Guided Search MIMO Detectors Aided by Lattice Reduction

Yuri Mendes Mostagi, Taufik Abrão & Paul Jean E. Jeszensky

***Abstract*— Under MIMO channels, the matched filter detection becomes inefficient to deal with high data throughput demanding systems. The performance or system capacity under conventional detection will be substantially degraded when the spatial diversity provided by multiple antennas can not be fully exploited and the detection process is unable to efficiently separate the signal from each antenna. The solution discussed in this paper seeks to establish more efficient detectors for MIMO systems with the aid of the lattice reduction (LR) technique. These detectors use information from the interfering signals in a way to improve the signal detection in the antenna of interest, thus providing advantages over the conventional system, at the expense of increasing complexity. The focus of this paper consists in comparing the characteristics of three representative sub-optimal detectors based on the maximum-likelihood function as well as on the guided search principle, previously analyzed in [1]. In this way, the complexity $\times$ performance trade-off for the sphere detector (SD), the QR decomposition-based detector (QRD) the greedy search detector (GSD) and its variants, all of them aided (or not) by the LR technique are analyzed and its potential of use in MIMO systems is put in perspective.**

***Keywords*— MIMO systems, ML estimation; sub-optimum detection, search algorithms, Lattice Reduction.**

I. Introduction

Systems with multiple transmitting antennas and multiple receiving antennas (MIMO) present a remarkable performance degradation under conventional detection process, which consist of the matching filter to the signal of each propagation branch between the transmitting and receiving antennas, due to the combination of effects of interference on the signal between the antennas, as well as the possible correlation between the received fading signals. Thus, the conventional receiver becomes inefficient in MIMO systems that require high data throughput (multiplexing gain).

There are well established solutions in literature to circumvent this problem, all of them consisting of maximum-likelihood detectors (MLD), which by transmitting the same information symbol over all antennas (diversity gain), improves the individual detection on each receiving antenna following by efficiently combining the signals from each one (spatial diversity), or under other hand, to use each antenna to transmit different symbols, and thus providing higher data throughput, providing in both case a clear advantage over the conventional SISO systems.

However, the MLD, which consists of a conventional receiver followed by a maximum-likelihood sequence detector, is impractical due to the fact that its complexity increases exponentially with the number of antennas (or users or problem dimension). Therefore, new methods have been proposed in order to overcome these disadvantages.

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The focus of this work are the sub-optimum guided search detectors for MIMO systems based on the maximum likelihood function. Among these, stand out the sphere detector (SD), the QR decomposition based detector (QRD-M) and the greedy search detector (GSD), and its association to the lattice reduction, in a way to improve the system performance and/or to reduce the complexity of this MIMO detectors, specially when occurs the combination of higher order modulation with large number of antennas.

II. System Model

The linear MIMO channel is defined by a generic transmitter transmitting simultaneously (in one symbol period, T_s) m symbols, $s_1, \ldots, s_m$, of a finite alphabet or constellation $\mathcal{D} \subset \mathbb{C}$. At the receiver, there are n signals, $y_1, \ldots, y_n$, one in each receiver antenna, received as a linear combination of the m input symbols plus the additive noise. It is usually assumed in the literature that the number of received signals in the n antennas exceeds the number of symbols transmitted by the m antennas, i.e., $n \geq m$. This ensures that the equations used in the detection process will not be under-determined [2]. The linear MIMO channel is described in a matrix notation as:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{v} \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{n\times m}$ is the channel matrix and $\mathbf{v} \in \mathbb{C}^{n}$ is the additive noise. The vectors $\mathbf{s} \in \mathcal{D}^{m}$ and $\mathbf{y} \in \mathbb{C}^{n}$ represents the transmitted symbols and the received signals, respectively [3]. If $\mathbf{H}$, $\mathbf{s}$, $\mathbf{y}$ and $\mathbf{v}$ are complex matrix and vectors they can be rewritten as:

