Enhancement of Data Rate for User Cooperation

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***Abstract*— In this paper, we propose a different scheme based on user cooperation diversity, in which we obtain an increase in the data rate. In particular, we present performance analysis using conventional CDMA (code division multiple access) implementation for two users. Cooperation among users has been shown to achieve significant gains as compared to a non-cooperative system. The results presented here show that the new scheme of cooperation achieves substantial increase of data rate, keeping the average bit error probability close to values obtained in [3]. Regarding the number of spreading codes, there are costs associated with our cooperative scheme, but even so our proposed strategy leads not only to an increase in throughput but also to a simpler system.**

Keywords*— *user cooperation, virtual MIMO, bit error probability.

I. INTRODUCTION

In wireless communications, the effects of multipath fading include constructive and destructive interference, which causes the signal attenuation to vary significantly over the course of a given transmission. In this case, diversity [1] plays an important role in combating the fading. Some well-know forms of diversity are spatial diversity, temporal diversity, and frequency diversity [4], in particular spatial diversity is specially effective at mitigating these multipath situation. Therefore, having multiple transmit antennas is desirable due to the advantages they can provide, but it may not be practical due to size, cost, or hardware limitations, specially in the uplink of a cellular system.

In recent years, a new method called user cooperation has emerged as a promising technique that allows to form a virtual multi-antenna transmitter and hence to reap the benefits of spatial diversity. The basic concept of cooperative communication was discussed in the pioneering work [2]-[3], where authors explained how diversity gains are achieved via the cooperation of in-cell users. That is, two mobile users in the same cell are responsible for transmitting not only their own information, but also the information of their partner. The cooperative system between two users can be modeled as a multiple access channel with interuser communication capability [5], as shown in Fig. 1. Results show that cooperation leads to an increase in the capacity region for both users as well as to a more robust system. The practical scheme presented in [2] focus on diversity gain in such a manner that each user sends *one* new bit per two symbol periods if they are cooperating.

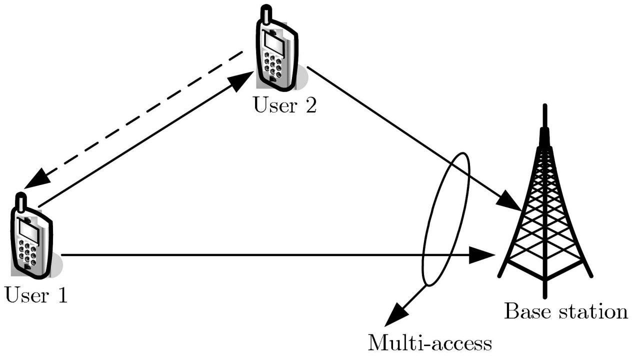

Fig. 1. Two-user cooperative system architecture.

Aiming to increase the data rate, in this paper, we propose a new scheme for user cooperation based on [3]. In fact, this new scheme is similar to implementation in [3], but the principal advantage is a significant increase in the number of transmitted bit/symbol, keeping the same bit error probability. The increase in the data rate comes at a cost of increasing the number of spreading codes used for the users.

II. CHANNEL MODEL

The channel model we use is depicted in [2] which is illustrated in Fig. 2 and can be mathematically expressed as

$$Y_0(t) = K_{10}X_1(t) + K_{20}X_2(t) + Z_0(t) \tag{1}$$

$$Y_1(t) = K_{21}X_1(t) + Z_1(t) \tag{2}$$

$$Y_2(t) = K_{12}X_2(t) + Z_2(t) \tag{3}$$

where $Y_0(t)$, $Y_1(t)$, and $Y_2(t)$ are the baseband models of the received signal at the BS, user 1, and user 2, respectively, during one symbol period. Also, $X_i(t)$ is the signal transmitted by user i under power constraint P_i, for i = 1, 2, and $Z_i(t)$ are white zero-mean Gaussian noise random processes with spectral height $N_i/2$ for i = 0, 1, 2, and the fading coefficients K_{ij} are Rayleigh with mean (ξ_{ij}). We also assume that the BS can track perfectly the variations in K_{10} and K_{20}, user 1 can track K_{21} and user 2 can track K_{12}.

