A Linearly Constrained IQRD-RLS Algorithm for Blind Multiuser Detection in CDMA Systems

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***Abstract*—In this paper we propose an inverse QR decomposition based recursive least squares algorithm (IQRD-RLS) for the linearly constrained minimum variance (LCMV) receiver for CDMA transmission systems. The proposed algorithm is numerically stable in finite precision environments and it is suitable for implementation in systolic arrays or DSP vector architectures. It is shown through computer simulations that the proposed algorithm outperforms, in terms of bit error rate, previously proposed IQRD-RLS based blind detection algorithms.**

***Index Terms*—CDMA, blind receiver estimation, linearly constrained minimum variance, IQRD-RLS filtering, constrained RLS.**

I. Introduction

Code division multiple access (CDMA) is a spread spectrum technique that allows multiple users to simultaneously share the same bandwidth. The most common CDMA system is the so-called direct sequence code division multiple access (DS-CDMA), which spreads the information bearing symbols over a wide frequency band, which is much greater than the coherence bandwidth. In recent years, block transmission systems have been widely studied in CDMA schemes and several CDMA based systems have appeared, such as single carrier and multicarrier CDMA systems.

Single carrier CDMA (SC-CDMA) is basically a DS-CDMA system with a guard interval between each symbol [1]. As in DS-CDMA, the chips are transmitted sequentially over the whole bandwidth allocated for that user. On the other hand, multicarrier CDMA (MC-CDMA) is based on the concatenation of DS-CDMA and OFDM technique [2]. The data symbols are spread and the chips are simultaneously transmitted, each one over a narrowband subchannel by the multicarrier modulation (frequency domain spreading). These two block transmission system can be used with cyclic prefix (CP) or zero padding (ZP) techniques as guard interval.

However, as mentioned before, the frequency band occupied by such CDMA based systems is much greater than the coherence bandwidth, and then suffers from multiple access interference (MAI), that arises even with the use of orthogonal codes. To deal with MAI, the use of multiuser detection is a well known approach. Blind adaptive linear receivers are interesting techniques for multiple access interference suppression, as they can be used in situations where a receiver loses track of the desired signal and a training sequence is not available.

A linearly constrained minimum variance (LCMV) receiver for blind user detection was proposed in [3]. This receiver presents, under ideal conditions, a performance close to the exact minimum mean square error (MMSE) solution, however, it was found that its performance degrades considerably due to channel estimation errors and other possible types of signal mismatch. Stochastic gradient algorithms and recursive least squares (RLS) adaptive algorithms were proposed for the LCMV receiver in [4].

It is well known that among the members of the RLS family, QR decomposition based RLS (QRD-RLS) algorithms have better numerically stability in limited precision environment [5] and they can be efficiently implemented in systolic arrays [6] or DSP vector architectures [7]. Constrained QR decomposition based RLS algorithms have been proposed in [8] and [9], nevertheless, they are only solutions to the minimum variance distortionless response (MVDR) receiver, as well as the algorithm proposed in [10] which is an implementation of the inverse QRD-RLS (IQRD-RLS) algorithm for the MVDR receiver. In [11], Chern et al. present an inverse QRD-RLS solution to the LCMV receiver.

In this paper we propose a linearly constrained minimum variance IQRD-RLS algorithm suitable for blind multiuser detection in CDMA transmission systems. The proposed algorithm can be viewed as an extension of the discussed in [11], the difference is that the adaptation of the Kalman *loss* [7] is performed by orthogonal hyperbolic plane rotations, and results in better numerical accuracy and enhanced performance in terms of bit error rate (BER) and signal-to-interference plus noise ratio (SINR).

This paper is structured as follows. Section II describes the CDMA based system model. In Section III the linearly constrained minimum variance receiver is introduced and its RLS solution is derived in Section IV. In Section V the proposed algorithm is derived. Some simulation experiments are presented in Section VI, while Section VII gives the conclusions.

