Diversity Exploitation in Multiuser Bidirectional Cooperative Cellular Networks

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***Resumo*— Este artigo investiga a ordem de diversidade de uma rede celular cooperativa bidirecional. Em nossa análise, assume-se que uma estação rádio-base (BS), equipada com N_B antenas, e uma dentre K estações móveis (MSs), de uma única antena, realizam comunicações bidirecionais com a ajuda de um relay amplifica-e-encaminha (AF) de uma única antena. Técnicas de *beamforming* são empregadas na estação rádio-base. Um limitante inferior em forma fechada para a probabilidade de *outage* é obtido, a partir do qual uma análise assintótica é realizada, revelando que a ordem de diversidade para o sistema em estudo iguala a $\min[N_B, K]$.**

***Palavras-Chave*— Técnicas de beamforming, comunicações cooperativas, ganho de diversidade, probabilidade de *outage*, diversidade multiusuário, *relaying* bidirecional.**

***Abstract*— This paper examines the diversity order of a multiuser bidirectional cooperative cellular network. In our analysis, we assume that one base station (BS), equipped with N_B antennas, and one out of K mobile stations (MSs) with single antennas, perform two-way transmissions with the help of a single-antenna amplify-and-forward (AF) relay. Beamforming techniques are employed at BS. A tight closed-form lower bound for the outage probability of the considered system is derived, from which asymptotic analysis is carried out, revealing that the diversity order equals to $\min[N_B, K]$.**

***Keywords*— Beamforming techniques, cooperative communications, diversity gain, outage probability, multiuser diversity, two-way relaying.**

I. INTRODUCTION

In wireless networks, antenna diversity techniques [1] have been commonly used to combat the deleterious effect of the fading. These techniques require the terminals to be of reasonable sizes so as to support multiple antennas; a requirement that proves to be unfeasible for future wireless terminals which are expected to be small and light. Recently, cooperative/collaborative diversity [2], [3] in wireless networks have gained much interest in the wireless research community due to its ability to mitigate fading through achieving spatial diversity, while resolving the difficulties of installing multiple antennas on small communication terminals. The basic idea is that, in addition to the direct signal from the source to the destination, multiple cooperative nodes (relays) collaborate together to relay the signal from the source node to the destination node. As a result, the latter can receive multiple independent copies of the same signal and can achieve diversity through the establishment of a virtual antenna array. Many other benefits can be attained from this strategy, e.g, expansion of the radio coverage without using high power levels at the source, increase of connectivity, and higher capacity.

In addition, according to transmission mode, cooperative relaying transmissions can be classified into two main categories, namely full-duplex relaying and half-duplex relaying. Full-duplex relaying allows the user terminals to receive and transmit at the same time in the same frequency band, whereas reception and transmission for half-duplex relaying are usually performed in time-orthogonal channels. Relying on the low cost of implementation when compared to full-duplex relaying protocols, half-duplex relaying protocols have been usually employed in practice even though they have lower spectral efficiency than full-duplex relaying due to the pre-log factor of $1/2$ in the sum rate expressions [3]. Then, in order to counteract this inconvenience inherent to half-duplex protocols, two-way relaying has recently attracted much research interest owing to its improved spectrum efficiency [4], [5]. More specifically, by supporting two opposite traffic flows from two different users at the same time, these protocols significantly enhance the spectrum efficiency.

On the other hand, it has been proven along the years that by letting only the 'best' user to transmit at a certain time, multiuser diversity (MUD) can be achieved in the form of increased system diversity order or increased total throughput [6]. Motivated by the important benefits acquired with MUD and two-way relaying, very recently the authors of [7] presented an application of MUD in multiuser bidirectional cooperative cellular networks by assuming that all terminals were single-antenna devices. However, because of the existence of only one single link between the base station (BS) and relay, the achieved diversity order was unity, which rendered the MUD not available in such systems. To solve this bottleneck, in this paper we consider that the BS is equipped with multiple (e.g., N_B) antennas and employs beamforming techniques [8] for exploiting system diversity. Both theoretical analysis and simulation results will show that our proposed scheme achieves a diversity order equals to $\min[N_B, K]$, where K denotes the number of mobile stations (MSs) in the system. As far as the authors are aware, up to now there are no works in the literature considering beamforming in multiuser two-way relaying systems.