$$\boldsymbol{\mathcal{H}} = \begin{bmatrix} \Re\{\mathbf{H}\} & -\Im\{\mathbf{H}\} \\ \Im\{\mathbf{H}\} & \Re\{\mathbf{H}\} \end{bmatrix} \quad \boldsymbol{\mathcal{S}} = \begin{bmatrix} \Re\{\mathbf{s}\} \\ \Im\{\mathbf{s}\} \end{bmatrix}$$

$$\boldsymbol{\mathcal{Y}} = \begin{bmatrix} \Re\{\mathbf{y}\} \\ \Im\{\mathbf{y}\} \end{bmatrix} \quad \boldsymbol{\mathcal{V}} = \begin{bmatrix} \Re\{\mathbf{v}\} \\ \Im\{\mathbf{v}\} \end{bmatrix}$$

where $\Re\{.\}$ and $\Im\{.\}$ are the real and imaginary operators, respectively [4].

Since the focus of this work is the MIMO detection, the channel matrix $\mathbf{H}$ will be considered perfectly known at the receiver, its complex values are described by a Rayleigh distribution for the magnitude and by an Uniform distribution for the phase. The transmitted symbols are modeled as random independent and identically distributed (*i.i.d*) variables over an alphabet of the constellation $\mathcal{D}$. The noise is modeled by a complex and circularly symmetrical Gaussian distribution, with zero mean and variance σ^2. The objective of the receiver is to estimate $\mathbf{s}$ from $\mathbf{y}$ and $\mathbf{H}$.

The maximum likelihood detector or optimum detector operationalizes, from (1), the test of all possible combination of symbols transmitted from m antennas, applying all possible values of candidate symbol $\mathbf{s}$ to the minimization of a cost function based on minimum Euclidean distance from $\mathbf{s}$ to the received signal, expressed by:

$$\hat{\mathbf{s}} = \min_{\mathbf{s}\in\mathcal{D}^m} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 \qquad (2)$$

However, this strategy results in exponential complexity with respect to the number of antennas and constellation size. If the constellation size of transmitted symbols is $\mathcal{M}$, e.g. in BPSK modulation, $\mathcal{M} = 2$, and exists m transmitting antennas, the detector need to search over a set of size $\mathcal{M}^m$. Under high order modulation formats, this complexity becomes prohibitive even for a moderate number of transmitting antennas [5].

III. Lattice Reduction (LR)

The LR is a mathematical concept utilized to solve many problems involving point lattices. In signal processing, specifically, the constellation formed by the symbols of a modulated signal can be seen as a lattice; in this way, with the LR one seeks for better ways to represent a lattice [6]. In the MIMO signal detection the LR can be used to improve the conditioning of the channel matrix, thus allowing to use simpler detectors, and consequently less computationally complex, maintaining acceptable performance, and also reducing the complexity of near optimum detectors.

The LR is performed in the pre-detection phase, by generating an uni-modular matrix $\mathbf{T}$ that multiplied by $\mathbf{H}$ results in a modified channel matrix with columns closer to the orthogonality condition, this matrix represents a signal basis with lower order than the original matrix $\mathbf{H}$ [6].

There are many definitions for the LR, depending on the reduction criteria adopted. In this work the chosen algorithm is the Lenstra-Lenstra-Lovász reduction (LLL or L^3) [7]; according to [6], LLL reduction shows a good trade-off between the quality of the results and complexity. The LLL uses the QR decomposition, reflections, translations and exchanges of the columns of the channel matrix, in a iterative way to obtain the channel matrix with reduced basis.