II. System Model

Let us consider the downlink of a synchronous multicarrier code division multiple access (MC-CDMA) system with K users, as depicted in Fig. 1. For user k, the transmitted symbols, $b_k(i)$, drawn from a complex signal constellation with zero mean and unitary average symbol energy, are first spreaded by a code $\boldsymbol{c}_k$ of M chips per symbol. The chips are grouped in blocks of length M (i.e., one symbol per block) and transmitted in multicarrier fashion by a $M \times M$ matrix $\boldsymbol{F}^H$, where $\boldsymbol{F}$ implements the normalized discrete Fourier transform, such that $\boldsymbol{F}\boldsymbol{F}^H = \boldsymbol{F}^H\boldsymbol{F} = \boldsymbol{I}_M$, and $\boldsymbol{I}_M$ represents the $M \times M$ identity matrix.

In order to allow interblock interference (IBI) suppression at the receiver, a length G cyclic prefix insertion is performed before transmission by a $P \times M$ matrix $\boldsymbol{T}$, detailed below, where $\mathbf{0}_{m\times n}$ represents an $m \times n$ null matrix and $P = M + G$; G must be at least the channel order to avoid IBI:

$$\boldsymbol{T} = \begin{bmatrix} \mathbf{0}_{G\times(M-G)} \,|\, \boldsymbol{I}_G \\ \boldsymbol{I}_M \end{bmatrix}$$

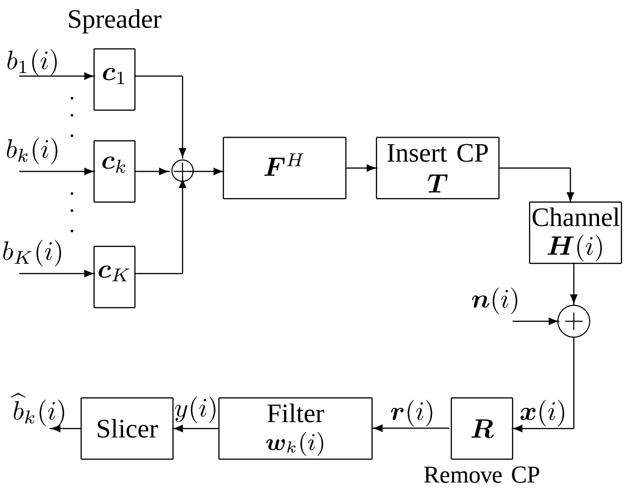

Figure 1. MC-CDMA downlink transmission system.

Each block of chips is then serially transmitted through a multipath channel, modeled here as a FIR filter with L taps whose gains are samples of the channel impulse response complex envelope. Assuming that during the i-th block duration the multipath channel impulse response remains constant, that is, $\boldsymbol{h}(i) = [h_0(i) \ldots h_{L-1}(i)]^T$, the transmission through the multipath channel can be represented by a $P \times P$ lower triangular Toeplitz convolution matrix $\boldsymbol{H}(i)$, whose first column is $[h_0(i) \;\ldots\; h_{L-1}(i)\; 0 \;\ldots\; 0]^T$.

The transmitted signal is corrupted by a complex white Gaussian noise vector $\boldsymbol{n}(i) = [n_0(i) \;\ldots\; n_{P-1}(i)]^T$ whose covariance matrix $\mathsf{E}\left[\boldsymbol{n}(i)\boldsymbol{n}^H(i)\right] = \sigma^2\boldsymbol{I}_P$, where $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian transpose, respectively. The operator $\mathsf{E}[\cdot]$ stands for ensemble average.

Finally, through the use of a matrix $\boldsymbol{R} = [\mathbf{0}_{M\times G}|\; \boldsymbol{I}_M]$, the receiver removes the cyclic prefix from the received signal to eliminate IBI. We can, therefore, represent the input to the detection filter by the M dimensional vector

$$\boldsymbol{r}(i) = \boldsymbol{R}\boldsymbol{H}(i)\boldsymbol{T}\boldsymbol{F}^H \sum_{k=1}^{K} \sqrt{\rho}_k \boldsymbol{c}_k b_k(i) + \boldsymbol{n}'(i) \tag{1}$$

where ρ_k is the average power of the transmitted symbol for user k and $\boldsymbol{n}'(i) = \boldsymbol{R}\boldsymbol{n}(i)$.