We consider a synchronous system where the mobiles can learn the phase of their respective signals at the BS, either

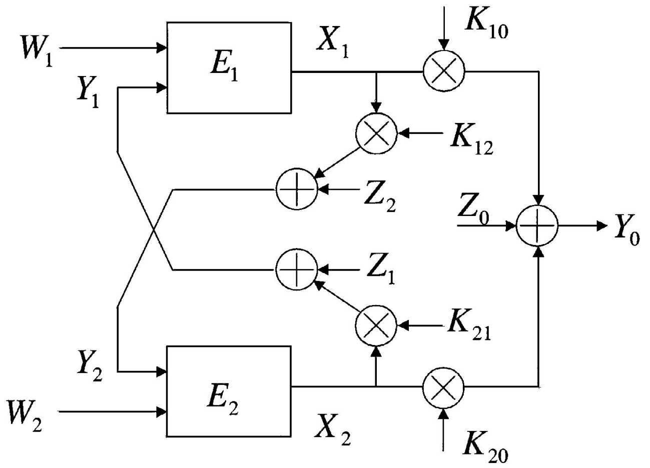

Fig. 2. Channel Model.

through a feedback link or a time-division duplexing (TDD) type system. On the other hand, in an asynchronous system we consider large time-bandwidth, where the receivers can track the phases of the users' signals. Thus, in both of above cases, due to phase knowledge, either at the transmitters or the BS, some residual error arises. However, in this paper, we assume any residual error as negligible and we do not consider it in our analysis.

III. COOPERATION STRATEGY

Our cooperation strategy is based on a conventional CDMA system. For a given coherence time of 2 symbols, the transmitted signal can be expressed as

$$\begin{aligned} X_1(t) &= \begin{cases} a_{11}b_1^{(1)}c_1(t)+a_{12}b_1^{(2)}c_3(t) \\ a_{13}b_1^{(1)}c_1(t)+a_{14}\hat{b}_{21}^{(1)}c_2(t)+a_{15}b_1^{(2)}c_3(t) \end{cases} \\ X_2(t) &= \begin{cases} a_{21}b_2^{(1)}c_2(t)+a_{22}b_2^{(2)}c_3(t) \\ a_{23}b_2^{(1)}c_2(t)+a_{24}\hat{b}_{12}^{(1)}c_1(t)-a_{25}b_2^{(2)}c_3(t) \end{cases} \end{aligned} \tag{4}$$

we denote the signal of user 1 $X_1(t)$ and the signal of user 2 $X_2(t)$, and we consider three spreading codes denoted by $c_1(t)$, $c_2(t)$ and $c_3(t)$. The two user's data bits are denoted $b_i^{(j)}$where i = 1, 2 are the user indices and j denotes the time index of information bits, $\hat{b}_{im}^{(j)}$ is the partner's estimate of user i's jth bit, where m is the partner of user i. Factors a_{ij} denote signal amplitudes, and hence represent power allocation to various parts of the signaling, while maintaining the average power constraint of P_i for user i, over two symbol periods. These constraints can be expressed as

$$\begin{aligned} a_{11}^2+a_{12}^2+a_{13}^2+a_{14}^2+a_{15}^2 &= P_1 \\ a_{21}^2+a_{22}^2+a_{23}^2+a_{24}^2+a_{25}^2 &= P_2 \end{aligned} \tag{5}$$

In the first interval, each user transmits their two firsts bits to the BS. Each user detects and estimates the first bit of the partner. In the second interval, both users transmit a linear combination of their two first bits and the partner's bit estimate, each one multiplied by the appropriate spreading code.

IV. CDMA IMPLEMENTATION

According to the above model we must calculate the various bit error probabilities associated with this scheme. We assume a CDMA system with spreading gain N_c and perfect orthogonality among the codes. In addition, in order to facilitate the implementation we will consider equal power allocation strategy.

The easiest way to visualize this is calculating the bit error probability separately. We must remember that each user transmits their two first bits in the first symbol period, and also transmits their two first bits besides the bit partner's estimate in the second symbol period. Therefore, we first calculate the bit error probability of the first bit (bit with cooperation) and then the bit error probability of the second bit (bit without cooperation). Finally, for simplicity, we focus on user 1, without loss of generality (user 2's error probabilities follows by symmetry).

A. Probability of bit error of bit with cooperation

1) Error Rate for First Period: During the first period, each user transmits only their own data, which is received and detected by the BS as well as by the partner. The signal transmited by user 1 is $X_1 = a_{11}b_1^{(1)}c_1 + a_{12}b_1^{(2)}c_3$. It is received by the BS accordig to $Y_0^1 = K_{10}X_1 + K_{20}X_2 + Z_0^1$, and by the partner according to $Y_2 = K_{12}X_1 + Z_2$. The partner uses Y_2 in order to form a hard estimate of $b_1^{(1)}$.