The algorithm depends on the δ parameter, with $\frac{1}{4} < \delta \leq 1$. The choice of the δ value affects the quality of the reduced basis and its computational complexity. Bigger values of δ results in better basis at cost of a higher complexity; a common choice is $\delta = \frac{3}{4}$, as suggested in [6]. Furthermore, the symbols vector $\mathbf{s}$, and the channel matrix $\mathbf{H}$, are transformed to a reduced basis applying the uni-modular matrix $\mathbf{T}$:

$$\mathbf{z} = \mathbf{T}^{-1}\mathbf{s} \qquad (3)$$

$$\widetilde{\mathbf{H}} = \mathbf{H}\mathbf{T} \qquad (4)$$

Finally, the matrix form of the MIMO channel with the channel matrix of reduced basis $\widetilde{\mathbf{H}}$ can be obtained:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{v} \Rightarrow \mathbf{y} = (\mathbf{H}\mathbf{T})\left(\mathbf{T}^{-1}\mathbf{s}\right) + \mathbf{v} \Rightarrow \mathbf{y} = \widetilde{\mathbf{H}}\mathbf{z} + \mathbf{v} \qquad (5)$$

Thus, with this new matrix form to describe the channel, from eq. (2) one can obtain a new minimization function for the MLD detector and the others analyzed in this work:

$$\hat{\mathbf{s}} = \min_{\mathbf{s}\in\mathcal{D}^m} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 \Rightarrow \hat{\mathbf{z}} = \min_{\mathbf{z}\in\mathcal{D}^m} \|\mathbf{y} - \widetilde{\mathbf{H}}\mathbf{z}\|^2 \qquad (6)$$

From the eq. (6), the structure of all the detector analyzed here can be modified to work with the LR.

IV. Sub-optimum Guided Search Detectors

A. MIMO Sub-optimum Detector Based on QR Decompositon

From eq. (2), we apply the $\mathbf{QR}$ decomposition [8] to the channel matrix $\mathbf{H}$

$$\mathbf{H} = \mathbf{Q}\begin{bmatrix}\mathbf{R}\\ \mathbf{0}\end{bmatrix}$$

where $\mathbf{Q} \in \mathbb{C}^{n\times m}$ is an orthogonal matrix, $\mathbf{R} \in \mathbb{C}^{m\times m}$ is an triangular superior matrix, and $\mathbf{0}$ is a matrix of zeros, dimension $(n-m)\times m$.

The QR decomposition of $\mathbf{H}$ is an orthogonal reduction to a triangular superior form. From the relation $\mathbf{H} = \mathbf{QR}$ and from the non-singularity of $\mathbf{R}$, we can conclude that the columns of $\mathbf{Q}$ forms an orthogonal base for $\mathcal{R}(\mathbf{H})$, where $\mathcal{R}(.)$ is the vectorial space operator. In this way, the matrix $\mathbf{P} = \mathbf{Q}\mathbf{Q}^{\mathbf{T}}$ is the orthogonal projection in $\mathcal{R}(\mathbf{H})$. Note that $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}$, with $\{.\}^H$ being the transpose conjugate operator and $\mathbf{I}$ is the identity matrix. In this way, pre-multiplying (2) by $\mathbf{Q}^H$ results in a tree shaped structure with depth m due to the triangular property of the $\mathbf{R}$ matrix.

$$\|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 = \|\mathbf{y} - \mathbf{QR}\mathbf{s}\|^2 = \|\mathbf{Q}^H\mathbf{y} - \mathbf{R}\mathbf{s}\|^2 \qquad (7)$$

To simplify, let be $\mathbf{x} = \mathbf{Q}^H\mathbf{y}$. So, the minimization problem becomes:

$$\hat{\mathbf{s}}_{\mathrm{ML}} = \min_{\mathbf{s}\in\mathcal{D}^m} \|\mathbf{x} - \mathbf{R}\mathbf{s}\|^2 \qquad (8)$$

After applying the QR decomposition and pre-multiplying by $\mathbf{Q}^H$, the M algorithm is applied to estimate the symbols in a sequential way [9], as described below.