The remainder of this paper is organized as follows. In Section II, the system and channel models are introduced. In Section III, the outage probability (OP) is investigated. Specifically, a tight closed-form lower bound for the outage

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probability of the considered system is derived, from which asymptotic analysis is carried out, revealing that the diversity order equals to $\min[N_B, K]$. Numerical results are provided in Section IV, followed by concluding remarks drawn in Section V.

Throughout this paper, vectors are represented by bold letters, $\|.\|_F$ denotes the Frobenius norm, $(\cdot)^\dagger$ symbolizes the conjugate transpose operator, Pr$(\cdot)$ denotes probability, E$[\cdot]$ indicates expectation and, $f_X(\cdot)$ and $F_X(\cdot)$ are the probability density function (PDF) and cumulative distribution function (CDF) of a variate X, respectively.

II. System and Channel Models

Consider a bidirectional cooperative cellular system in which one BS intends to exchange messages with one out of K mobile stations MS$_k$ ($k = 1, 2, \ldots, K$) by using a half-duplex amplify-and-forward (AF) relay $\mathcal{R}$. In this case, the MSs and $\mathcal{R}$ are assumed to be single-antenna devices, whereas the BS is equipped with N_B antennas for exploiting spatial diversity. As in [7], we assume that due to the unsatisfactory quality of the channel between MSs and BS, there is no direct link between them and the communication can be performed only through the relay. This latter introduces variable gain on the received signal regardless of the amplitude on the incoming signals, resulting hence in an output signal with fixed power.

For MUD, at a certain time, the 'best' MS (e.g., MS$_k$), among the K potential ones, is selected and the detailed selection standard will be given later. Afterwards, a two-phase bidirectional transmission begins [7]. In the first phase, BS and MS$_k$ transmit their respective signals to $\mathcal{R}$ simultaneously. Specifically, for the BS, its message x_1 is weighted with a $N_B \times 1$ transmit beamforming vector $\mathbf{w}_t$ to form the transmit signal vector [8][1], whereas for the MS$_k$, its message x_2 is transmitted directly. Herein, we assume that x_l, $l = 1, 2$, has an average power normalized to unity, i.e., E$[|x_l|^2] = 1$. Hence, the received signal at $\mathcal{R}$ can be expressed as

$$y_\mathcal{R} = \mathbf{h}_1^\dagger \mathbf{w}_t x_1 + h_{2,k} x_2 + n, \tag{1}$$

where $\mathbf{h}_1$ denotes the $N_B \times 1$ channel vector between BS and $\mathcal{R}$ with independent and identically distributed Rayleigh fading entries, $h_{2,k}$ is the Rayleigh fading coefficient pertaining to the link $\mathcal{R} \leftrightarrow$ MS$_k$, and n stands for the additive white Gaussian noise (AWGN) arriving at $\mathcal{R}$ with mean power N_0. As in [8], we consider $\mathbf{w}_t = \frac{\mathbf{h}_1}{\|\mathbf{h}_1\|_F}$. In the second phase, $\mathcal{R}$ amplifies $y_\mathcal{R}$ with a variable gain given by

$$G = \sqrt{\frac{1}{\|\mathbf{h}_1\|_F^2 + |h_{2,k}|^2 + N_0}} \tag{2}$$

and forwards it to both BS and MS$_k$. Another choice for the variable gain is to assume an ideal/hypothetical relay that is able to invert the channels regardless of their magnitude. In such a case, the gain can be rewritten as

$$G = \sqrt{\frac{1}{\|\mathbf{h}_1\|_F^2 + |h_{2,k}|^2}}. \tag{3}$$

At BS, the received signal is multiplied by a $1 \times N_B$ receive beamforming vector $\mathbf{w}_r = \frac{\mathbf{h}_1^\dagger}{\|\mathbf{h}_1\|_F}$ [8], leading to

$$y_{1,k} = \mathbf{w}_r \mathbf{h}_1 G y_\mathcal{R} + \mathbf{w}_r \mathbf{n}_1, \tag{4}$$

where $\mathbf{n}_1$ denotes the $N_B \times 1$ AWGN vector with each entry having mean power N_0. On the other hand, the received signal at MS$_k$ can be written as

$$y_{2,k} = h_{2,k} G y_\mathcal{R} + n_2, \tag{5}$$

in which n_2 indicates the AWGN added at MS$_k$ with mean power N_0[2]. Finally, self-interferences are suppressed, respectively, at BS and MS$_k$ [7]. Therefore, making use of the gain given in (2), the end-to-end signal-to-noise ratios (SNRs) pertaining to the links BS $\rightarrow \mathcal{R} \rightarrow$ MS$_k$ and MS$_k \rightarrow \mathcal{R} \rightarrow$ BS can be formulated, respectively, as