The partner's hard estimate of $b_1^{(1)}$ is given by $\hat{b}_{12}^{(1)} = sign\left((1/N_c)c_1^T Y_2\right)$, resulting in a probability of bit error equals to

$$P_{e_{12}} = Q\left(K_{12}a_{11}\frac{\sqrt{N_c}}{\sigma_2}\right) \tag{6}$$

where $P_{e_{12}}$ is the probability of $b_1^{(1)}$ estimate by user 2, $\sigma_2^2 = N_2/(2T_c)$, T_c is the chip period, and $N_2/2$ is the spectral height of $Z_2(t)$. On the other hand, the BS forms a soft decision statistic by calculating

$$y_0^1 = \frac{1}{N_c}c_1^T Y_0^1 \tag{7}$$

2) Error Rate for Second Period: During the second period, the two users transmit a cooperative signal to the BS, the transmitted signals of the two partners are

$$\begin{aligned} X_1 &= a_{13}b_1^{(1)}c_1 + a_{14}\hat{b}_{21}^{(1)}c_2 + a_{15}b_1^{(2)}c_3 \\ X_2 &= a_{23}b_2^{(1)}c_2 + a_{24}\hat{b}_{12}^{(1)}c_1 - a_{25}b_2^{(2)}c_3 \end{aligned} \tag{8}$$

The BS receives these signals according to $Y_0^2 = K_{10}X_1 + K_{20}X_2 + Z_0^2$ and forms a soft decision statistic by calculating

$$y_0^2 = \frac{1}{N_c} c_1^T Y_0^2 \tag{9}$$

The BS's combined decision statistics for user 1 are given by

$$\begin{aligned} y_0^1 &= K_{10} a_{11} b_1^{(1)} + n^1 \\ y_0^2 &= K_{10} a_{13} b_1^{(1)} + K_{20} a_{24} \hat{b}_{12}^{(1)} + n^2 \end{aligned} \tag{10}$$

where $\hat{b}_{12}^{(1)}$ is user 2's estimate of b_1, with an error probability given by (6). Also, n^1 and n^2 are statistically independent and both distributed according to $N(0,\ \sigma_0^2/N_c)$.

We consider the suboptimum detector proposed in [3], the λ- MRC, given by

$$\hat{b}_1^{(1)} = sign\left(\left[\begin{array}{cc} K_{10}a_{11} & \lambda\left(K_{10}a_{13} + K_{20}a_{24}\right) \end{array}\right] y\right) \tag{11}$$

where $y = \left[\begin{array}{cc} y_0^1 & y_0^2 \end{array}\right]^T \sqrt{N_c}/\sigma_0$ and $\lambda \in [0{,}1]$ is a measure of the BS's confidence in the bits estimated by the partner. The probability of bit error for this detector, given a λ, is given by

$$P_{e_1} = (1 - P_{e_{12}})\, Q\left(\frac{v_\lambda^T v_1}{\sqrt{v_\lambda^T v_\lambda}}\right) + P_{e_{12}}\, Q\left(\frac{v_\lambda^T v_2}{\sqrt{v_\lambda^T v_\lambda}}\right) \tag{12}$$

where $v_\lambda = \left[\begin{array}{cc} K_{10}a_{11} & \lambda\left(K_{10}a_{13} + K_{20}a_{24}\right) \end{array}\right]^T$, $v_1 = \left[\begin{array}{cc} K_{10}a_{11} & \lambda\left(K_{10}a_{13} + K_{20}a_{24}\right) \end{array}\right]^T \sqrt{N_c}/\sigma_0$, and $v_2 = \left[\begin{array}{cc} K_{10}a_{11} & \lambda\left(K_{10}a_{13} - K_{20}a_{24}\right) \end{array}\right]^T \sqrt{N_c}/\sigma_0$.

B. Probability of bit error of bit without cooperation

1) Error Rate for First Period: As describe before, during the first period, each user transmits only their own data. The signal transmited by user 1 is $X_1 = a_{11} b_1^{(1)} c_1 + a_{12} b_1^{(2)} c_3$, and by user 2 is $X_2 = a_{21} b_2^{(1)} c_2 + a_{22} b_2^{(2)} c_3$. It is received by the BS accordig to $Y_0^1 = K_{10}X_1 + K_{20}X_2 + Z_0^1$, thus the BS forms a soft decision statistic by calculating

$$y_0^1 = \frac{1}{N_c} c_3^T Y_0^1 \tag{13}$$

2) Second Period: During the second period, the two users transmit a cooperative signal to the BS. The signal transmited by user 1 is $X_1 = a_{13} b_1^{(1)} c_1 + a_{14} \hat{b}_{21}^{(1)} c_2 + a_{15} b_1^{(2)} c_3$, and by user 2 is $X_2 = a_{23} b_2^{(1)} c_2 + a_{24} \hat{b}_{12}^{(1)} c_1 - a_{25} b_2^{(2)} c_3$. The BS receives these signals according to $Y_0^2 = K_{10}X_1 + K_{20}X_2 + Z_0^2$, and forms a soft decision statistic by calculating