Note that the discrete Fourier transform usually present at the receiver was not applied. Actually, this operation is embedded in the receiver filter that will be derived in the next section. The observation vector $\boldsymbol{r}(i)$ in (1) for a synchronous K-user system can be rewritten as

$$\boldsymbol{r}(i) = \sum_{k=1}^{K} \sqrt{\rho}_k \boldsymbol{C}_k \boldsymbol{h}(i) b_k(i) + \boldsymbol{n}'(i) \tag{2}$$

where $\boldsymbol{C}_k$ is an $M \times L$ code related circulant matrix for user k, containing circularly-shifted versions of the k-th user transformed spreading sequence, $\boldsymbol{F}^H\boldsymbol{c}_k$ [12]. Note that although a MC-CDMA system was modeled, a similar signal model in (2) applies for other CDMA based systems [12].

III. Linearly Constrained Minimum Variance Receivers

Considering an observation vector $\boldsymbol{r}(i)$ in the form of (2), the design of a receiver filter $\boldsymbol{w}_k(i)$ based on the minimum variance (MV) criterion uses the output energy as a cost function to be minimized [3]:

$$J_{MV}(\boldsymbol{w}_k) = \mathsf{E}\left[|\boldsymbol{w}_k^H\boldsymbol{r}(i)|^2\right] = \boldsymbol{w}_k^H\boldsymbol{R}_{\boldsymbol{rr}}\boldsymbol{w}_k \tag{3}$$

where $\boldsymbol{R}_{\boldsymbol{rr}} = \mathsf{E}\left[\boldsymbol{r}(i)\boldsymbol{r}^H(i)\right]$ is the autocorrelation matrix of the observed vector. In order to avoid the trivial solution, $\boldsymbol{w}_k = \mathbf{0}$, and anchor the desired user signal, the minimization problem in (3) is subject to the linear set of constraints $\boldsymbol{C}_k^H\boldsymbol{w}_k = \boldsymbol{g}_k$, where $\boldsymbol{g}_k$ is a parameter vector, that can be appropriately chosen or can be the result of some optimization problem [3]. In this paper we assume that $\boldsymbol{g}_k$ is a given vector. For the case of unknown vector $\boldsymbol{g}_k$, a numerically stable and robust algorithm is proposed in [13].

Using the method of Lagrange multipliers, the optimum receiver filter is obtained as [4]:

$$\boldsymbol{w}_{k,MV} = \boldsymbol{R}_{\boldsymbol{rr}}^{-1}\boldsymbol{C}_k(\boldsymbol{C}_k^H\boldsymbol{R}_{\boldsymbol{rr}}^{-1}\boldsymbol{C}_k)^{-1}\boldsymbol{g}_k. \tag{4}$$

In practical situations the matrix $\boldsymbol{R}_{\boldsymbol{rr}}$ is unknown at the receiver, and must be estimated. A commonly used approach is the use of the weighted least-squares as objective function, which renders the so-called RLS algorithm, as shown in the next section.

IV. Linearly Constrained Minimum Variance Weighted Least-Squares Receiver

The least-squares solution to the minimum variance receiver is the vector $\boldsymbol{w}_k(i)$ that minimizes the output error in the weighted least-squares sense, subject to a set of linearly constraints, i.e.,

$$\boldsymbol{w}_k(i) = \arg\min_{\boldsymbol{w}} \sum_{j=1}^{i} \lambda^{i-j} \left|\boldsymbol{w}^H\boldsymbol{r}(j)\right|^2 \tag{5}$$

subject to

$$\boldsymbol{C}_k^H\boldsymbol{w}_k(i) = \boldsymbol{g}_k \tag{6}$$

where $0 \ll \lambda < 1$ is the forgetting factor. Using the method of Lagrange multipliers, the optimum receiver filter in the least-squares sense is obtained as [14]:

$$\boldsymbol{w}_k(i) = \boldsymbol{R}_{\boldsymbol{rr}}^{-1}(i)\boldsymbol{C}_k(\boldsymbol{C}_k^H\boldsymbol{R}_{\boldsymbol{rr}}^{-1}(i)\boldsymbol{C}_k)^{-1}\boldsymbol{g}_k \quad (7)$$

where

$$\begin{aligned} \boldsymbol{R}_{\boldsymbol{rr}}(i) &= \sum_{j=1}^{i}\lambda^{i-j}\boldsymbol{r}(j)\boldsymbol{r}^H(j) \\ &= \lambda\boldsymbol{R}_{\boldsymbol{rr}}(i-1) + \boldsymbol{r}(i)\boldsymbol{r}^H(i). \end{aligned} \quad (8)$$

