

Performance of Single Carrier and Multicarrier With PAPR Reduction Technique in MIMO Systems

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Abstract—We have analyzed the spatial modulation (SM) OFDM and V-BLAST OFDM systems equipped with particle swarm optimization (PSO)-based partial transmit sequence (PTS) PAPR reduction technique and not ideal side information (SI) transmission; while comparing such transmission schemes with the SM single-carrier (SC) and V-BLAST-SC systems. We observe that under 2×4 antennas the SM-SC and V-BLAST SC outperform the SM-OFDM and V-BLAST OFDM. For 4×4 antennas the V-BLAST OFDM system, in the linear region or not, results in a better performance than V-BLAST SC. On the other hand, the SM-OFDM system performs better than SM SC system under low saturation.

Keywords—CCDF, PAPR, spatial modulation (SM), V-BLAST, OFDM, single-carrier (SC), IBO, high power amplifier (HPA), partial transmit sequence (PTS), particle swarm optimization.

I. INTRODUCTION & SYSTEM MODEL

There are widely available methods to reduce the peak-to-average power ratio (PAPR) for OFDM systems that use multiple signal representation (MSR) techniques, among them selective mapping (SLM) and partial transmit sequence (PTS). The main drawback of MSR techniques is the necessity of the side information (SI) to be transmitted; this process causes some loss on the throughput. The literature shows that PTS outperforms SLM techniques [1]. A heuristic optimization technique namely particle swarm optimization (PSO) has been evoked aiming to reduce substantially the complexity of the PTS technique. Besides, the single-carrier (SC) transmission system combined with SM and V-BLAST systems can overcome the PAPR problem.

The combination of OFDM and multiple-input multiple-output (MIMO) schemes add high-gain diversity and ability to cope with multipath fading, but it presents the disadvantage of having high levels of PAPR. Thus, there is a need to reduce PAPR in order to prevent the HPA operating in nonlinear region or there is a signal clipping that reduces the BER performance of such systems. The SM-OFDM system [2] looks promising to have less complexity feature than the V-BLAST OFDM system [3]. Hence, there is a need in exploring PAPR reduction techniques for SM OFDM systems, since such systems exhibit greater sensitivity to the signal in the HPA clipping than V-BLAST OFDM technique for a same spectral efficiency and number of antennas [4].

The author in [1] shows PTS technique outperforms SLM technique. In the PTS technique different approaches can be adopted for generating phases sequences. We can have binary PTS wherein phases assume $\pm\pi$ values or use random PTS in which the phases are generated randomly. To reduce the complexity in searching optimal transmission phase, a heuristic PSO optimization step [5], [6] can be introduced in

combination with the PTS technique. On the other hand, MSR techniques present some disadvantages; the main of them is the system throughput reduction because the receiver needs at least good channel estimations on the side information. If there is an error in SI recovering at the reception, the OFDM symbol will be detected erroneously. To avoid high PAPR we can combine the SM and V-BLAST systems with the single carrier (SC) transmission systems [7].

The contribution of this work is to analyze SM OFDM and V-BLAST OFDM systems performance deploying PSO-based PTS technique aiming to reduce PAPR and assuming ideal and no ideal SI recovering against SM-SC and V-BLAST-SC systems performance. PSO-based PTS techniques for PAPR reduction [5], [6] do not exhibit significant PAPR reduction in MIMO systems. Indeed, unlike our analysis, [7] does not presents PAPR reduction for multicarrier schemes; besides, it does not analyze the V-BLAST system performance.

Measuring the efficiency of PAPR reduction methods and its effects on the performance of SM OFDM and V-BLAST OFDM systems by mean of the complementary cumulative distribution function (CCDF) of PAPR curves, one can compare the signal after the reduction technique. Then, the BER performance is obtained for the SM and V-BLAST MIMO transmission schemes considering different input back-off (IBO) values.