$$\gamma_{1,k} = \frac{g_1 g_{2,k}/N_0^2}{g_1/N_0 + 2g_{2,k}/N_0 + 1}, \tag{6a}$$

$$\gamma_{2,k} = \frac{g_1 g_{2,k}/N_0^2}{2g_1/N_0 + g_{2,k}/N_0 + 1}, \tag{6b}$$

where $g_1 \triangleq \|\mathbf{h}_1\|_F^2$ and $g_{2,k} \triangleq |h_{2,k}|^2$. Accordingly, g_1 conforms to a Gamma distribution [6] with mean Ω_1 so that its PDF is given by

$$f_{g_1}(x) = \frac{x^{N_B-1}}{\Gamma(N_B)\Omega_1^{N_B}} \exp\left(-\frac{x}{\Omega_1}\right), \tag{7}$$

where $\Gamma(\cdot)$ represents the Gamma function [9, Eq. (8.310.1)] and $g_{2,k}$ conforms to independent exponential distributions with mean $\Omega_{2,k}$. The expressions for the end-to-end SNRs presented in (6) are not easily mathematically tractable due to the complexity in finding the required first-order statistics associated with them. Fortunately, if we make use of the relay gain given in (3), (6) can be tightly bounded by

$$\gamma_{1,k} = \frac{g_1 g_{2,k}/N_0^2}{g_1/N_0 + 2g_{2,k}/N_0}, \tag{8a}$$

$$\gamma_{2,k} = \frac{g_1 g_{2,k}/N_0^2}{2g_1/N_0 + g_{2,k}/N_0}. \tag{8b}$$

It is worth noting that Eqs. (8a) and (8b) have the advantage of mathematical tractability over that in (6) in addition to be a tight upper bound for the form in (6), specially at high average SNR, as the numerical examples will illustrate later on.

Based on above, the transmission rates pertaining to the links BS $\rightarrow \mathcal{R} \rightarrow$ MS$_k$ and MS$_k \rightarrow \mathcal{R} \rightarrow$ BS can be expressed as $R_{1,k} \triangleq \frac{1}{2}\log_2(1+\gamma_{1,k})$ and $R_{2,k} \triangleq \frac{1}{2}\log_2(1+\gamma_{2,k})$, respectively. It is worth noting that since both $R_{1,k}$ and $R_{2,k}$ are strict monotonic increasing functions with respect to $g_{2,k}$ [7], the MS with the maximum $R_{1,k}$ and $R_{2,k}$ satisfies to $k^* = \arg\max_{k\in\{1,\ldots,K\}} [g_{2,k}]$. Thus, for the MUD-based MS selection, MS$_{k^*}$ is naturally chosen to perform the two-way

[1]The channel state information (CSI) is supposed to be available at BS.

[2]It is noteworthy that we assume perfect channel reciprocity between BS and MS$_k$. This assumption is valid when the local oscillators of the BS, relay, and MS are perfectly synchronized. The imperfect channel reciprocity and calibration remain for future work.

communications with BS, resulting respectively in

$$R_{1,k^*} \triangleq \frac{1}{2}\log_2(1+\gamma_{1,k^*}), \tag{9a}$$

$$R_{2,k^*} \triangleq \frac{1}{2}\log_2(1+\gamma_{2,k^*}). \tag{9b}$$

In what follows, a tight closed-form lower bound for the outage probability of the system under studied will be derived, from which insightful discussions will be provided.

III. Outage Probability Analysis

An important performance measure in wireless systems is the outage probability. Assuming a multiuser bidirectional cooperative cellular network, such a metric is defined as the probability that either R_{1,k^*} or R_{2,k^*} falls below a predefined threshold rate $\Re$ bit/s/Hz, being mathematically expressed as

$$\begin{aligned} P_{\text{out}} &= \Pr\left(\min[\gamma_{1,k^*},\gamma_{2,k^*}] < 2^{2\Re}-1 \triangleq \rho\right) \\ &= 1-\Pr(\gamma_{1,k^*}>\rho,\gamma_{2,k^*}>\rho) \\ &\geq 1-\underbrace{\Pr\left(\frac{g_1g_2/N_0^2}{g_1/N_0+2g_2/N_0}>\rho,\frac{g_1g_2/N_0^2}{2g_1/N_0+g_2/N_0}>\rho\right)}_{\Xi}, \end{aligned} \tag{10}$$

