Throughput Performance Analysis of Some Half and Full Duplex Cooperative Schemes

Hirley Alves, Gustavo Fraidenraich and Richard Demo Souza

***Abstract*— In this paper, we compare Full-Duplex (FD) and Half-Duplex (HD) relaying, in terms of outage probability and throughput. We consider a practical relay, where loop interference between transmitted and received signal is taken into account. Two models for FD transmission are considered: Block Markov encoding and multi-hop encoding. For HD transmission, the space-time cooperation is considered. Surprisingly, our results show that HD relaying can outperform FD relaying, specially if loop interference is non negligible. Such results hold even if power and rate allocation are carried out.**

***Keywords*— Cooperation, Full-duplex, Half-duplex**

I. INTRODUCTION

Cooperative relaying has been proposed as an alternative to exploit the spatial diversity of the wireless channel. Through cooperation among nodes even single antenna devices can achieve spatial diversity, as shown in [1]–[3]. Probably the most known cooperative protocols are the amplify-and-forward (AF) and the decode-and-forward (DF) [1], including their variants (selective and incremental protocols [1]). Moreover, the cooperative protocols can operate either in a half-duplex (HD) or full-duplex (FD) fashion.

In the HD mode the relay cannot transmit and receive simultaneously. The HD cooperative protocols are spectrally inefficient [4], [5], in the sense that two time slots (or two frequencies) are used to transmit a message from source to destination. By its turn, in the FD mode the relay simultaneously transmits and receives in the same frequency. The authors in [6] show that FD cooperative protocols achieve, in general, a higher capacity than HD cooperative protocols. In [7] the authors assume a capacity achieving Block Markov transmission scheme, and then derive the outage probability of the ideal FD scheme. However, ideal FD operation is often not possible. In practice, transmitted power is normally much larger than received power [6], which turns the isolation a difficult task. Practical FD relay models were proposed in [8], [9], where a loop interference is assumed between the transmitted and received signals. Moreover, multi-hop FD transmission considering that the destination is able to cancel out the interference from the source signal is analyzed in [8]. The case without cancellation was addressed in [9].

In this paper we consider a practical FD relay, with loop interference, which may operate under one of two schemes: i) Block Markov transmission, which is a quite complex approach; ii) multi-hop FD without interference cancellation, which is much simpler and practical. We compare the performance, in terms of outage probability and throughput, of the FD relaying schemes to that of a high performance HD incremental DF (IDF) cooperative scheme: the incremental redundancy space-time (IR-ST) method. In IR-ST the relay only cooperates if requested by the destination. Once requested, the relay sends additional parity bits, together with the source, by means of a space-time codeword, which is then appropriately combined by the destination with the first source transmission [10]. We also analyze the performance of the FD and HD relaying schemes, in terms of throughput, when power allocation (PA) and rate allocation (RA) are carried out. Our results show that, when the loop interference is taken into account, IR-ST can outperform both FD relaying schemes, even with PA and RA.

The rest of this paper is organized as follows. Section II presents the system model. Section III introduces the outage and throughput analysis of the FD and HD schemes. Section IV investigates the impact of doing power and rate allocation. Finally, Section V concludes the paper.

II. SYSTEM MODEL

Consider a system with three cooperating terminals: source (S), relay (R) and destination (D), as shown in Fig 1. The S-D, R-D, and S-R channels are all subject to quasi-static Rayleigh fading, with zero mean and unit variance. We assume perfect channel state information (CSI) at the receivers. The channel noise is considered to be a complex AWGN (additive white Gaussian noise) with variance $N_0/2$ per dimension. We consider that R can operate either in FD or HD mode.

In the FD mode, the source broadcasts the message $x_{\rm S}$, which is heard by both D and R. At the same time R sends a message $x_{\rm R}$ to D. Moreover, since we assume the presence of a loop interference in the relay, the transmission from R to D interferes in the reception at R of the message sent by S. Following the above, the received signals at the relay and at the destination can be written as:

$$y_{\rm R} = \sqrt{P_{\rm S}\,\kappa_{\rm SR}}\,h_{\rm SR}\,x_{\rm S} + \sqrt{P_{\rm R}}h_{RR}\,x_{\rm R} + n_{\rm R}, \quad (1)$$
$$y_{\rm D} = \sqrt{P_{\rm R}\,\kappa_{\rm RD}}h_{\rm RD}\,x_{\rm R} + \sqrt{P_{\rm S}\,\kappa_{\rm SD}}\,h_{\rm SD}\,x_{\rm S} + n_{\rm D}, \quad (2)$$

where $h_{\rm SD}$, $h_{\rm SR}$ and $h_{\rm RD}$ are the complex fading channel coefficients of the S-D, S-R and R-D links, respectively, while $h_{\rm RR}$ is the complex fading coefficient of the loop interference [9]. Vectors $n_{\rm R}$ and $n_{\rm D}$ are the noise at R and D, while κ_{ij} is the path loss coefficient between nodes i and j.