1) The M Algorithm: Beginning with the last element of $\mathbf{s}$, s_m, the algorithm calculates the metric in (8) for all possible values of $s_m \in \mathcal{D}^m$ using

$$|x_m - r_{m,m}\hat{s}_m|^2, \qquad (9)$$

where $r_{m,m}$ is the (m,m)th element of $\mathbf{R}$. The metrics of this nodes are ordered and it holds only the M nodes with smaller values; the others are discarded. The surviving nodes are then extended to each of $\mathcal{M}$ symbols, resulting in $M\mathcal{M}$ branches; again, from this new branches, only the M branches with the smaller values is saved and then expanded again to more $\mathcal{M}$ branches, until the process reaches the last layer (m). Fig. 1 illustrates the process for a system with $m = n = 3$, $M = 2$ branches, and quaternary modulation, $\mathcal{M} = 4$.

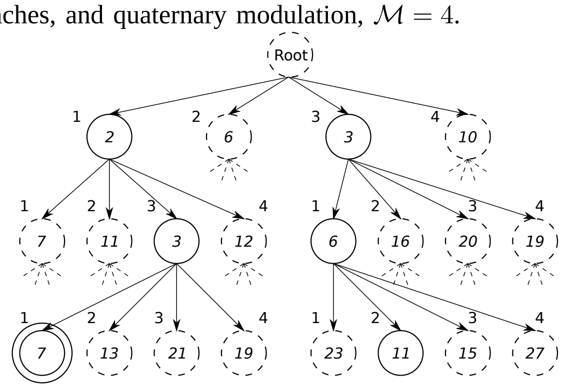

Fig. 1. M algorithm. The numbers outside the circles are the symbols of the constellation (nodes), those inside the circles are the accumulated metric until that node. The solid circles are the M nodes chosen by the algorithm in each layer, while the doted ones are the excluded nodes. Doted lines indicate the not expanded branches. The double lined circle shows the obtained solution.

The branches values are calculated utilizing a modified minimization function metric from (8). For a tree with length i, $1 \leq i \leq m$, the metric for each branch becomes:

$$|x_{m-i+1} - \mathbf{R}_{m-i+1}\bar{\mathbf{s}}_i|^2, \tag{10}$$

where x_i is the i-th element of $\mathbf{x}$, $\mathbf{R}_i$ is the i-th row of $\mathbf{R}$ and $\bar{\mathbf{s}}_i$ is the vector with the appropriated nodes of a particular branch.

B. Sub-optimum MIMO Sphere Detector

The sphere detector (SD), searches over the nodes $s \in \mathcal{D}^m$ of the lattice that are inside of a hyper-sphere of radius d, centered at the received vector $\mathbf{y}$ [10]. In this way, the search space is smaller and as a consequence, the final computational complexity is smaller too.

The SD must determine which points of the constellation (nodes) are inside of the search sphere, although if the detector have to test the Euclidean distance of all nodes $\mathbf{s}$ in order to determine which one lies on the search sphere of radius d, an exhaustive search still exists. Hence, it is hard to determine which lattice nodes lay inside of the *m*-dimensional sphere, but it is trivial to do it in a uni-dimensional case $m = 1$. In this way, the algorithm can go from the dimension k to the dimension $k+1$. This means that the nodes at the dimension m and radius d can be determined iteratively by determining all the nodes contained into hyper-spheres of smaller dimensions $(1, 2, \ldots, m)$ and the same radius d. Consequently, the SD search method can be represented by a tree, as in the case of the QRD-M, the branches of the k-th layer of the tree corresponds to the nodes of the lattice that lays inside the sphere of radius d and dimension k [11]. An example of the SD search in a system with $m = n = 3$, search radius $d = 6$, and binary modulation can be seen in the Fig. 2.