in which $g_2 \triangleq \max_{k\in\{1,\ldots,K\}}[g_{2,k}]$. As in [10], we define $E \triangleq 1/N_0$ as system SNR[3]. Now, let $\alpha \triangleq 1/g_1$ and $\beta \triangleq 1/g_2$, then Ξ can be rewritten as

$$\begin{aligned} \Xi &= \Pr\left(\beta<\frac{E}{\rho}-2\alpha,\beta<\frac{E}{2\rho}-\frac{\alpha}{2}\right) \\ &= \iint_D f_\alpha(\alpha)f_\beta(\beta)d\alpha d\beta \\ &= \int_0^{\frac{E}{3\rho}}\left(\int_\beta^{\frac{E}{2\rho}-\frac{\beta}{2}} f_\alpha(\alpha)d\alpha\right)f_\beta(\beta)d\beta \\ &\quad+\int_0^{\frac{E}{3\rho}}\left(\int_\alpha^{\frac{E}{2\rho}-\frac{\alpha}{2}} f_\beta(\beta)d\beta\right)f_\alpha(\alpha)d\alpha, \end{aligned} \tag{11}$$

where the PDF of α can be readily attained by using the Fundamental Theorem [11, Sec. 5-2]. On the other hand, the PDF of β can be obtained from [12, Theorem 1] and [11, Sec. 5-2]. Unfortunately, to the best of the authors' knowledge, closed-form solutions are not available for (11). However, when the integration regions D of (11) are slightly enlarged, it follows that

$$\begin{aligned} \Xi &\leq \int_0^{\frac{E}{2\rho}}\int_0^{\alpha} f_\alpha(\alpha)f_\beta(\beta)d\beta d\alpha+\int_0^{\frac{E}{2\rho}}\int_0^{\beta} f_\alpha(\alpha)f_\beta(\beta)d\alpha d\beta \\ &= \underbrace{\int_0^{\frac{E}{2\rho}} f_\alpha(\alpha)F_\beta(\alpha)d\alpha}_{J_1}+\underbrace{\int_0^{\frac{E}{2\rho}} F_\alpha(\beta)f_\beta(\beta)d\beta}_{J_2}, \end{aligned} \tag{12}$$

where the CDFs of α and β can be readily attained by integrating appropriately their corresponding PDFs and knowing that [9, Eq. (3.381.3)]

$$\int_u^\infty x^{\nu-1}\exp(-\mu x)dx = \mu^{-\nu}\Gamma(\nu,\mu u), \tag{13}$$

where $\Gamma(\cdot,\cdot)$ denotes the upper incomplete Gamma function [9, Eq. (8.350.2)]. In what follows, both J_1 and J_2 will be determined. Firstly, by substituting the PDF of α and the CDF of β into J_1 and utilizing [9, Eq. (3.381.3)], we have

$$\begin{aligned} J_1 &= \frac{1}{\Gamma(N_B)\Omega_1^{N_B}}\sum_{m=1}^{K}\sum_{\substack{S_m\subseteq\{1,\ldots,K\}\\|S_m|=m}}(-1)^{m+1} \\ &\times\left(\frac{1}{\Omega_1}+\xi_{S_m}\right)^{-N_B}\Gamma\left(N_B,\frac{2\rho}{E}\left(\frac{1}{\Omega_1}+\xi_{S_m}\right)\right), \end{aligned} \tag{14}$$

where $\xi_{S_m} \triangleq \sum_{j\in S_m}\frac{1}{\Omega_{2,j}}$. Next, plugging the PDF of β and the CDF of α into J_2, using [9, (3.381.3)] and the following identity [9, Eq. (8.352.4)]

$$\Gamma(n,x) = (n-1)!\exp(-x)\sum_{m=0}^{n-1}\frac{x^m}{m!}, \tag{15}$$

one can attain at

$$\begin{aligned} J_2 &= \sum_{m=1}^{K}\sum_{\substack{S_m\subseteq\{1,\ldots,K\}\\|S_m|=m}}(-1)^{m+1}\xi_{S_m}\sum_{l=0}^{N_B-1}\frac{(1/\Omega_1)^l}{l!} \\ &\times\left(\frac{1}{\Omega_1}+\xi_{S_m}\right)^{-l-1}\Gamma\left(l+1,\frac{2\rho}{E}\left(\frac{1}{\Omega_1}+\xi_{S_m}\right)\right). \end{aligned} \tag{16}$$

Now, by summarizing the above results, we arrive at the proposition as below.

