test signal and the joint interfering signal exceeds a certain threshold during a given portion of the transmission period t_w, $0 < t_w < p$, to lock the receiver [9], [10]. In such a case, a test packet is only destroyed if $w_s/w_n \leq z$ during t_w, with $n > 0$, where z is the capture ratio, and w_s and w_n are the test packet power and the joint interference power at the receiver's antenna, respectively. Values for z and the capture window t_w depend on the modulation and the coding employed by the network, among other things. For a typical narrowband FM receiver, a z value of 6 dB is suggested in [11]. The details about estimation of the values of z and t_w are beyond the scope of this paper.

Let the random variable Z be defined as the signal-to-interference ratio

$$Z \triangleq \frac{W_s}{W_n}, \quad Z \geq 0 \tag{1}$$

where $W_s \geq 0$ and $W_n \geq 0$ are random variables representing the desired signal power and the interference power, respectively. Assuming that W_s and W_n are statistically independent, the resulting probability density function (PDF) can be expressed as [12]

$$f_Z(z) = \int_0^{\infty} y f_{W_s}(zy) f_{W_n}(y) dy \tag{2}$$

where $f_{W_s}(.)$ and $f_{W_n}(.)$ are the PDFs of the desired signal power and the interference power, respectively. The cumulative distribution function (CDF) is then expressed as

$$F_Z(z_0) = \text{Prob}\left\{\frac{W_s}{W_n} \leq z_0\right\} = \int_0^{z_0} f_Z(z) dz. \tag{3}$$

Let the number of packets generated in the network for new messages plus retransmissions be Poisson distributed, with mean generation rate of λ packets per second. The mean offered traffic is then expressed as $G = p\lambda$ packets per transmission period. Given the transmission of an arbitrary test packet, the probability of it being overlap by n other packets is given by [12]

$$R_n = \frac{(Ga)^n}{n!} e^{-Ga} \tag{4}$$

where τ is the worst case propagation delay and $a = \tau/p$ its normalised version. Finally, the unconditional probability of a test packet being able to capture the receiver in an arbitrary transmission period may be expressed by

$$P_{capt}(z_0) = 1 - \sum_{n=1}^{\infty} R_n F_Z(z_0). \tag{5}$$

C. Channel Throughput

Let U, B and I be random variables representing, respectively, the duration of the successful transmission, the duration of the busy period and the duration of the idle period. Let E{U}, E{B} and E{I} be their respective expected values. Clearly, the channel throughput can be expressed by $S = E\{U\}/(E\{B\} + E\{I\})$. For nonpersistent CSMA, Kleinrock and Tobagi have shown that [8]

$$E\{B\} = p + 2\tau - \frac{p}{G}\left(1 - e^{-Ga}\right) \text{ and } E\{I\} = \frac{p}{G}. \tag{6}$$

It can also be seen that

$$E\{U\} = p P_{capt}(z_0) \tag{7}$$

where $P_{capt}(.)$ is the probability of receiver's capture and also it represents the probability of a successful transmission. Using the results of (6)-(7), the throughput can be written as

$$S = \frac{G P_{capt}(z_0)}{G(1+2a) + e^{-Ga}} \tag{8}$$

D. Nakagami Fading Channel

In a Nakagami fading channel, PDF of the signal envelope r is given by [13]

$$f_R(r) = \frac{2r^{2m-1}}{\Gamma(m)} \left(\frac{m}{\overline{w}}\right)^m \exp\left(-\frac{mr^2}{\overline{w}}\right) \tag{9}$$

where $\overline{w} = E\{r^2\}$ is the mean square value, m is a fading parameter, and $\Gamma(.)$ is the gamma function [14, Eq. 6.1.1]. For m=1 , the Nakagami distribution reduces to the Rayleigh PDF while $m \to \infty$ corresponds to a non-fading situation. If we define the signal power $w = r^2$, its PDF can be expressed as

$$f_W(w) = \frac{w^{m-1}}{\Gamma(m)} \left(\frac{m}{\overline{w}}\right)^m \exp\left(-\frac{mw}{\overline{w}}\right) \tag{10}$$

where $\overline{w} = E\{w\}$ is the mean power.

E. Interference Signal

In a wireless system, interference typically results from (supposedly uncorrelated) signals arriving at the receiver's antenna from multiple transmitters. Depending on how these random signals combine during the observation interval, one of two scenarios might occur [15]: coherent addition or incoherent addition.