The direct implementation of (7) is computationally intensive because involves, among other matrix operations, two matrix inversions. In [14] is proposed an implementation that uses Kalman recursions to compute recursively $\boldsymbol{R}_{\boldsymbol{rr}}^{-1}(i)$ and the associated matrices needed in the receiver filter estimation. Nevertheless, Kalman recursions are based on the matrix inversion lemma which is known to be numerically instable in finite precision environments. To overcome this problem, a numerical stable linearly constrained IQRD-RLS algorithm is proposed in the next section.

V. A Linearly Constrained Minimum Variance IQRD-RLS Algorithm

In this section propose a linearly constrained IQRD-RLS algorithm. The proposed algorithm is better explained as a two stages algorithm, where the first stage performs the rank-one update of $\boldsymbol{R}_{\boldsymbol{rr}}^{-1}(i)$ in similar way as in the IQRD-RLS algorithm [15] while the second stage performs a rank-one matrix downdate in order to compute the desired matrices.

Let $\boldsymbol{R}_{\boldsymbol{rr}}(i)$ be updated as in (8) and $\boldsymbol{U}(i)$ be its the Cholesky factor, such that $\boldsymbol{R}_{\boldsymbol{rr}}(i) = \boldsymbol{U}^H(i)\boldsymbol{U}(i)$. Then $\boldsymbol{U}(i)$ can be updated from $\boldsymbol{U}(i-1)$ by an orthogonal matrix, $\boldsymbol{Q}_\theta(i)$, as

$$\begin{bmatrix} \boldsymbol{0}^T \\ \boldsymbol{U}(i) \end{bmatrix} = \boldsymbol{Q}_\theta(i)\begin{bmatrix} \boldsymbol{r}^H(i) \\ \lambda^{1/2}\boldsymbol{U}(i-1) \end{bmatrix} \quad (9)$$

where $\boldsymbol{Q}_\theta(i)$ is constructed by planar rotations that annihilates $\boldsymbol{r}^H(i)$ over $\lambda^{1/2}\boldsymbol{U}(i-1)$ [16].

However, in (7) the inverse of the autocorrelation matrix is desired. It is worth emphasizing that $\boldsymbol{R}_{\boldsymbol{rr}}^{-1}(i) = \boldsymbol{U}^{-1}(i)\boldsymbol{U}^{-H}(i)$, and thus, it is desired to update matrix $\boldsymbol{U}^{-H}(i)$ instead of matrix $\boldsymbol{U}(i)$, as in the unconstrained IQRD-RLS algorithm [15].

In [7], Pan and Plemmons prove that the same matrix $\boldsymbol{Q}_\theta(i)$ used to update $\boldsymbol{U}(i)$ can be used to update $\boldsymbol{U}^{-H}(i)$ by the following expression:

$$\begin{bmatrix} \boldsymbol{z}^H(i) \\ \boldsymbol{U}^{-H}(i) \end{bmatrix} = \boldsymbol{Q}_\theta(i)\begin{bmatrix} \boldsymbol{0}_{M\times 1}^T \\ \lambda^{-1/2}\boldsymbol{U}^{-H}(i-1) \end{bmatrix} \quad (10)$$

where $\boldsymbol{z}(i) = -\delta(i)\boldsymbol{R}_{\boldsymbol{rr}}^{-1}(i)\boldsymbol{r}(i)$ and $\boldsymbol{Q}_\theta(i)$ is a matrix that implements successive Givens rotations that annihilates vector $\boldsymbol{a}(i) = \lambda^{-1/2}\boldsymbol{U}^{-H}(i-1)\boldsymbol{r}(i)$ such that

$$\begin{bmatrix} \delta(i) \\ \boldsymbol{0}_{M\times 1} \end{bmatrix} = \boldsymbol{Q}_\theta(i)\begin{bmatrix} 1 \\ -\boldsymbol{a}(i) \end{bmatrix}, \quad (11)$$

with $\delta^2(i) = 1 + \boldsymbol{a}^2(i)$.