SM-OFDM and V-BLAST OFDM System Model. Channel with N_t transmitting antennas and N_r receiving antennas [4], where the i^{th} receiving antenna and the j^{th} transmitting antenna link is assumed multipath block-fading, with a channel impulse response of length L , given by $\mathbf{h}_{ij}(k) = \left(\sum_{l=1}^L \rho_l\right)^{-1/2} \left[\sqrt{\rho_1}h_{ij}^{(1)}(k) \cdots \sqrt{\rho_L}h_{ij}^{(L)}(k)\right]$ where ρ_l is the power of the l^{th} path, $h_{ij}^{(l)}(k)$ is a zero-mean unitary variance complex white Gaussian random variable representing the Rayleigh fading of the l^{th} path while the coefficients are kept constant during the transmission of each OFDM symbol, indexed by k ; channel coefficients are assumed independent.

The transmitted signal on the j^{th} antenna is originated from an OFDM symbol that can be written as $\mathbf{s}_j(k) = \mathbf{F}^H \mathbf{x}_j(k)$, where $\mathbf{s}_j(k)$ is the k^{th} OFDM symbol, $(\cdot)^H$ is transposed conjugate of complex matrix, \mathbf{F}^H is the unitary inverse discrete Fourier transform (IDFT) matrix of dimension $N \times N$, $\mathbf{x}_j(k) = [x_{0,j}(k), \dots, x_{N-1,j}(k)]^T$ is the block of N transmitted symbols. Then, a cyclic prefix (CP) of length $L - 1$ is appended, giving origin to the signal $\mathbf{s}'_j(k)$. This signal is passed through a high power-amplifier (HPA), which is modeled, in baseband, as a memory-less hard-limiter HPA:

$$\tilde{u} = \begin{cases} v, & \text{for } |v| < A \\ A \exp \left[j \arctan \left(\frac{\text{Im}(v)}{\text{Re}(v)} \right) \right], & \text{for } |v| > A \end{cases} \quad (1)$$

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where A is the amplifier amplitude saturation, \tilde{u} and v are the output and input of the HPA, respectively [8].

As we use a CP of length $L-1$ and the HPA is memoryless, after removal of the CP at the reception, the time domain received signal of the k^{th} OFDM symbol for the i^{th} receiving antenna can be written as: $\mathbf{y}_i(k) = \sum_{m=1}^{N_t} \mathbf{h}_{im}(k) \otimes \tilde{\mathbf{s}}_m(k) + \eta_i(k)$ where \otimes is the circular convolution operator, $\tilde{\mathbf{s}}_j(k)$ is the output of the HPA for $\mathbf{s}_j(k)$ and η_i is the additive noise vector which elements are complex white Gaussian random variables with zero-mean and variance σ_η^2 .

By applying the discrete Fourier transform to the received signal, we obtain $\tilde{\mathbf{y}}_i(k) = \mathbf{F}\mathbf{y}_i(k) = \sum_{m=1}^{N_t} \mathbf{H}_{ij}(k)\tilde{\mathbf{x}}_m(k) + \mathbf{F}\eta_i(k)$ where $\tilde{\mathbf{x}}_m(k)$ is a encoded symbols vector with the distortion caused by the clipping, \mathbf{F} is the unitary discrete Fourier matrix of dimension $N \times N$,

$$\begin{aligned} \mathbf{H}_{ij}(k) &= \text{diag} \left\{ \mathbf{F} \left[h_{ij}^{(1)}(k), \dots, h_{ij}^{(L)}(k), 0, \dots, 0 \right]^T \right\} \\ &= \text{diag} \left\{ \left[H_{ij}^{(1)}(k), \dots, H_{ij}^{(N)}(k) \right]^T \right\} \end{aligned} \quad (2)$$

and $\mathbf{F}\eta_i(k)$ is also an additive complex Gaussian noise with zero-mean and variance σ_η^2 .