Coherent addition occurs if the carrier frequencies are equal and if the random phase fluctuations are small during the capture time t_w. For instance, coherent addition might happen when the deviation caused by the phase modulation is very small, and the observation interval is short compared to the modulation rate. For the Nakagami channel, analysis of coherent addition of signals is rather intricate and it is a matter still under investigation by the authors.

Incoherent addition occurs if the phases of the individual signals fluctuate significantly due to mutually independent modulation [7], [16]. In this case, the interference power w_n experienced during the observation interval is the sum of the individual signals' powers w_i, i.e.,

$$w_n = \sum_{i=1}^{n} w_i = \sum_{i=1}^{n} \overline{x_i(t) x_i^*(t)} \tag{11}$$

where $x_i^*(.)$ is the complex conjugate of phasor $x_i(.)$. Considering the current work, where the signal power is a random variable, the PDF of the joint interference power is therefore the convolution of the PDFs of all contributing signal powers.

III. Analytical Results

For the calculations presented in this section, let (10) represent the desired signal power PDF as well as, with different parameters, the signal power PDF of an individual component of the interference signal. Note that due to the lack of space, some intermediate steps in the derivation of the formulae presented below may be omitted. Also, for the remaining of this paper and wherever applicable, the subscripts s, i and n are used to represent the desired signal variables, the interference signal's individual component variables, and the joint interference signal variables, respectively. Finally, let $\tilde{z}_0$ be defined as $\tilde{z}_0 \triangleq z_0/(\overline{w}_s/\overline{w}_n)$.

A. Incoherent Interference

For the incoherent addition, assume that the interference signal is composed of n i.i.d. variables. As a result, the joint interference signal power PDF is the n-fold convolution of the individual signal power PDF which, on its turn, is expressed by (10). This calculation gives that

$$f_{W_n}(w_n) = \frac{w_n^{m_n-1}}{\Gamma(m_n)} \left(\frac{m_n}{\overline{w}_n}\right)^{m_n} \exp\left(-\frac{m_n w_n}{\overline{w}_n}\right) \quad (12)$$

where m_i and $m_n = nm_i$ are the individual and the joint fading parameters, respectively, and $\overline{w}_n = E\{w_n\} = nE\{w_i\}$ is the joint mean power. It can be seen from (10) and (12) that both signal power and interference power are described by the same distribution, except that they have distinct parameters.

Using the appropriated expressions in (2), and after some manipulation, the signal-to-interference PDF may be expressed as

$$\begin{aligned} f_Z(z) = & \frac{1}{B(m_s, m_n)} \frac{1}{z} \left(\frac{m_s}{m_n} \frac{\overline{w}_n}{\overline{w}_s} z\right)^{m_s} \\ & \cdot \left(1 + \frac{m_s}{m_n} \frac{\overline{w}_n}{\overline{w}_s} z\right)^{-m_n - m_s} \end{aligned} \quad (13)$$

where $B(.)$ is the beta function [14, Eq. 6.2.2]. The corresponding CDF may be expressed as [17]

$$F_Z(z_0) = I_{x_0}(m_s, m_n) \quad (14)$$

where $I_x(.)$ is the regularized incomplete beta function [14, Eq. 6.6.2], and

$$x_0 = \left(1 + \frac{m_n}{m_s} \frac{\overline{w}_s}{\overline{w}_n} \frac{1}{z_0}\right)^{-1}. \quad (15)$$

As a special case, for Rayleigh fading channels $m_s = m_i = 1$ (which results in $m_n = n$). Using these values in (14), it can be shown that

$$F_Z(z_0) = 1 - \left(\frac{n}{n + z_0 \overline{w}_n / \overline{w}_s}\right)^n. \quad (16)$$

If the result above is further simplified by assuming that $\overline{w}_s = \overline{w}_i = \overline{w}_n / n$, then the same expression presented in [7] is found.

B. Spatial Coverage

The analysis presented so far assumes that the components of the interference signal have equal mean power $\overline{w}_i$, $i = 1, \ldots, n$. This restriction limits the results to systems where perfect power control is employed or to terminals placed at a fixed distance from the receiving antenna (i.e., on a circular ring) and in an environment without any shadowing effects. Let us now extend the model presented above to include the case of packets arriving with different mean power, e.g., from terminals spread across a radio cell and at different transmission distances to the receiver's antenna. Therefore, the statistical behaviour of the packet mean power needs to be specified and taken into account.