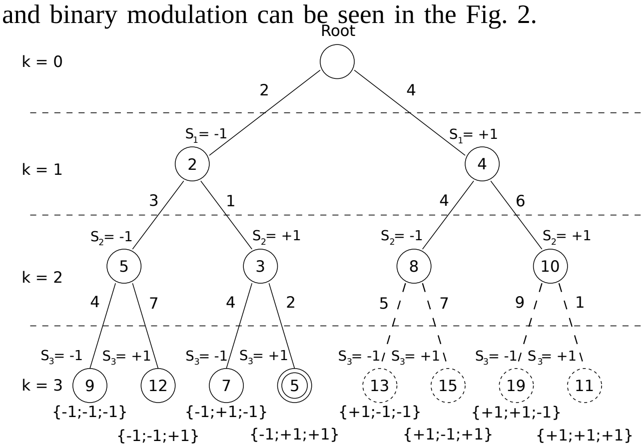

Fig. 2. SD search tree, $d = 6$. Numbers at the side of each branch are lengths; numbers inside the nodes are the accumulated metrics; double lined circle indicates the optimal solution; doted branches are non-visited nodes.

The point $\mathbf{Hs}$ is inside the sphere of radius d if and only if:

$$d^2 \geq \|\mathbf{y} - \mathbf{H}\hat{\mathbf{s}}\|^2 \tag{11}$$

So, the main problem must be broken into sub-problems; instead of trying to determine the points of the constellation that lay inside of the hyper-dimensional search sphere, it determine the points that lay inside of multiple uni-dimensional spheres. In order to proceed that, the QR decomposition is applied to $\mathbf{H}$ in (11). From the pre-multiplication by $\mathbf{Q}^H$, as in (8), one can immediately obtain:

$$\|\mathbf{x} - \mathbf{R}\hat{\mathbf{s}}\|^2 \leq d^2 \tag{12}$$

The SD core consists of the enumeration method [12], which enumerates the possible symbols that lay inside the sphere, based on the conditional observation:

$$\begin{aligned} \text{If :} \quad & \mathbf{p} \stackrel{\triangle}{=} \mathbf{x} - \mathbf{R}\hat{\mathbf{s}}, \\ \text{Then:} \quad & \|\mathbf{p}_{l_k}\|^2 \;>\; d^2 \;\Rightarrow\; \|\mathbf{p}\|^2 \;> d^2 \end{aligned} \tag{13}$$

where $\mathbf{p}_{l_k} \in \mathbb{C}^k$ is the vector composed by the last k components of $\mathbf{p}$. Hence, at each new iteration, the algorithm executes a search with depth k in the tree of search with m layers: for $k = 1$, $\mathbf{p}_{l_k}$ will be composed by the component m of $\mathbf{p}$; for $k = 2$, $\mathbf{p}_{l_k}$ will be composed by the components m and $m-1$ of p, and so on. Due to the upper triangular structure of $\mathbf{R}$, the vector $\|\mathbf{p}_{l_k}\|$ will depends only on $\hat{\mathbf{s}}_{l_k}$, where $\hat{\mathbf{s}}_{l_k} \in \mathcal{D}^k$ is the vector composed by the last k components of $\hat{\mathbf{s}}$. Hence, stating that for some vector $\hat{\mathbf{s}} \in \mathcal{D}^m$ with index k, $\|\mathbf{p}_{l_k}\|^2 > d^2$, any other vector $\tilde{\mathbf{s}} \in \mathcal{D}^m$ for which $\tilde{\mathbf{s}}_{l_k} = \hat{\mathbf{s}}_{l_k}$ can be excluded from the search. The SD uses this observation to enumerate in an efficient way all the points in the hyper-sphere supplied by the equation (12). After this enumeration the vector with the possible symbols is saved, and those that have the lower value based on the modified MLD equation (8) will be the chosen as the output symbol of the algorithm.

1) Radius of the Sphere: In order to achieve high efficiency with SD, a critical parameter namely the radius of the search sphere (d) must be adjusted, or in case of an algorithm with iterative upgradable search radius, the initial radius. It is essential to define d carefully, therefore, in the case of it being too large the search will result in a exponential complexity with the number of antennas or users, without showing any advantage over the MLD. In other way, in the case of a too small radius, the algorithm will have a great chance of not finding any point inside the search sphere.