H. Alves and R. D. Souza are with CPGEI, Federal University of Technology - Paraná (UTFPR), Curitiba, PR 80230-901, Brazil, richard@utfpr.edu.br, hyalves@gmail.com.

G. Fraidenraich is with the University of Campinas (Unicamp), Campinas, SP 13083-852, Brazil, gf@decom.fee.unicamp.br

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In the HD mode the transmissions are orthogonal in time, and we assume the presence of a return channel. Therefore, in this scenario the nodes operate under the IDF protocol. In the first time slot, S broadcasts a message to R and D:

$$y_{\mathrm{R}} = \sqrt{P_{\mathrm{S}}\,\kappa_{\mathrm{SR}}}\,h_{\mathrm{SR}}\,x_{\mathrm{S}} + n_{\mathrm{R}}, \quad (3)$$

$$y_{\mathrm{D}} = \sqrt{P_{\mathrm{S}}\,\kappa_{\mathrm{SD}}}\,h_{\mathrm{SD}}\,x_{\mathrm{S}} + n_{\mathrm{D}}. \quad (4)$$

If an error is detected, D requires a retransmission. In such a case, R cooperates with S and the received signal at D is:

$$y'_{\mathrm{D}} = \sqrt{P_{\mathrm{S}}\,\kappa_{\mathrm{SD}}}\,h_{\mathrm{SD}}\,x'_{\mathrm{S}} + \sqrt{P_{\mathrm{R}}\,\kappa_{\mathrm{RD}}}\,h_{\mathrm{RD}}\,x'_{\mathrm{R}} + n_{\mathrm{D}}, \quad (5)$$

where x'_{S} and x'_{R} are symbols from a space-time codeword jointly transmitted by S and R. The signals y'_{D} and y_{D} are combined at D and a new decoding attempt is carried out.

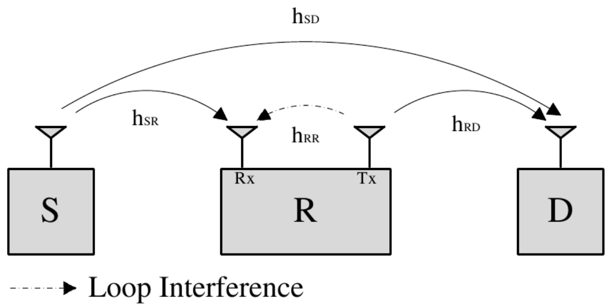

Fig. 1. System model with Source, Relay and Destination.

III. Outage and Throughput Analysis

A. *Multi-hop Full-Duplex Relaying*

First we analyze the non-cooperative multi-hop FD mode (FD-MH), in which S broadcasts a message and R forwards it to D. Note that the transmission from S is actually seen as interference at D. Thus, the Signal-to-Noise Ratio (SNR) at R is given by:

$$\gamma_{\mathrm{R}} = \frac{|h_{\mathrm{SR}}|^2\ P_{\mathrm{S}}\ \kappa_{\mathrm{SR}}}{|h_{\mathrm{RR}}|^2\ P_{\mathrm{R}} + 1}, \quad (6)$$

and at D is:

$$\gamma_{\mathrm{D}} = \frac{|h_{\mathrm{RD}}|^2\ P_{\mathrm{R}}\ \kappa_{\mathrm{RD}}}{|h_{\mathrm{SD}}|^2\, P_{\mathrm{S}}\ \kappa_{\mathrm{SD}} + 1}, \quad (7)$$

where P_{S} and P_{R} are the source and relay transmit power, respectively. Note that, neither R nor D use the interference cancellation technique.

An outage occurs when $\mathcal{I}_{ij} < \mathcal{R}$, where $\mathcal{I}_{ij}$ is the mutual information in the i to j link, and $\mathcal{R}$ is the attempted information transmission rate. Supposing complex Gaussian inputs and unitary bandwidth [11], the mutual information of the S-R and R-D links, respectively, are given by:

$$\mathcal{I}_{\mathrm{SR}} = \log_2\left(1 + \gamma_{\mathrm{R}}\right), \quad (8)$$

$$\mathcal{I}_{\mathrm{RD}} = \log_2\left(1 + \gamma_{\mathrm{D}}\right), \quad (9)$$

The overall outage probability of the multi-hop FD mode in terms of mutual information is:

$$\begin{aligned}\mathcal{P}_{\mathrm{FD,MH}} &= \Pr\{\mathcal{I}_{\mathrm{SR}} < \mathcal{R}\} + \Pr\{\mathcal{I}_{\mathrm{SR}} > \mathcal{R}\}\Pr\{\mathcal{I}_{\mathrm{RD}} < \mathcal{R}\} \\ &= \mathcal{P}_{\mathrm{SR}} + (1 - \mathcal{P}_{\mathrm{SR}})\mathcal{P}_{\mathrm{RD}} \quad (10)\end{aligned}$$

where $\Pr\{\theta\}$ is the probability of the event θ.