SM-OFDM Receiver. The receiver of the SM-OFDM system for n^{th} subcarrier deploys the criterion of maximum likelihood (ML) detection for the estimation of transmitted bits, defined by the index q with $1 \leq q \leq M$, and the corresponding OFDM modulator (or transmitting antenna) j from which it was transmitted. The ML detection criterion based on exhaustive search for the SM-OFDM is obtained as [9]:

$$[\hat{j}_{ML}, \hat{q}_{ML}]_n = \arg \min_{j,q} (\sqrt{\gamma} \|\mathbf{g}_{j,q}\|^2 - 2\Re \{ \bar{\mathbf{y}}_n^H \mathbf{g}_{j,q} \}) \quad (3)$$

where $\bar{\mathbf{y}}_n$ is the received signal for the n^{th} subcarrier, γ is the signal to noise ratio (SNR) in each reception antenna, $\bar{\mathbf{h}}_j$ is the j^{th} column of $\bar{\mathbf{H}}_n$, $\mathbf{g}_{j,q} = \bar{\mathbf{h}}_j x_q$ and the MIMO channel matrix is constituted by the coefficients of multipath block-fading channel of each subcarrier [4]:

$$\bar{\mathbf{H}}_n = \begin{bmatrix} H_{11}^{(n)} & H_{12}^{(n)} & \dots & H_{1N_t}^{(n)} \\ H_{21}^{(n)} & H_{22}^{(n)} & \dots & H_{2N_t}^{(n)} \\ \vdots & \vdots & \ddots & \vdots \\ H_{N_r1}^{(n)} & H_{N_r2}^{(n)} & \dots & H_{N_rN_t}^{(n)} \end{bmatrix} \quad (4)$$

V-BLAST OFDM Receiver. The V-BLAST OFDM system considered herein also adopts the ML criterion to estimate the transmitted information bits. The ML estimate for the n^{th} subcarrier can be written as [3]:

$$\hat{\mathbf{x}}_{n,ML} = \arg \min_{\mathbf{x}_n} \|\bar{\mathbf{y}}_n - \bar{\mathbf{H}}_n \mathbf{x}_n\|^2 \quad (5)$$

where \mathbf{x}_n is a column vector of dimension N_t in which the elements take M -QAM symbols values.

A. Performance Measures

The degree of backoff under specified operating conditions, calculated as the ratio of the minimum input power for a single carrier which gives maximum output power, corresponding to saturation (P_{sat}), to the total average input power under the specified conditions ($P_{\text{avg}}^{\text{in}}$). IBO is defined in decibels as

[10]: $\text{IBO}_{\text{dB}} = 10 \log_{10} \left(\frac{P_{\text{sat}}}{P_{\text{avg}}^{\text{in}}} \right)$ where, due to the HPA model defined in (1), the $P_{\text{sat}} = A^2$ and $P_{\text{avg}}^{\text{in}}$ is the average signal strength before the HPA, which is given by $\mathcal{E} \left\{ |s_{n,j}(k)|^2 \right\}$, where $s_{n,j}(k)$ is the n^{th} output sample of the OFDM modulator for the j^{th} antenna and k^{th} symbol, $\mathcal{E} \{ \cdot \}$ is the expectation operator over n and k .

Using the CCDF for the PAPR, both SM-OFDM and V-BLAST OFDM transmitting schemes will be compared with respect to a certain level of PAPR threshold, defined by: $\text{CCDF}(\text{PAPR}_{\text{th}}) = \Pr(\text{PAPR}_j > \text{PAPR}_{\text{th}})$ where $\Pr(\cdot)$ is the probability of occurrence and the PAPR for j^{th} antenna can be obtained by $\text{PAPR}_j = \left(\max_{0 \leq n \leq N-1} \left\{ |s_{n,j}|^2 \right\} \right) / (P_{\text{avg}}^{\text{in}})$.