A better way to determine the radius is the pruning procedure, when the algorithm reaches the last node of a branch of the search tree with an accumulated metric M, we can suppose that the solution of (8) must be inside of the sphere $\|\mathbf{x} - \mathbf{R}\hat{\mathbf{s}}\|^2 \leq M$. So, in the case of $M < d$ the algorithm can make $d = M$, and continue the search with a smaller search radius. With this method the search tree applies successive pruning which is able to reduce the number of visited nodes in comparison to the original [3]. Under pruning procedure, the initial radius is defined as $d = \infty$, and then it is updated every time that the algorithm finds a branch that have the Euclidean distance from the received vector smaller than the actual search radius. Furthermore, the critical task of finding an appropriate initial radius, and a function to update the radius are eliminated.

C. Sub-optimum MIMO Greedy Search Detector

In a same way as the other detectors of this work the greedy search detector (GSD) starts with the QR decomposition of the channel matrix ($\mathbf{H}$), taking into account eq. (8). The GSD takes advantage of the superior triangular characteristic of $\mathbf{R}$ to calculate the Euclidean distance step-by-step, from the antenna

m until the first one, as shown in Fig.3. In this flow chart, the nodes represents the modulation symbols. The GSD method is organized into m stages, each stage represents one antenna; at each stage exists C nodes. Each node is connected to C nodes of the previous stage and to C nodes of the next stage, with exception of the stage 1 and m, because in stage 1 does not have a previous stage and is connected to the root node; in the stage m has no next stage and it is connected to the final node. Between this nodes there is the partial Euclidean distance, i.e., the metric of the previous nodes added to the metric of the current node, until that at the end of the mth stage, the algorithm is able to compute the total metric for a specific candidate vector.

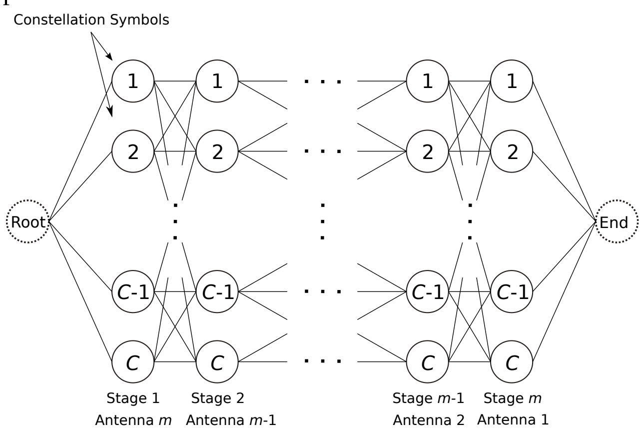

Fig. 3. Flow chart of the GSD for a generic system with m antennas and modulation with C symbols.

Two distinct phases will be executed in the GSD detection process: a) the phase of nodes reduction, followed by b) the phase of branches extension [13]. On the first one a reduction of the quantity of candidate nodes is performed through a tree search similar to that performed by the SD and QRD-M. On the second phase the branches extension is performed, i.e., from the last stage to the first, the algorithm performs, at the current stage, a symbol swap by the others possible symbols, been capable to form a list of vectors from where it will select the one that better satisfies (8).

V. Performance and Complexity Analysis

Considering a wireless MIMO communication system with $m = n = 4$, high order modulation (16-QAM) under flat Rayleigh fading, Monte-Carlo simulations results have been obtained in order to analyze and compare the performance-complexity trade-off of the three detectors with and without the LR technique aiding.

A. Performance of the MIMO Detectors

Fig. 4 presents the symbol error rate (SER) versus the SNR curves. Unlike the results analyzed in [1], where QPSK modulation was utilized, in this work, the GSD under high order modulation has been shown unsatisfactory performance, even under LR aiding, the GSD was able to achieve only a marginal performance improvement. On the other hand, the QRD-M had been able to achieve near-ML performance for $M = 128$, as one can see from Fig. 4. However, with $M = 127$, the performance of this detector was far from near-ML performance; even with the aid of the LR, the improvement of performance was only marginal. The performance of the SD remained near-ML, as expected. In this way, on the LR aided version of the SD (LR-SD), the improvement can be noted in terms of complexity.