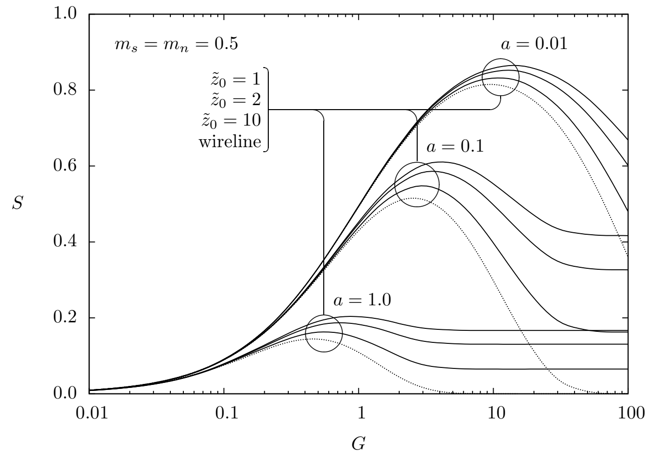

Fig. 1. Throughput curves assuming incoherent addition of Nakagami-fading interfering packets and constant mean packet power, with $m_s = m_n = 0.5$. The dotted lines correspond to the original (wireline) channel where $z_0 \to \infty$.

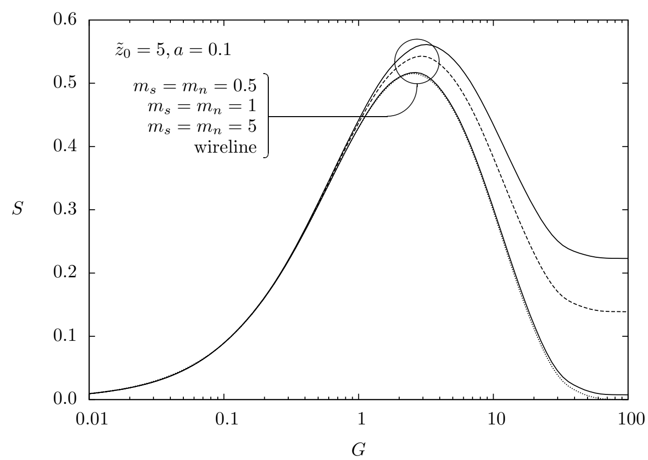

Fig. 2. Throughput curves assuming incoherent addition of Nakagami-fading interfering packets and constant mean packet power with $\tilde{z}_0 = 5$ and $a = 0.1$. The dashed line correspond to the Rayleigh channel where $m_s = m_n = 1$. The dotted line correspond to the original (wireline) channel where $z_0 \to \infty$.

The mean power of a packet received from a terminal at a distance d is of the general form [18]

$$\overline{w} = k\, d^{-\alpha} \quad (17)$$

where α gives the channel attenuation with the distance, and k is a value that depends on, among other things, the transmit power and the height and gain of the antennas. Typical values of the exponent α are $\alpha = 2$ in free space and $\alpha = 4$ in urban land mobile cellular systems. Using a similar approach to that presented in [7], let $\rho \triangleq d\, k^{-1/\alpha}$ be defined and used to rewrite (17) as

$$\overline{w} = \rho^{-\alpha}. \quad (18)$$

Let the offered traffic density function $G(\rho)$ be defined as the number of packets offered per transmission period per

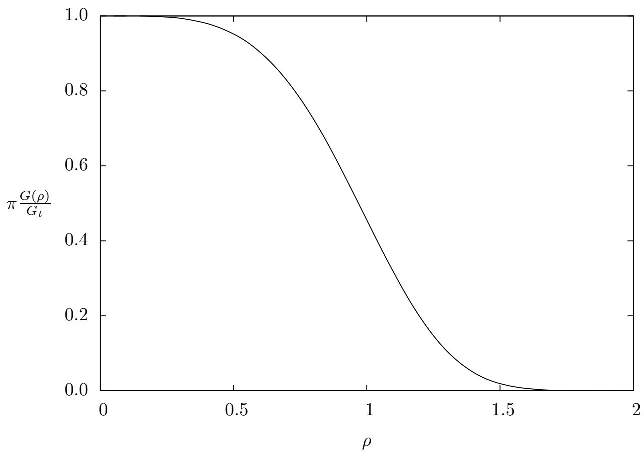

Fig. 3. Normalised offered traffic density defined by (25).