The Kalman gain can be computed as [15]:

$$\begin{aligned} \boldsymbol{\kappa}(i) &= \boldsymbol{R}_{\boldsymbol{rr}}^{-1}(i)\boldsymbol{r}(i) \\ &= -\boldsymbol{z}(i)/\delta(i). \end{aligned} \quad (12)$$

Note that $\boldsymbol{z}(i)$ is not computed directly, but it is a consequence of (10).

From (10), as $\boldsymbol{Q}_\theta(i)$ is orthogonal and $\boldsymbol{R}_{\boldsymbol{rr}}^{-1}(i) = \boldsymbol{U}^{-1}(i)\boldsymbol{U}^{-H}(i)$, we have:

$$\boldsymbol{R}_{\boldsymbol{rr}}^{-1}(i) = \lambda^{-1}\boldsymbol{R}_{\boldsymbol{rr}}^{-1}(i-1) - \boldsymbol{z}(i)\boldsymbol{z}^H(i) \quad (13)$$

now, pre-multiplying (13) by $\boldsymbol{C}_k^H$ and pos-multiplying by $\boldsymbol{C}_k$ we get that $\boldsymbol{\Omega}_k(i) = \boldsymbol{C}_k^H\boldsymbol{R}_{\boldsymbol{rr}}^{-1}(i)\boldsymbol{C}_k$ can be recursively computed as

$$\boldsymbol{\Omega}_k(i) = \lambda^{-1}\boldsymbol{\Omega}_k(i-1) - \boldsymbol{\alpha}_k(i)\boldsymbol{\alpha}_k^H(i). \quad (14)$$

where $\boldsymbol{\alpha}_k(i) = \boldsymbol{C}_k^H\boldsymbol{z}(i)$. Observe that (14) is a rank-one downdate process and the QR decomposition method based in (9) does not apply due the negative sign of $\boldsymbol{\alpha}_k(i)\boldsymbol{\alpha}_k^H(i)$ [7]. Downdating algorithms by orthogonal plane rotations are well studied in [16], [17], [18], however, a more computationally efficient scheme is the use of hyperbolic rotations [19], [7], as shown next.

A. Downdating $\boldsymbol{\Omega}_k^{-1}(i)$

For this purpose, we say that a matrix $\boldsymbol{P}_k(i)$ is pseudo-orthogonal if

$$\boldsymbol{P}_k(i)\boldsymbol{\Phi}\boldsymbol{P}_k(i) = \boldsymbol{\Phi} \quad (15)$$

for some signature matrix $\boldsymbol{\Phi} = \text{diag}(\pm 1)$. Now, let us define $\boldsymbol{D}_k(i)$ as the Cholesky factor of $\boldsymbol{\Omega}_k(i)$ such that $\boldsymbol{\Omega}_k(i) = \boldsymbol{D}_k^H(i)\boldsymbol{D}_k(i)$.

The key observation is that a sequence of hyperbolic orthogonal plane rotations can be found, resulting in a pseudo-orthogonal matrix $\boldsymbol{P}_k(i)$ with respect to $\boldsymbol{\Phi} = \text{diag}(-1, \boldsymbol{I}_L)$ so that [7]

$$\boldsymbol{P}_k(i)\begin{bmatrix} \boldsymbol{\alpha}_k^H(i) \\ \lambda^{-1/2}\boldsymbol{D}_k(i-1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{0}_{L\times 1}^T(i) \\ \boldsymbol{D}_k(i) \end{bmatrix} \quad (16)$$

where $\boldsymbol{P}_k(i)\boldsymbol{\Phi}\boldsymbol{P}_k(i) = \boldsymbol{\Phi}$, and the downdated Cholesky factor $\boldsymbol{D}_k(i)$ satisfies

$$\boldsymbol{D}_k^H(i)\boldsymbol{D}_k(i) = \lambda^{-1}\boldsymbol{D}_k^H(i-1)\boldsymbol{D}_k(i-1) - \boldsymbol{\alpha}_k(i)\boldsymbol{\alpha}_k^H(i) \quad (17)$$