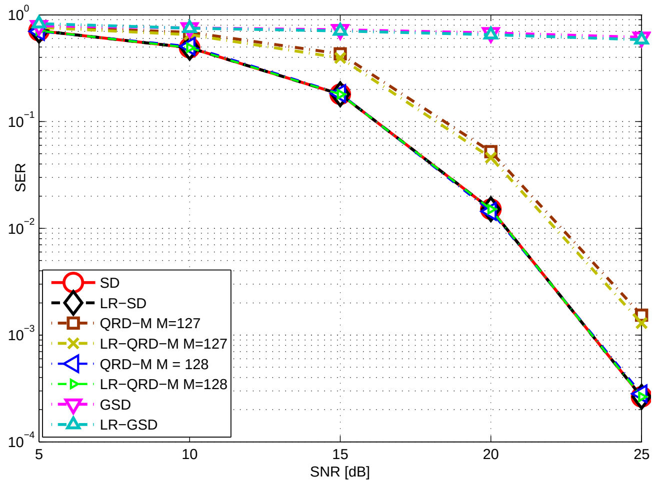

Fig. 4. SER for the SD, QRD-M and GSD detectors in $m = n = 4$ system and 16-QAM modulation.

B. Computational Complexity

The complexity of the MIMO detection algorithms with and without the aid of LR was analyzed in terms of the real operations terms. The three algorithms discussed herein use the QR decomposition procedure in its guided search mechanisms. The complexity of the LR is in the first line of Tab. I; the LR uses the QR decomposition so can be seen that its complexity is very close to the QR decomposition itself. Hence, the final complexity of the detectors aided by the LR is the detector's complexity plus the complexity of the QR decomposition and the LR procedure.

From Tab. I, it is evident that the QR decomposition complexity, with $\mathcal{O}(m^3)$ order, is dominant in determining the complexity of the central search functions of the QRD-M and GSD, that show a complexity of order $\mathcal{O}(m^2\mathcal{M}M)$ and $\mathcal{O}(m^2\mathcal{M}^2)$, respectively. Furthermore the SD in it's best case (lower complexity), show a complexity of order $\mathcal{O}(m^2\mathcal{M})$.

TABLE I

Real Operations Complexity for the three MIMO detectors.

LR	$\frac{2}{3}m^3 + 29m^2 + \frac{5}{3}m - 3$
QR	$\frac{2}{3}m^3 + m^2 + \frac{1}{3}m - 2$
QRD-M	$\left[\left(m^2 + 7m\right)M + 6\right]\mathcal{M}$
GSD	$\left(m^2 + 7m\right)\mathcal{M}^2 + \left(m^2 + 5m + 6\right)\mathcal{M}$
SD	$\left(m^2 + 5m\right)\mathcal{M}\gamma^m$

Since the QR decomposition is a common step to the three detectors, the complexity of this step is prominent. For the remaining steps, the QRD-M shows a fixed complexity in relation to the SNR, depending only on the M size, quantity of transmitting antennas, m, and the modulation order, $\mathcal{M}$. Obviously the higher M is, more branches will be expanded, higher will be the complexity but better will be the quality of the solution given by the algorithm. On the other hand, the GSD also has fixed complexity regarding the SNR, being dependent only on the m and $\mathcal{M}$.

The SD complexity is variable, stochastic and dependent of the channel condition and noise level [10], besides the quantity of antennas and modulation order. Depending on the combination of this factors, the complexity can be incremented from quadratic polynomial (best case) to exponential [14]. Therefore, it is hard and complex to determine a closed expression to describe the SD complexity. The expression obtained here is a simplifying expression, however it is sufficient to accomplish the proposed analysis for this work. Hence, Tab. I presents in addition to the m and $\mathcal{M}$, the parameter γ, that represents the dependence of the SD to the channel conditions (or noise level), being $\gamma \propto \mathrm{SNR}^{-1}$.