The outage probability of the S-R link can be found as:

$$\mathcal{P}_{\mathrm{SR}} = 1 - \frac{\exp\left(-\frac{2^{\mathcal{R}}-1}{P_{\mathrm{S}}\,\kappa_{\mathrm{SR}}\,\pi_{\mathrm{SR}}}\right) P_{\mathrm{S}}\,\kappa_{\mathrm{SR}}\,\pi_{\mathrm{SR}}}{P_{\mathrm{S}}\,\kappa_{\mathrm{SR}}\,\pi_{\mathrm{SR}} + (2^{\mathcal{R}} - 1)\,P_{\mathrm{R}}\pi_{\mathrm{RR}}}, \quad (11)$$

where $\pi_{\mathrm{SR}} = E\left[|h_{\mathrm{SR}}|^2\right]$ and $\pi_{\mathrm{RR}} = E\left[|h_{\mathrm{RR}}|^2\right]$. Likewise, the outage probability of the R-D link is:

$$\mathcal{P}_{\mathrm{RD}} = 1 - \frac{\exp\left(-\frac{2^{\mathcal{R}}-1}{\pi_{\mathrm{RD}}\,P_{\mathrm{R}}\,\kappa_{\mathrm{RD}}}\right) P_{\mathrm{R}}\,\kappa_{\mathrm{RD}}\,\pi_{\mathrm{RD}}}{P_{\mathrm{R}}\,\kappa_{\mathrm{RD}}\,\pi_{\mathrm{RD}} + (2^{\mathcal{R}} - 1)\,P_{\mathrm{S}}\,\kappa_{\mathrm{SD}}\,\pi_{\mathrm{SD}}}, \quad (12)$$

where $\pi_{\mathrm{SD}} = E\left[|h_{\mathrm{SD}}|^2\right]$ and $\pi_{\mathrm{RD}} = E\left[|h_{\mathrm{RD}}|^2\right]$. Then the overall outage probability is:

$$\begin{aligned}\mathcal{P}_{\mathrm{FD,MH}} &= 1 - \frac{\exp\left(\frac{P_{\mathrm{R}}\,\kappa_{\mathrm{RD}}\,\pi_{\mathrm{RD}}\left(1-2^{\mathcal{R}}\right) + P_{\mathrm{S}}\kappa_{\mathrm{SR}}\pi_{\mathrm{SR}}\left(1-2^{\mathcal{R}}\right)}{\pi_{\mathrm{SR}}\pi_{\mathrm{RD}}\,P_{\mathrm{S}}\,\kappa_{\mathrm{SR}}\,P_{\mathrm{R}}\,\kappa_{\mathrm{RD}}}\right)}{(P_{\mathrm{R}}\,\kappa_{\mathrm{RD}}\,\pi_{\mathrm{RD}} + (2^{\mathcal{R}} - 1)P_{\mathrm{S}}\,\kappa_{\mathrm{SD}}\,\pi_{\mathrm{SD}})} \times \\ &\quad \frac{P_{\mathrm{S}}\,\kappa_{\mathrm{SR}}\,P_{\mathrm{R}}\,\kappa_{\mathrm{RD}}\,\pi_{\mathrm{SR}}\pi_{\mathrm{RD}}}{(P_{\mathrm{S}}\,\kappa_{\mathrm{SR}}\,\pi_{\mathrm{SR}} + (2^{\mathcal{R}} - 1)P_{\mathrm{R}}\ \pi_{\mathrm{RR}})}. \quad (13)\end{aligned}$$

Note that when $P_{\mathrm{S}} = P_{\mathrm{R}} = P$ goes to infinity, there is an error floor given by:

$$\lim_{P\to\infty}\mathcal{P}_{\mathrm{FD,MH}} = 1 - \frac{\pi_{\mathrm{RD}}\pi_{\mathrm{SR}}}{(\pi_{\mathrm{SR}} + (2^{\mathcal{R}} - 1)\pi_{\mathrm{RR}})\,(\pi_{\mathrm{RD}} + (2^{\mathcal{R}} - 1)\pi_{\mathrm{SD}})}. \quad (14)$$

The throughput, which is the average spectral efficiency seen at D, can be written as:

$$\mathcal{T}_{\mathrm{FD}} = \mathcal{R}\left(1 - \mathcal{P}_{\mathrm{FD,MH}}\right). \quad (15)$$

B. *Block Markov Full-Duplex Relaying*

The capacity for the relay channel is still an open problem. In view of this unanswered issue, the best achievable rate known in the literature is attained when the Block Markov enconding technique [7], [12]–[14] is employed.