Pan and Plemmons in [7] proved that the same matrix $\boldsymbol{P}_k(i)$ used to downdate $\boldsymbol{D}_k(i)$ can be used to downdate $\boldsymbol{D}_k^{-H}(i)$, as

$$\begin{bmatrix} \boldsymbol{v}_k^H(i) \\ \boldsymbol{D}_k^{-H}(i) \end{bmatrix} = \boldsymbol{P}_k(i)\begin{bmatrix} \boldsymbol{0}_{L\times 1}^T \\ \lambda^{1/2}\boldsymbol{D}_k^{-H}(i-1) \end{bmatrix} \quad (18)$$

where $\boldsymbol{P}_k(i)$ is constructed from hyperbolic plane rotations matrices such that

$$\begin{bmatrix} q_k(i) \\ \boldsymbol{0}_{L\times 1} \end{bmatrix} = \boldsymbol{P}_k(i)\begin{bmatrix} 1 \\ \boldsymbol{b}_k(i) \end{bmatrix}, \quad (19)$$

$$\boldsymbol{b}_k(i) = \lambda^{1/2}\boldsymbol{D}_k^{-H}(i-1)\boldsymbol{\alpha}_k(i) \quad (20)$$

and $q_k(i) = \sqrt{1 - \|\boldsymbol{b}_k(i)\|^2}$. In similar way that in the updating process, the Kalman *loss* can be computed as [7]:

$$\begin{aligned} \boldsymbol{\eta}_k(i) &= \boldsymbol{\Omega}_k^{-1}(i)\boldsymbol{\alpha}_k(i) \\ &= -\boldsymbol{v}_k(i)/q_k(i). \end{aligned} \tag{21}$$

Tab. I shows an algorithm to construct a matrix $\boldsymbol{P}_k(i)$ that implements an hyperbolic Householder transform. More robust algorithms for hyperbolic Householder transforms can be found in the literature [20], [21], [22], as well systolic implementations of the algorithms [23].

Table I
HYPERBOLIC HOUSEHOLDER TRANSFORM [5]

1) Define $\boldsymbol{\Phi} = \mathrm{diag}(1, -\boldsymbol{I}_L)$
2) Define $\beta_1(i)$ the first element of $\boldsymbol{\beta}_k(i) = [1 \;\; \boldsymbol{b}_k^T(i)]^T$
2) Let $\boldsymbol{e}_1$ the first column of the identity matrix, then set
$$\boldsymbol{x}_k(i) = \boldsymbol{\Phi}\boldsymbol{\beta}_k(i) + \left(\mathrm{sign}(\beta_1(i))\sqrt{\boldsymbol{\beta}_k^H(i)\boldsymbol{\Phi}\boldsymbol{\beta}_k(i)}\right)\boldsymbol{e}_1$$
3) Set $\beta_k(i) = 2/\left(\boldsymbol{x}_k^H(i)\boldsymbol{\Phi}\boldsymbol{x}_k(i)\right)$
4) Compute $\boldsymbol{P}_k(i)$ as
$$\boldsymbol{P}_k(i) = \boldsymbol{\Phi} - \beta_k(i)\boldsymbol{x}_k(i)\boldsymbol{x}_k^H(i)$$

Finally, the receiver filter can be computed as

$$\boldsymbol{w}_k(i) = \boldsymbol{R}_{\boldsymbol{rr}}^{-1}(i)\boldsymbol{C}_k\boldsymbol{\Omega}_k^{-1}(i)\boldsymbol{g}_k. \tag{22}$$

In blind user detection the receiver filter estimation is not always of interest as it is the receiver filter output, in that case some computational cost can be saved, as the output signal can be computed as

$$\begin{aligned} y(i) &= \boldsymbol{w}_k^H(i)\boldsymbol{r}(i) \\ &= \frac{1}{\delta(i)q_k(i)}\boldsymbol{g}_k^H(i)\boldsymbol{v}_k(i). \end{aligned} \tag{23}$$

In Tab. II is summarized the proposed constrained IQRD-RLS algorithm for blind multiuser detection.