$$y_0^2 = \frac{1}{N_c} c_3^T Y_0^2 \tag{14}$$

Therefore, the BS's combined decision statistic are given by

$$\begin{aligned} y_0^1 &= K_{10} a_{12} b_1^{(2)} + K_{20} a_{22} b_2^{(2)} + n^1 \\ y_0^2 &= K_{10} a_{15} b_1^{(2)} - K_{20} a_{25} b_2^{(2)} + n^2 \end{aligned} \tag{15}$$

where n^1 and n^2 are statistically independent and both distributed according to $N(0,\ \sigma_0^2/N_c)$.

Since we are using equal power allocation strategy, we consider the followings detectors employed by user 1 and user 2, respectively

$$\hat{b}_1^{(2)} = sign\left(y_0^1 + y_0^2\right) \tag{16}$$

$$\hat{b}_2^{(2)} = sign\left(y_0^1 - y_0^2\right) \tag{17}$$

The bit error probability for this detector is given by (see Appendix)

$$P_{e_2} = Q\left(2K_{10}a_{12}\frac{\sqrt{N_c}}{\sigma_0\sqrt{2}}\right) \tag{18}$$

since we consider equal power allocation, all the power allocation factors will assume the same value, thus the probability given by (18) is only valid for this scheme.

V. NUMERICAL RESULTS

In this section, we carry out analytical comparisons to examine the performance of the proposed cooperative system. As it is obvious from the above, we try to improve the strategy in [3], therefore we must compare it with our proposed scheme. It is clear that equal power allocation is not the best way that minimizes the bit-error rate, but it is a good way to compare our system. Moreover, even though the best value for is a function of channel condition, for a fair comparison we will use a fixed value for both schemes.

Initially, Fig. 3 shows the simulation results of the probability of bit error to both schemes, in the case of [3], it considers three symbols periods, each of the periods with an average power of P, in the scheme proposed we consider two symbols periods but with the same average power constraint, that is, we use a total power of $2P$ versus $3P$ to transmit two different bits. Also, in order to estimate the cooperative bit, we are considering the λ-MRC detector given in [3] with $\lambda = 1$. Moreover, due to the reciprocity of the channel, we assume that K_{12} and K_{21} are equal, and for simplicity of analysis that K_{10} and K_{20} are equal also. Thus, the curves in Fig. 3 correspond to the average probability of error of the two first bits of user 1, where we can observe that both of curves have similar performance in terms of bit error probability, and at high SNR region, our proposed system has better performance.

On the other hand, Fig, 4 shows the bit error probability to several values of interuser channel, keeping the uplink channels constant. We can see clearly that cooperative mode is not beneficial in all cases. When fading coefficients of interuser channel and uplink channel are close, it is better to avoid cooperation. Nullifying the factors $a_{14} = 0$ and $a_{24} = 0$ turns the system noncooperative. Therefore, we do not need to vary the number of periods to obtain a noncooperative

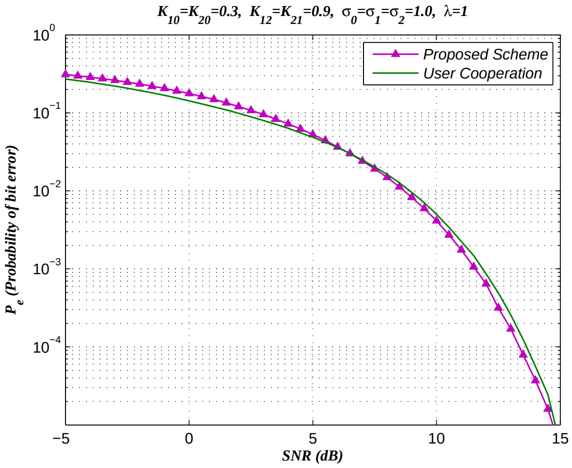

Fig. 3. Comparison of the probability bit error between user cooperation diversity implementation and proposed scheme.