The Block Markov (FD-BM) mutual information is:

$$\mathcal{I}_{\mathrm{FD\text{-}BM}} = \min\left(\mathcal{I}_{\mathrm{SR,BM}}, \mathcal{I}_{\mathrm{MAC}}\right) \quad (16)$$

where the mutual information $\mathcal{I}_{\mathrm{SR}}^{\mathrm{BM}}$ is given by

$$\mathcal{I}_{\mathrm{SR,BM}} = \log_2\left(1 + \left(1 - \rho^2\right)\gamma_{\mathrm{R}}\right), \quad (17)$$

while the variable ρ is the correlation coefficient between the source and the relay messages [7], [14]. In fact, ρ can be chosen in order to maximize $\mathcal{I}_{\mathrm{FD\text{-}BM}}$.

The multiple access channel (MAC) mutual information formed by R-D and S-D links is:

$$\begin{aligned}\mathcal{I}_{\mathrm{MAC}} &= \log_2\Big(1 + P_{\mathrm{S}}\,\kappa_{\mathrm{SD}}\,|h_{\mathrm{SD}}|^2 + P_{\mathrm{R}}\,\kappa_{\mathrm{RD}}\,|h_{\mathrm{RD}}|^2 \\ &\quad + 2\sqrt{P_{\mathrm{S}}\,\kappa_{\mathrm{SD}}\,P_{\mathrm{R}}\,\kappa_{\mathrm{RD}}}\,\mathrm{Re}\left(\rho h_{\mathrm{SD}}\,h^*_{\mathrm{RD}}\right)\Big), \quad (18)\end{aligned}$$

with $\mathrm{Re}(.)$ denoting the real part and $(.)^*$ denoting the complex conjugate. Thus, the overall outage probability of FD-BM can be written as:

$$\mathcal{P}_{\mathrm{FD,BM}} = \mathcal{P}\{\min\left(\mathcal{I}_{\mathrm{SR,BM}}, \mathcal{I}_{\mathrm{MAC}}\right) \leq \mathcal{R}\} = 1 - \mathcal{P}^C_{\mathrm{MAC}}\,\mathcal{P}^C_{\mathrm{SR,BM}}. \quad (19)$$

The complementary outage probability of the S-R link is:

$$\mathcal{P}_{\text{SR,BM}}^{C}=\frac{\exp\left(-\frac{2^{\mathcal{R}}-1}{P_{\text{S}}\,\kappa_{\text{SR}}\,\pi_{\text{SR}}(1-\rho^2)}\right)P_{\text{S}}\,\kappa_{\text{SR}}\,\pi_{\text{SR}}}{P_{\text{S}}\,\kappa_{\text{SR}}\,\pi_{\text{SR}}+\frac{(2^{\mathcal{R}}-1)}{(1-\rho^2)}P_{\text{R}}\pi_{\text{RR}}}, \tag{20}$$

while the complementary outage probability of the MAC channel is [14]:

$$\mathcal{P}_{\text{MAC}}^{C}=\frac{\alpha e^{\frac{-\left(2^{\mathcal{R}}-1\right)}{\alpha}}-\beta e^{\frac{-\left(2^{\mathcal{R}}-1\right)}{\beta}}}{\alpha-\beta}, \tag{21}$$

where α and β are, respectively, given by:

$$\alpha=\frac{a}{2}+\sqrt{b} \tag{22}$$

$$\beta=\frac{a}{2}-\sqrt{b} \tag{23}$$

and

$$a=\left(P_{\text{R}}\,\kappa_{\text{RD}}\,\pi_{\text{RD}}+P_{\text{S}}\,\kappa_{\text{SD}}\pi_{\text{SD}}\right) \tag{24}$$

$$b=\frac{a^2}{4}-P_{\text{R}}\,\kappa_{\text{RD}}P_{\text{S}}\,\kappa_{\text{SD}}\pi_{\text{RD}}\pi_{\text{SD}}\left(1-\rho^2\right) \tag{25}$$

Note that the outage probability of the S-R link is $\mathcal{P}_{\text{SR,BM}}=1-\mathcal{P}_{\text{SR,BM}}^{C}$ and the MAC channel is $\mathcal{P}_{\text{MAC}}=1-\mathcal{P}_{\text{MAC}}^{C}$.