unit of area at distance ρ. The total offered traffic can be calculated by

$$G_t = 2\pi \int_0^\infty \rho\, G(\rho)\, d\rho. \tag{19}$$

The spatial CDF of the offered traffic as a function of the distance ρ can be expressed as

$$\begin{aligned} F_G(\rho) &= \text{Prob}\{\text{packet generated within distance } \rho\} \\ &= \frac{2\pi}{G_t} \int_0^\rho u\, G(u)\, du \end{aligned} \tag{20}$$

and the corresponding PDF is

$$f_G(\rho) = \frac{2\pi}{G_t} \rho\, G(\rho). \tag{21}$$

The PDF of the received packet mean power

$$f_{\overline{W}}(\overline{w}) = f_G(\rho) \left| \frac{d\rho}{d\overline{w}} \right| \tag{22}$$

is calculated using (18) and (21) and can be expressed as

$$f_{\overline{W}}(\overline{w}) = \frac{2\pi}{\alpha\, G_t} \overline{w}^{\,1+2/\alpha}\, G(\overline{w}^{\,-1/\alpha}). \tag{23}$$

The capture probability for spatial coverage is now considered. Given an arbitrary spatial traffic density $G(\rho)$, (23) can be used to calculate the PDF of the test packet mean power $f_{\overline{W}_s}(\overline{w}_s)$. The PDF of the mean interference power of n packets $f_{\overline{W}_n}(\overline{w}_n)$ is calculated by convolving (23) n times. With these results and assuming that the signal power and the interference power are statistically independent, the CDF of the signal-to-interference ratio can be calculated as

$$F_Z(z_0) = \int_0^{z_0} dz \int_0^\infty d\overline{w}_n \int_0^\infty f_Z(z)\, f_{\overline{W}_n}(\overline{w}_n)\, f_{\overline{W}_s}(\overline{w}_s)\, d\overline{w}_s \tag{24}$$

where $f_Z(.)$ is given by (2). With (5) and (24), the capture probability is then calculated. As an example, let us use the quasi-constant traffic density given in [7], expressed as

$$G(\rho) = \frac{G_t}{\pi} \exp\left(-\frac{\pi}{4}\rho^4\right), \quad \rho \geq 0, \tag{25}$$

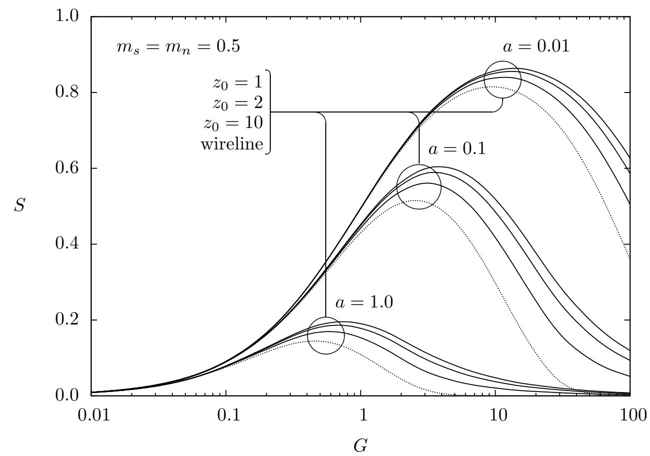

Fig. 4. Throughput curves assuming incoherent addition of Nakagami-fading interfering packets and spatial coverage, with $m_s = m_n = 0.5$. The dotted lines correspond to the original (wireline) channel where $z_0 \to \infty$.