VI. SIMULATION RESULTS

In this section we assess the performance of the proposed constrained IQRD-RLS blind detection algorithm in terms of the signal-to-interference plus noise ratio (SINR) and bit error rate (BER). The simulation results are for downlink BPSK synchronous MC-CDMA systems with $K = 10$ active users that employ Gold sequences of length $M = 31$. The guard interval length is the same that the channel order, i.e., $L - 1 = G = 3$. Regarding power distribution, we simulate near-far scenario where the interferes have a power 20 dB above desired user. The forgetting factor was set to $\lambda = 0.997$. The results are an average of 100 experiments.

We compare the proposed algorithm with the linearly constrained minimum variance fast recursive least squares (LCMV FRLS) algorithm in [14], the linearly constrained minimum variance IQRD-RLS (LCMV IQRD-RLS) algorithm in [11] and the linearly constrained constant modulus IQRD-RLS receiver (LCCM IQRD-RLS) in [24].

In Fig. 2 we plot the SINR for all the algorithms. We use a fixed channel modeled as an FIR filter, $\boldsymbol{h} =$

Table II
PROPOSED CIQRD-RLS ALGORITHM

1) Form the matrix-vector product:
$$\boldsymbol{a}(i) = \lambda^{-1/2}\boldsymbol{U}^{-H}(i-1)\boldsymbol{r}(i)$$
2) Compute $\boldsymbol{Q}_\theta(i)$ such that
$$\begin{bmatrix} \delta(i) \\ \boldsymbol{0}_{M\times 1} \end{bmatrix} = \boldsymbol{Q}_\theta(i)\begin{bmatrix} 1 \\ -\boldsymbol{a}(i) \end{bmatrix}$$
3) Apply $\boldsymbol{Q}_\theta(i)$ to $[\boldsymbol{0}_{M\times 1} \;\; \lambda^{-1/2}\boldsymbol{U}^{-1}(i-1)]^H$ forming
$$\begin{bmatrix} \boldsymbol{z}^H(i) \\ \boldsymbol{U}^{-H}(i) \end{bmatrix} = \boldsymbol{Q}_\theta(i)\begin{bmatrix} \boldsymbol{0}_{M\times 1}^T \\ \lambda^{-1/2}\boldsymbol{U}^{-H}(i-1) \end{bmatrix}$$
5) Form the matrix-vector product:
$$\boldsymbol{\alpha}_k(i) = \boldsymbol{C}_k^H\boldsymbol{z}(i)$$
$$\boldsymbol{b}_k(i) = \lambda^{1/2}\boldsymbol{D}_k^{-H}(i-1)\boldsymbol{\alpha}_k(i)$$
6) Compute $\boldsymbol{P}_k(i)$ such that (see Tab. I)
$$\begin{bmatrix} q_k(i) \\ \boldsymbol{0}_{L\times 1} \end{bmatrix} = \boldsymbol{P}_k(i)\begin{bmatrix} 1 \\ \boldsymbol{b}_k(i) \end{bmatrix}$$
7) Apply $\boldsymbol{P}_k(i)$ to $[\boldsymbol{0}_{L\times 1} \;\; \lambda^{-1/2}\boldsymbol{D}_k^{-1}(i-1)]$ forming
$$\begin{bmatrix} \boldsymbol{v}_k^H(i) \\ \boldsymbol{D}_k^{-H}(i) \end{bmatrix} = \boldsymbol{P}_k(i)\begin{bmatrix} \boldsymbol{0}_{L\times 1}^T \\ \lambda^{1/2}\boldsymbol{D}_k^{-H}(i-1) \end{bmatrix}$$
6) Compute the receiver filter output and detect the transmit symbol
$$y(i) = \frac{1}{\delta(i)q_k(i)}\boldsymbol{g}_k^H(i)\boldsymbol{v}_k(i)$$
$$\hat{b}_k(i) = \mathsf{disc}\{y(i)\},$$
where $z = \mathsf{disc}\{x\}$ is the symbol of the signal constellation closer to x.
7) Alternatively compute
$$\boldsymbol{w}_k(i) = \boldsymbol{R}_{\boldsymbol{rr}}^{-1}(i)\boldsymbol{C}_k\boldsymbol{\Omega}_k^{-1}(i)\boldsymbol{g}_k$$

$[0.7297 \;\; 0.5166 \;\; 0.3657 \;\; 0.2589]^T$, and a signal to noise ratio of 30 dB with respect to the desired user power level. The results reveal that the LCMV FRLS and the LCMV IQRD-RLS algorithms exhibits numerical instability as they diverge after a number of transmitted symbols, while the LCCM IQRD-RLS and the proposed algorithm are stable, although the proposed algorithm outperforms the LCCM IQRD-RLS.