Proposition 1: For the multiuser bidirectional cooperative cellular network under study, a closed-form lower bound for the outage probability can be derived as

$$P_{\text{out}} \geq 1-J_1-J_2 \triangleq P_{\text{out}}^{\text{LB}}, \tag{17}$$

where J_1 and J_2 are determined by (14) and (16), respectively. ∎

Proposition 2: In high SNR regime, i.e., $E\to\infty$, $P_{\text{out}}^{\text{LB}}$ can be asymptotically expressed as

$$P_{\text{out}}^{\text{LB}} \simeq \begin{cases} \left(\frac{1}{E}\right)^{N_B}\left(\frac{2\rho}{\Omega_1}\right)^{N_B}\frac{1}{(N_B)!}, & \text{if } N_B<K \\ \left(\frac{1}{E}\right)^{K}\prod_{j=1}^{K}\left(\frac{2\rho}{\Omega_{2,j}}\right), & \text{if } N_B>K \\ \left(\frac{1}{E}\right)^{N_B}\left[\left(\frac{2\rho}{\Omega_1}\right)^{N_B}\frac{1}{(N_B)!}+\prod_{j=1}^{K}\left(\frac{2\rho}{\Omega_{2,j}}\right)\right], & \text{if } N_B=K \end{cases} \tag{18}$$

Proof: From [12, Theorem 1], we have that if W_n, $n=1,2,\ldots,M$, are M statistically independent and not necessarily identically distributed exponential random variables, each with parameter $\lambda_n = 1/\text{E}[W_n]$, then the PDF of $\Psi = \max[W_1,W_2,\ldots,W_M]$, for $\psi\geq 0$, is given by

$$f_\Psi(\psi) = \sum_{m=1}^{M}\sum_{\substack{S_m\subseteq\{1,\ldots,M\}\\|S_m|=m}}(-1)^{m+1}\left(\sum_{j\in S_m}\lambda_j\right)e^{-\psi\sum_{j\in S_m}\lambda_j}. \tag{19}$$

[3] In order to evaluate the asymptotic outage behavior, which is crucial for the determination of the system diversity order, in the subsequent analysis we will set $E\to\infty$.

Now, we turn our attention for the proof of *Proposition 2*. As $E \to \infty$, by taking the Taylor's series expansion of (17) at $1/E = 0$, with the aid of [9, Eq. (8.352.4)] and (19), and employing the binomial theorem [13, Eq. (3.1.1)], it follows, after some algebraic manipulations, that

$$P_{\text{out}}^{\text{LB}} \simeq \left(\frac{1}{E}\right)^{N_B} \left(\frac{2\rho}{\Omega_1}\right)^{N_B} \frac{1}{(N_B)!} + \left(\frac{1}{E}\right)^{K} \prod_{j=1}^{K} \left(\frac{2\rho}{\Omega_{2,j}}\right), \tag{20}$$

which leads to (18). ■

From Proposition 2, it is clear that the achieved diversity order of the considered system is $\min[N_B, K]$. For practical system configurations, when the number of antennas at BS (i.e., N_B) is not less than the number of MSs, the MUD gain (i.e., K) can be fully exploited. In particular, when $N_B = 1$ as considered in [7], the system diversity order reduces to unity.

IV. Numerical Results

In this section, we present illustrative numerical examples for the outage probability expressions obtained previously. Monte Carlo simulation results are also provided in order to validate our analysis. For illustration purposes and without loss of generality, we assume $\Omega_{2,l} = \Omega_2$ for $l = 1, \ldots, K$. Also, the end-to-end threshold rate is set to $\Re = 1$ bit/s/Hz. Figs. 1, 2, and 3 depict the outage probability versus system SNR E assuming different set of parameters, which yields distinct diversity gains. It can be observed that the proposed lower bound is very tight especially in the medium and high SNR regions. Besides, for all the cases, the asymptotic curves and the corresponding simulated ones overlap in high SNR region, validating therefore the Proposition 2.