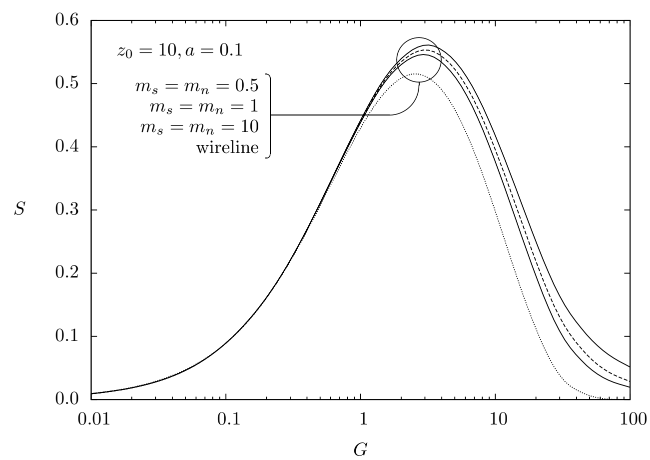

Fig. 5. Throughput curves assuming incoherent addition of Nakagami-fading interfering packets and spatial coverage with $z_0 = 10$ and $a = 0.1$. The dashed line correspond to the Rayleigh channel where $m_s = m_n = 1$. The dotted line correspond to the original (wireline) channel where $z_0 \to \infty$.

and depicted in Fig. 3. Note that the traffic density is roughly constant inside the cell of radius $\rho = 1$, falling rapidly beyond the cell boundary. If we select $\alpha = 4$, it can be seen that

$$f_{\overline{W}_s}(\overline{w}_s) = \frac{1}{2\overline{w}_s^{3/2}} \exp\left(-\frac{\pi}{4\overline{w}_s}\right) \tag{26}$$

and

$$f_{\overline{W}_n}(\overline{w}_n) = \frac{n}{2\overline{w}_n^{3/2}} \exp\left(-\frac{\pi n^2}{4\overline{w}_s}\right). \tag{27}$$

Using these results in (24), and considering $f_Z(.)$ given by

(13), the signal-to-interference CDF is given by

$$F_Z(z_0) = \sec(m_s\pi) \cdot \left[\frac{c_0^{m_s}}{m_s} \frac{{}_2F_1\left(m_s, m_s+m_n, m_s+1, c_0\right)}{B(m_s,m_n)} - 2\sqrt{c_0}\frac{\Gamma\left(m_n+\frac{1}{2}\right)}{\Gamma(m_s)\Gamma(m_n)\Gamma\left(-m_s+\frac{3}{2}\right)} \cdot {}_3F_2\left(\frac{1}{2},1,m_n+\frac{1}{2},\frac{3}{2},-m_s+\frac{3}{2},c_0\right)\right] \quad (28)$$

where ${}_2F_1(.)$ is the Gauss hypergeometric function [14, Eq. 15.1.1], ${}_3F_2(.)$ is a generalized hypergeometric function [19, Eq. 9.14.1], and

$$c_0 = n^2 z_0 \frac{m_s}{m_n}. \quad (29)$$

IV. Numerical Results

A. Perfect Power Control

With the results obtained in Section III-A, using (5), (8) and (14), it is possible to calculate the capture probability and the channel throughput considering the perfect power control. Figs. 1 and 2 present the channel throughput for a number of different conditions. As expected, lower values for the propagation delay a brings higher throughput figures. Throughput is also higher with lower values for the capture threshold z_0, which is related to the receiver's ability to cope with the interference. A lower value for z_0 means that the receiver is able to detect the test signal even in the presence of higher interference power levels. The fading parameter m has also an impact on the throughput. Less severe fading conditions (indicated by higher values of m) result in lower throughput figures. In other words, higher m values push the throughput results closer to those obtained for the original (wireline) channel model.

B. Spatial Coverage

With the results obtained in Section III-B, applying the results of (4) and (28) in (5), and the latter in (8), the throughput can be calculated. Fig. 4 presents the throughput for $m_s = m_n = 0.5$ and various values of the capture threshold z_0 and the propagation delay a. As expected, the lower the value of z_0, the higher the throughput obtained. The same behaviour is also observed with the value of a. Fig. 5 presents the curves of throughput for various values of m_s and m_n. In a way similar to the result presented above, the lower fading intensity also translates into lower throughput. However, for the current analysis it seems that m has a somewhat minor impact on the throughput.

V. Conclusions

This paper investigates the throughput performance of CSMA in a packet radio network and Nakagami fading environment. Analytical and numerical results are presented considering the signal capture model with incoherent addition of interfering signals, as well as the case of unequal average power levels. The results showed that lower values for the capture threshold z_0 result in higher throughput figures. This is an expected outcome since z_0 is related to the ability of a receiver to detect the intended signal among the interfering signals. Also, the results showed that the fading intensity, represented by the Nakagami distribution m parameter, has a somewhat smaller influence in the throughput performance.

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