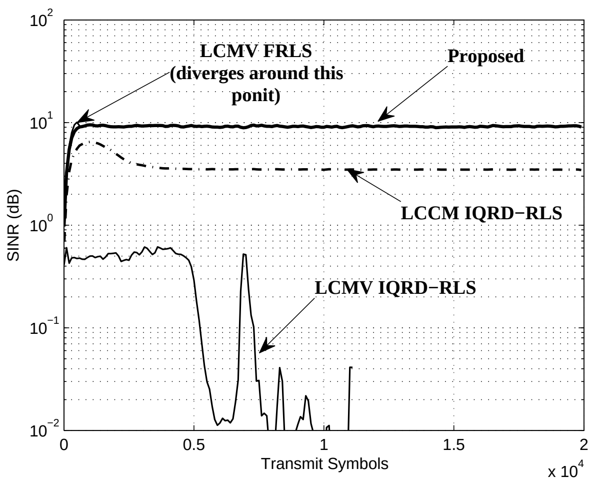

Figure 2. SINR vs. number of symbols for a fixed channel.

In the second experiment we simulate the receiver in a time-variant scenario. For the time-variant channel, the sequence of channel coefficients, $h_l(i) = p_l\alpha_l(i)$ $(l = 0, 1, 2, \ldots, L-1)$ is obtained with Clarke's model. This procedure corresponds to the generation of independent sequences of correlated unit power complex Gaussian random variables ($\mathsf{E}\left[|\alpha_l^2(i)|\right] = 1$) with the path weights p_l normalized so that $\sum_{l=1}^{L_{\mathrm{p}}} |p_l|^2 = 1$. In this work $p_0 = 0.7297$, $p_1 = 0.5166$, $p_2 = 0.3657$ and $p_3 = 0.2589$. The results are shown in terms of the normalized Doppler frequency $(f_d T)$, where f_d is the Doppler frequency and T is the symbol duration. In the simulations a $f_d T = 0.0001$ was assumed.

Fig. 3 shows the resulting BER of the LCCM IQRD-RLS and the proposed algorithm. It can be seen that for small values of E_b/N_0 the two algorithms have almost the same performance. As the E_b/N_0 increase the proposed algorithm presents lower BER. This results from the fact that at higher signal to noise ratio, matrices of LCCM IQRD-RLS tend to be more ill-conditioned and then, numerical instabilities appears.

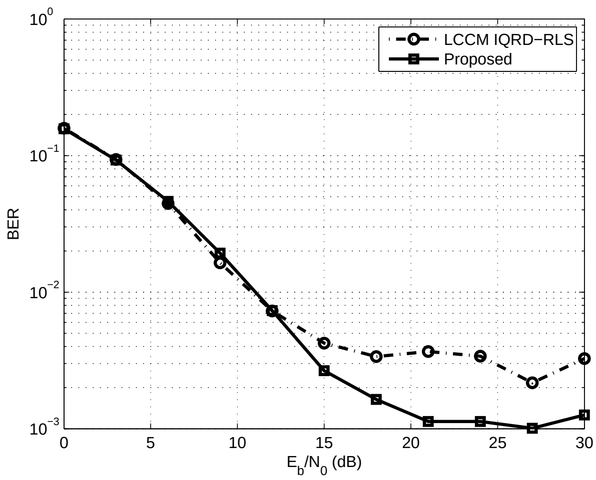

Figure 3. BER vs. E_b/N_0(dB) for time-invariant channel.

VII. CONCLUSIONS

In this paper we proposed an IQRD-RLS algorithm for blind multiuser detection in CDMA systems. The proposed algorithm is suitable for implementations on systolic arrays or DSP vector architectures and, as shown through computer simulations, the proposed algorithm has better numerical behavior in finite precision environments if compared with other constrained IQRD-RLS blind detection algorithms. This numerical robustness results in better performance in terms of BER and SINR.

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