The error floor when $P_{\text{R}}=P_{\text{S}}=P$ is given by:

$$\lim_{P\to\infty}\mathcal{P}_{\text{FD,BM}}=\frac{\left(2^{\mathcal{R}}-1\right)\pi_{\text{RR}}}{(2^{\mathcal{R}}-1)\,\pi_{\text{RR}}+\pi_{\text{SR}}\,(1-\rho^2)}, \tag{26}$$

while the throughput is:

$$\mathcal{T}_{\text{FD,BM}}=\mathcal{R}\left(1-\mathcal{P}_{\text{FD,BM}}\right). \tag{27}$$

C. IR-ST Half-Duplex Relaying

In the HD mode the outage probability of the S-D link is:

$$\mathcal{P}_{\text{SD}}=\mathcal{P}\left\{\gamma_{\text{SD}}<2^{R}-1\right\}=1-e^{\frac{1-2^{\mathcal{R}}}{\gamma_{\text{SD}}}}, \tag{28}$$

where $\gamma_{\text{SD}}=\frac{P_{\text{S}}\,\kappa_{\text{SD}}\,|h_{\text{SD}}|^2}{N_0}$. Similarly,

$$\mathcal{P}_{\text{SR}}^{\text{IR}}=\mathcal{P}\left\{\gamma_{\text{SR}}<2^{\mathcal{R}}-1\right\}=1-e^{\frac{1-2^{\mathcal{R}}}{\gamma_{\text{SR}}}}. \tag{29}$$

where $\gamma_{\text{SR}}=\frac{P_{\text{S}}\,\kappa_{\text{SR}}\,|h_{\text{SR}}|^2}{N_0}$. In case of a requested retransmission, in the IR-ST scheme both source and relay retransmit the same new coded bits in perfect synchronism, so that the receiver can concatenate the originally received packet and the combined retransmitted packets to form a single packet of lower rate. In that case, after the retransmissions, the mutual information at the receiver is:

$$\mathcal{I}_{\text{IR-ST}}=\log_2\left(1+\gamma_{\text{SD}}\right)+\log_2\left(1+\gamma_{\text{SD}}+\gamma_{\text{RD}}\right). \tag{30}$$

where $\gamma_{\text{RD}}=\frac{P_{\text{R}}\,\kappa_{\text{RD}}\,|h_{\text{RD}}|^2}{N_0}$. It is possible to write the outage probability as:

$$\mathcal{P}_{IR-ST}=\frac{1}{\gamma_{\text{RD}}\gamma_{\text{SD}}}\times\int_{1}^{2^{\mathcal{R}}}\int_{0}^{\sqrt{z}-1}\frac{e^{-\frac{z}{\gamma_{\text{SD}}(w+1)}+\frac{w+1}{\gamma_{\text{RD}}}-\frac{w}{\gamma_{\text{SD}}}}}{w+1}dwdz. \tag{31}$$

Then the throughput for IR-ST is:

$$\mathcal{T}_{\text{IR-ST}}=R\left(1-\mathcal{P}_{\text{SD}}\right)+\frac{\mathcal{R}}{2}\mathcal{P}_{\text{SD}}\,\mathcal{P}_{\text{SR}}^{\text{IR}}\left(1-\frac{\mathcal{P}_{\text{SD,2}}}{\mathcal{P}_{\text{SD}}}\right)+\frac{\mathcal{R}}{2}\mathcal{P}_{\text{SD}}\left(1-\mathcal{P}_{\text{SR}}^{\text{IR}}\right)\left(1-\frac{\mathcal{P}_{\text{IR-ST}}}{\mathcal{P}_{\text{SD}}}\right), \tag{32}$$

where $\mathcal{P}_{\text{SD,2}}$ is the outage after two consecutive source transmissions (since we assume a long term quasi-static fading channel, $\mathcal{P}_{\text{SD,2}}$ is just 3 dB better than $\mathcal{P}_{\text{SD}}$ in terms of SNR). Two consecutive source transmissions may happen in the case that D could not decode the packet after the first transmission, and that R was also unable to decode S message, which happens with probability $(\mathcal{P}_{\text{SD}}\,\mathcal{P}_{\text{SR}})$. Moreover,

$$\frac{\mathcal{P}_{\text{IR-ST}}}{\mathcal{P}_{\text{SD}}}=\mathcal{P}\left\{\mathcal{I}_{\text{IR-ST}}<\mathcal{R}|\mathcal{I}_{\text{SD}}<\mathcal{R}\right\}$$

is the probability that an error occurs after the destination applies IR-ST in the source and relay transmissions, given that an error occurred after the original source transmission. Likewise, $\frac{\mathcal{P}_{\text{SD,2}}}{\mathcal{P}_{\text{SD}}}=\mathcal{P}\left\{\mathcal{I}_{\text{SD,2}}<\mathcal{R}|\mathcal{I}_{\text{SD}}<\mathcal{R}\right\}$.