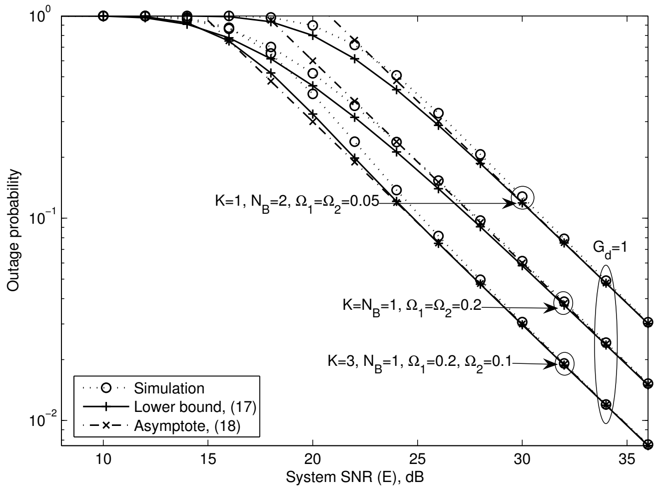

Fig. 1. Outage probability versus system SNR E for $G_d = 1$; G_d denotes diversity order.

V. Conclusions

In this paper, by deploying multiple antennas at BS and making use of beamforming techniques, a tight closed-form

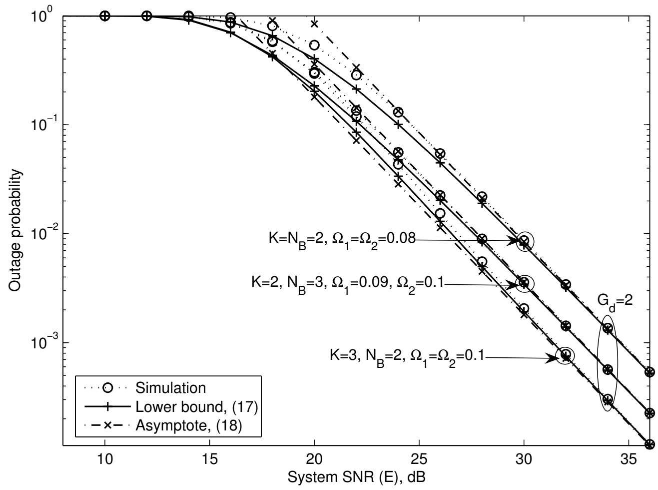

Fig. 2. Outage probability versus system SNR E for $G_d = 2$; G_d denotes diversity order.

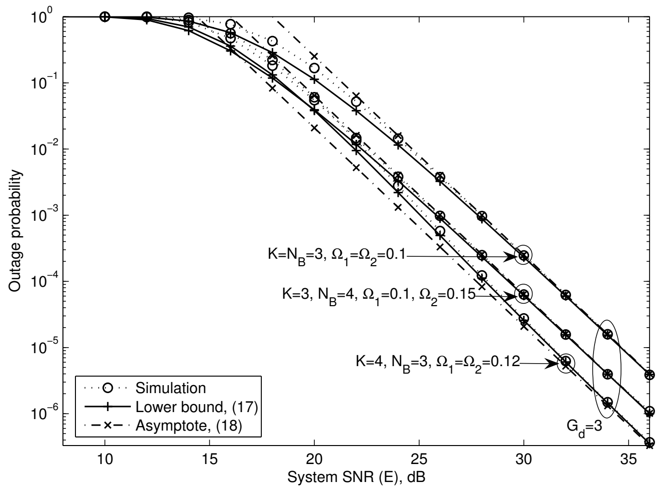

Fig. 3. Outage probability versus system SNR E for $G_d = 3$; G_d denotes diversity order.

lower bound for the outage probability of multiuser bidirectional cooperative cellular networks was derived. Our analysis revealed that the system diversity can be successfully exploited, which shows therefore the practical usefulness of the proposed scheme.

Acknowledgment

Prof. Daniel Benevides da Costa would like to thank the financial support by the Ceará Council of Scientific and Technological Development (FUNCAP) under Grant no. BP1-0031-00090.01.00/10.

Mr. Haiyang Ding and Prof. Jianhua Ge would like to thank the financial support by the Program for Changjiang Scholars and Innovative Research Team in University under Grant no. IRT0852, by the Natural Science Foundation of Guangdong Province under Grant no. U0635003, by the National Major Specialized Project of Science and Technology of China under

Grant no. 2009ZX03003-003/004, by the "111" project under Grant no. B08038, and by the National Natural Science Foundation of China under Grant no. 61001128 and Grant no. 61001207.

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