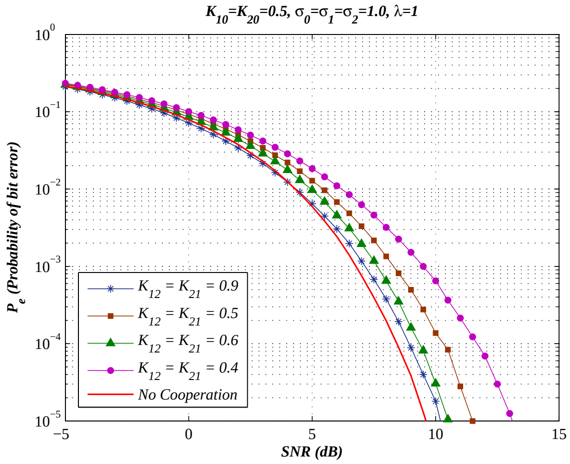

Fig. 4. Comparison of the probability bit error between user cooperation diversity implementation and proposed scheme.

system, this is a significant advantage with respect to the other system.

Finally, the most important benefit of our proposed scheme, is based on throughput, this is because the performance criterion is the number of successfully received bits/transmission. Fig. 5 shows the achievable throughput under the same conditions presented in Fig. 3, where we can clearly see that for high SNR values both schemes are close to reaching their maximum throughput, that is, two new bits per two symbol periods in our proposed scheme, as compared to two new bits per three symbol periods of the other scheme.

VI. CONCLUSIONS

We have developed in this paper a new method of cooperative scheme for mobile users, where we take the user cooperation concept in order to modify it and achieving greater performance. Results indicate that user cooperation is beneficial under certain channel conditions and can result in substantial gains over a noncooperation strategy.

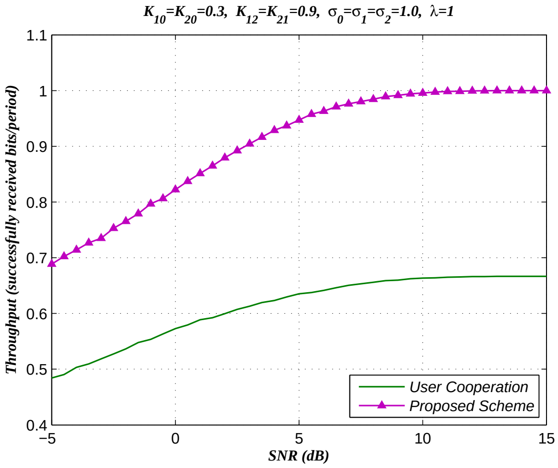

Fig. 5. Comparison of the probability bit error between user cooperation diversity implementation and proposed scheme.

Note that since we use the same strategy for the cooperative bit, we can achieve the same benefits of [2] to decrease the total power to achieve the same rate pair (keeping the same bit error probability) obtained in a non-cooperative scheme. However, there are some differences in our proposed system, one of them is that we only use two symbol periods, this involve a big difference between the factors of power allocation for the same power constraint in each system. The other one is the number of spreading codes, we use three codes for two users versus two spreading codes for two users in the case of a low bit-rate CDMA system. If we consider a high bit-rate CDMA system presented in [3], where users achieve a high data rate by virtue of having more than one spreading code, our proposed scheme also has an advantage, that one spreading code is shared by both of users, which reduces the number of codes used in the system.

In terms of probability of bit error, as compared to the strategy in [3], where we consider the especial case of 1 symbol period without cooperation and two symbols periods with cooperation, our proposed scheme obtain results very close, which is precisely what we wanted. Also, results show that interuser link quality plays a significant role in determining the optimum level of cooperation.

Finally, the aim of this paper was to increase the throughput of cooperative scheme in [2], since are well-known advantages of cooperative strategies. Fig. 5 shows in terms of throughput, that our proposed strategy is always above of the implementation in [3], this is due to use of an additional spreading code. In addition, the spectral efficiency of each user improves because, due to cooperation diversity, the channel code rates can be increased. Thus, despite these costs, our analysis demonstrated significant performance enhancements.

APPENDIX
DERIVATION OF (18)

The detector presented in (16), is given by $\hat{b}_1^{(2)} = sign\left(y_0^1 + y_0^2\right)$, where

$$y_0^1 + y_0^2 = K_{10}a_{12}b_1^{(2)} + K_{20}a_{22}b_2^{(2)} + K_{10}a_{15}b_1^{(2)} - K_{20}a_{25}b_2^{(2)} + n^1 + n^2 \quad (19)$$

since we assume equal power allocation, we have $a_{12} = a_{22} = a_{15} = a_{25}$. Then, (19) becomes

$$y_0^1 + y_0^2 = 2K_{10}a_{12}b_1^{(2)} + n^1 + n^2 \quad (20)$$

where $n^1 + n^2 \sim N(0,\ 2\sigma_0^2/N_c)$. Therefore, the probability of error is given by

$$P_e = Q\left(2K_{10}a_{12}\frac{\sqrt{N_c}}{\sigma_0\sqrt{2}}\right) \quad (21)$$

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