Thus, interesting, even when one adds the complexity of the LR to the complexity of SD, what, apparently raises the total complexity, in practice it does not happens, due to the LR effect over the detection step. Therefore, due to the LR procedure, the reduced channel matrix is closer to the orthogonality condition, requiring a fewer branches expansions in order to the algorithm find the near-ML solution. This can be seen in Fig. 5, which presents, in a complementary way to the Tab. I, the equivalent operations obtained from the computational time taking into consideration the MatLab implementations of the three detectors. This computational time has been converted into the number of equivalent real operations, simply dividing this time by the average time necessary to the execution of one real sum.

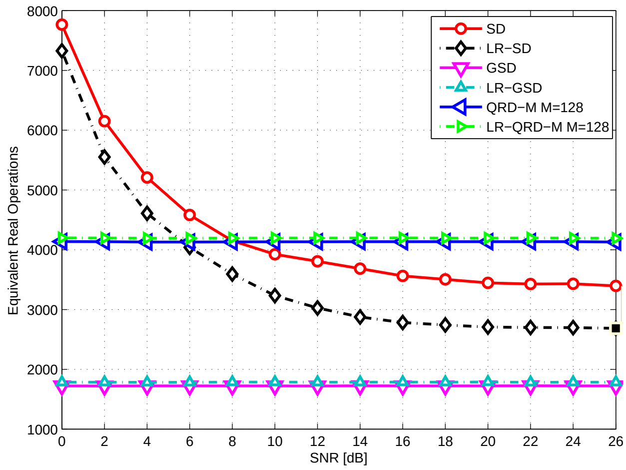

Fig. 5. Equivalent Real Operations obtained from the equivalent computational time *versus* SNR. System with $m = n = 4$ and 16-QAM modulation.

From the Fig. 5 one can see the advantage of the LR-SD over the SD; i.e., for the SNR $= 26$ dB, the LR-SD performed nearly 20% less operations. At a lower SNR level this proportion is hold, while for the lowest SNR region, the difference is slightly smaller, but the LR-SD always presents advantage in terms of computational complexity reduction regarding the SD with no LR aiding.

However, with the LR aiding the other detectors have presented slightly higher complexities; this occurs due to the addition of operations caused by the operations of the LR, that is not compensated by the amount of extended branches, since that in this detectors the number of expanded branches is fixed. In the QRD-M case, however, it was expected that with the LR aid would be possible to use lower M values in order to achieve near-ML performance with lower complexity. Thus, one can conjecture that in a system with a higher order modulation and/or higher quantity of antennas the reduction of the complexity or improvement in the performance of a MIMO QRD-M receiver, when aided by the LR, will be more remarkable. Even so, in low SNR situations, the LR QRD-M complexity remain lower than the SD one, been an option for a system operating under this configuration.

In terms of performance and complexity, the numerical results allows us to conclude that among the analyzed MIMO detectors, the SD remains as the best option. With the LR aiding, its complexity is substantially reduced, making its use even more advantageous in practical communication systems.

VI. Conclusion

In order to find alternatives to the exponential complexity inherent to the MLD detector, three sub-optimal detectors suitable for MIMO systems have been analyzed, with(out) the LR aiding. The QRD-M, GSD and SD algorithms were analyzed in terms of SER performance $\times$ SNR, as well as computational complexity, characterized by the number of real operations required. The best performance-complexity trade-off was obtained by the LR-SD, which presented near-ML performance with lower complexity regarding the one of the SD, QRD-M and LR-QRD-M. The GSD presented poor performance, been not suitable for the system configurations analyzed. The QRD-M was suitable only for low SNR situations, where its complexity results lower than the SD and LR-SD.

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