The throughput of a single direct transmission will be used as a reference in the next section and it is given by:

$$\mathcal{T}_{\text{dir}}=\mathcal{R}\left(1-\mathcal{P}_{\text{SD}}\right). \tag{33}$$

IV. NUMERICAL RESULTS

We assume a log-distance path loss model with decay exponent 4, and that the transmit power of the source and the relay are the same, $P_{\text{S}}=P_{\text{R}}$, at least when power allocation is not carried out. We also suppose that R is positioned in a straight line between S and D. Thus, normalizing the distance between S and D to the unit, then $d_{\text{RD}}=1-d_{\text{SR}}$. In the results we considered $d_{\text{SR}}=0.5$. Based on [9], we consider three levels of loop interference: the ideal case, in which $\pi_{\text{RR}}=0$ ($-\infty$dB); and the more practical cases of $\pi_{\text{RR}}=-8$ dB and $\pi_{\text{RR}}=-16$dB. The correlation coefficient for the Block Markov case is assumed to be null, $\rho=0$, since the conclusions for this value of d_{SR} do not vary for $\rho\neq 0$.

Fig. 2 presents the outage probability as a function of $\overline{\gamma}_{SD}=\frac{P_{\text{S}}\,\kappa_{SD}}{N_0}$, when $\mathcal{R}=2$ bits/s/Hz. From the figure we can notice that IR-ST outperforms FD methods and direct transmission. For FD relaying the performance decreases significantly with the increase of the loop interference. In Fig. 3 we consider a similar scenario, but when $\mathcal{R}=6$ bits/s/Hz. From the figure we can see that IR-ST increases its advantage over the other schemes with the increase in the attempted rate. Therefore, at least in terms of outage probability, HD is considerably superior than FD relaying. That is reasonable since in FD mode D sees a superposition of S and R signals, which increases the error probability at D.

On the other hand, when the throughput is the metric considered, as shown in Fig. 4, we can see that FD-BM can considerably outperform IR-ST. However, for higher values of $\mathcal{R}$ and for practical values of the loop interference the performance of FD relaying considerably decreases. For $\mathcal{R}=6$ bits/s/Hz and $\pi_{\text{RR}}=-8$ dB the IR-ST scheme outperforms FD-BM even in terms of throughput, specially in the high

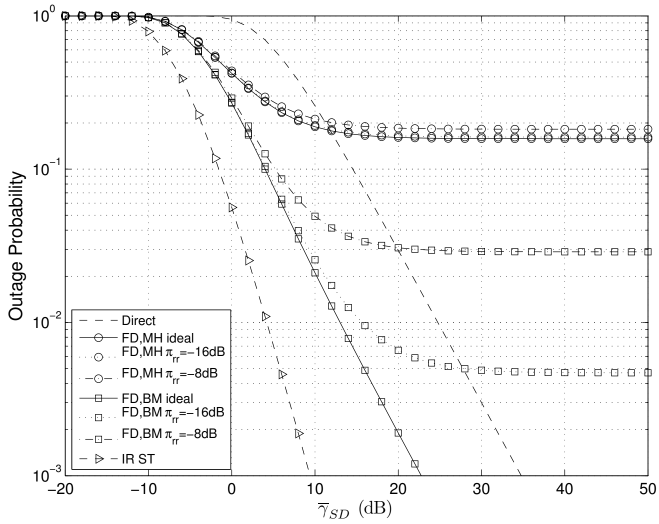

Fig. 2. Outage probability as a function of the SNR for $\mathcal{R}$ =2 bits/s/Hz.

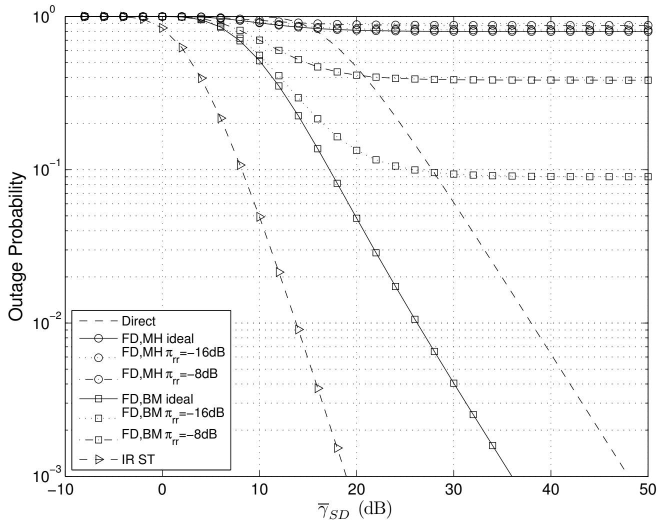

Fig. 3. Outage probability as a function of the SNR for $\mathcal{R}$ =6 bits/s/Hz.

SNR region. Moreover, the performance of the simple FD-MH scheme can be considerably worse than that of the other methods.

A. Power and Rate Allocation

Now, we investigate the impact of allocating power between source and relay, and also of allocating rate. The choice of power and rate is such that maximizes the throughput. The problem can be formalized as:

$$\begin{aligned} \max_{\mathcal{R},P_{\mathrm{S}}^{\star}} \quad & \mathcal{T} \\ \text{subject to} \quad & P_{\mathrm{S}}^{\star}+P_{\mathrm{R}}^{\star}\leq 2P \\ & \mathcal{R}_{min}\leq\mathcal{R}\leq\mathcal{R}_{max} \end{aligned} \tag{34}$$

where $\mathcal{T}$ can be $\mathcal{T}_{\text{FD,MH}}$, $\mathcal{T}_{\text{FD,BM}}$, $\mathcal{T}_{\text{IR-ST}}$ or $\mathcal{T}_{\text{dir}}$, and P is the used power for direct transmission. The maximization can be performed with respect to $\mathcal{R}$, $P_{\mathrm{S}}^{\star}$, or both.

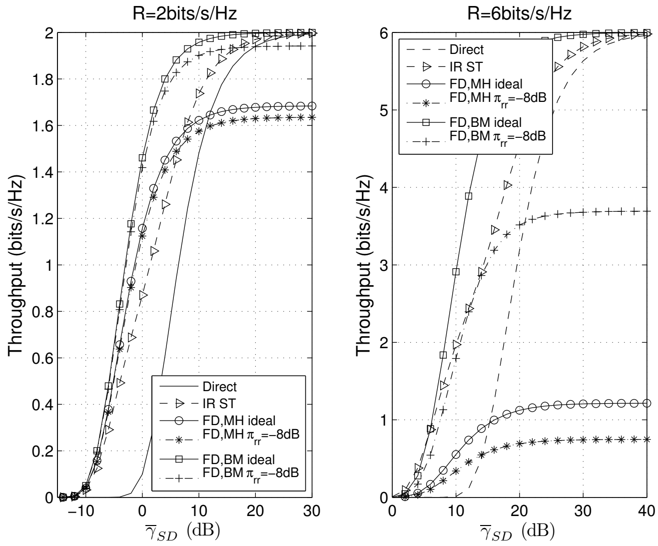

Fig. 4. Throughput as a function of SNR for $\mathcal{R}$ = 2 bits/s/Hz and $\mathcal{R}$ =6 bits/s/Hz.

Since our goal is to compare the different HD and FD schemes, we do not focus on the proposal of a particular PA and RA solution, but we resort to numerically efficient algorithms. For RA we considered that $\mathcal{R}$ could vary from $\mathcal{R}_{min}$ = 1 bits/s/Hz to $\mathcal{R}_{max}$ = 10 bits/s/Hz. At each SNR value we numerically determine the attempted rate $\mathcal{R}$ which maximizes the throughput. For the PA, we determine the values of $P_{\mathrm{S}}^{\star}$, and therefore $P_{\mathrm{R}}^{\star}$ since $P_{\mathrm{S}}^{\star}+P_{\mathrm{R}}^{\star}=2\,P$, which maximize the throughput. When PA and RA are carried out at the same time, the two parameters ($P_{\mathrm{S}}^{\star}$, and $\mathcal{R}$) are jointly numerically optimized.

Fig. 5 shows the throughput with PA and RA. From the figure, we can see that the ideal FD-BM relaying outperforms IR-ST. Nevertheless, when we consider the loop interference, the performance of IR-ST becomes very competitive, specially from the mid to high SNR region. As we know from Fig. 3, at high spectral efficiency or $\pi_{\text{RR}}\neq 0$ dB the outage probability of the FD-MH scheme is close to the unit. Therefore, the performance of FD-MH even with PA and RA is worse than the direct transmission.

In Fig. 6 we present the power allocated to S as a function of the SNR. We can notice that most of the available power is allocated to the source ($P_{\mathrm{S}}^{\star}$) in the IR-ST and FD-BM schemes, consequently less power is allocated to the relay. However, in the FD-MH scheme $P_{\mathrm{S}}^{\star}$ decreases and more power is allocated to the relay. In terms of RA, both schemes perform alike, except for the FD-MH scheme which employs a less aggressive RA strategy, due to the reasons discussed before. To reinforce this conclusion on the FD-MH scheme we show in Fig.7 the error floor as a function of the spectral efficiency for FD schemes. In the FD-MH case, the outage probability floor gets close to the unit even for high spectral efficiency.

V. Final Comments

We analytically evaluated the performance of some FD and HD cooperative schemes in terms of outage probability and

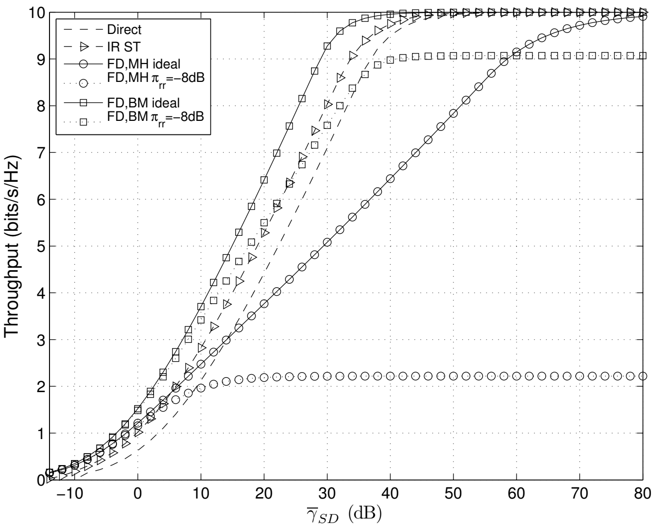

Fig. 5. Throughput as function of SNR with PA and RA.

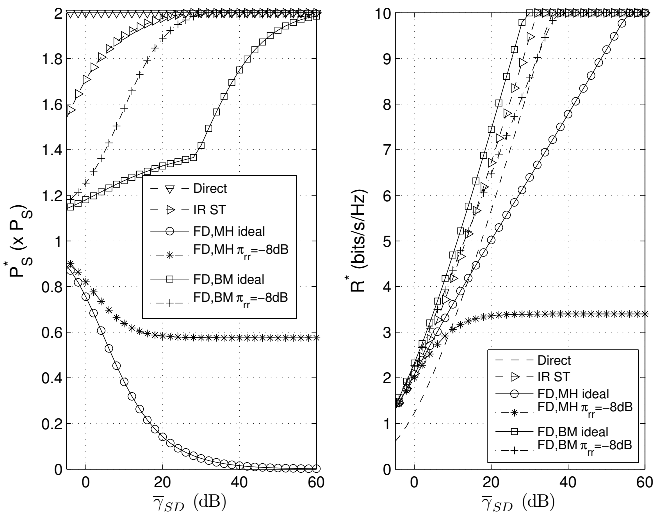

Fig. 6. Power (left) and rate (right) allocated to the source.

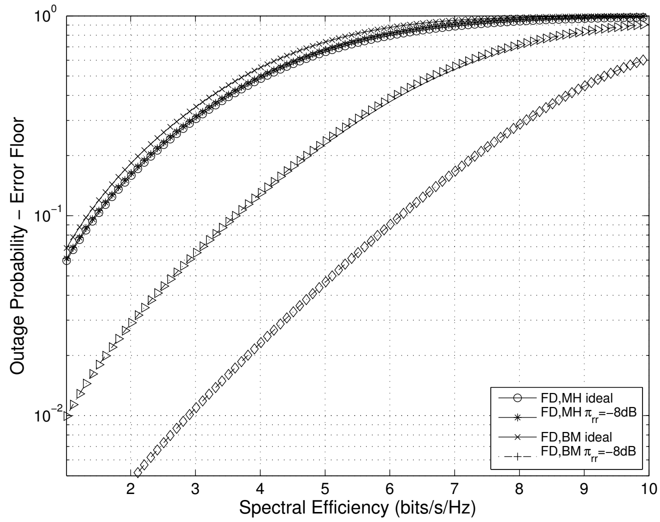

Fig. 7. Outage probability - (Error Floor) as a function of spectral efficiency, $\mathcal{R}$, for FD schemes.

throughput, under power and rate allocation. In the FD schemes we considered a practical relay model which takes into account the loop interference caused by the relay transmitted signal into the relay received signal. Our results show that only the ideal (without loop interference) Block Markov FD relaying outperforms the IR-ST HD scheme. When the loop interference is considered, the IR-ST HD relaying scheme can outperform FD relaying, even when resource allocation is carried out. As a future work we intend to investigate the impact of several issues as: relay position; relay selection; network topology; etc.

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