B. Partial Transmit Sequences (PTS) Method

PTS method divides each OFDM symbol given by $\mathbf{x}(k)$ into U sub-blocks, i.e., $\mathbf{x}(k)^{(u)}$, $u = 1, \dots, U$, [11]. Mathematically, we can represent $\mathbf{x}(k) = \sum_{u=1}^U \mathbf{x}(k)^{(u)}$. Multiply each new OFDM symbol generated, i.e., $\mathbf{x}(k)^{(u)}$ by a complex rotation factor given by $b(k)^u$ with $|b(k)^u| = 1$, i.e., $b(k)^u = e^{j\varphi(k)^u}$ results $\check{\mathbf{x}}(k) = \sum_{u=1}^U b(k)^u \mathbf{x}(k)^{(u)}$ which is able to describe the same information as $\mathbf{x}(k)$ if the set $\{b(k)^{(v)}, v = 1, \dots, V\}$ is known on the receiver side. From the linearity property of the IDFT, the sub-blocks can be turned into U IDFT separate, parallel, resulting in $\check{\mathbf{s}}(k) = \sum_{u=1}^U b(k)^u \mathbf{s}(k)^u$ where $\mathbf{s}(k)^u = \text{IDFT} \{ \mathbf{x}(k)^u \}$.

Finally, the optimization process of the set $\{b(k)^{(u)}\}$ can be written as

$$\begin{aligned} & \left[\tilde{b}(k)^1, \dots, \tilde{b}(k)^U \right] = \\ & = \arg \min_{\{b(k)^1, \dots, b(k)^U\}} \left(\max_{0 \leq n < N} \left| \sum_{u=1}^U b(k)^u s_n(k)^u \right| \right), \end{aligned} \quad (6)$$

resulting in *optimal transmission sequence* [11] given by $\check{\mathbf{s}}(k) = \sum_{u=1}^U \tilde{b}(k)^u \mathbf{s}(k)^u$ and thus, the lowest peak average.

II. HEURISTIC APPLIED TO PAPR REDUCTION

The PSO algorithm applied to PTS optimization sequence for PAPR-OFDM reduction problem [6] deploys a size optimization dimension U , has the position and velocity vector at the i^{th} stage represented by $\mathbf{b}(i) = \{b(i)^1, b(i)^2, \dots, b(i)^U\}$ and $\mathbf{v}(i) = \{v(i)^1, v(i)^2, \dots, v(i)^U\}$, respectively. PSO algorithm starts with a vector group of random phases factors that are adjusted to generate optimal values. In each iteration, the vectors are updated by tracing two best vectors positions. The first refers to the best individual position $\mathbf{b}(i)^p$ that is the best solution reached by each individual so far. The other is the best overall position $\mathbf{b}(i)^g$ which represents the best position among all individuals until now. At the $(t+1)^{th}$ iteration, the velocity and position vectors are updating, respectively, as

$$\begin{aligned} \mathbf{v}_i(t+1) &= w \mathbf{v}_i(t) + c_1 r_1 [\mathbf{b}_i^p(t) - \mathbf{b}_i(t)] \\ &\quad + c_2 r_2 [\mathbf{b}_i^g(t) - \mathbf{b}_i(t)] \end{aligned} \quad (7)$$

$$\mathbf{b}_i(t+1) = \mathbf{b}_i(t) + \mathbf{v}_i(t+1), \quad (8)$$

where index i indicates the i^{th} individual, the constant c_1 and c_2 represent the terms of local and global acceleration, r_1 and r_2 represent two random variables with uniform distribution between $[0, 1]$ and w is the weight of inertia constant

A. PAPR Reduction Using the PSO-based PTS technique in SM and V-BLAST Systems

The performance of systems using random sequence and the PSO-based PTS PAPR reduction technique were compared. The CCDF of PSO-based PTS PAPR reduction technique depicted in Fig. 2 and Table I, confirm an average PAPR improvement in SM and V-BLAST OFDM systems of ≈ 3.5 dB and 4.0dB, respectively, at 0.1% CCDF. The greatest mitigation of the V-BLAST system is explained by the smaller constellation of this system in relation to the SM system.

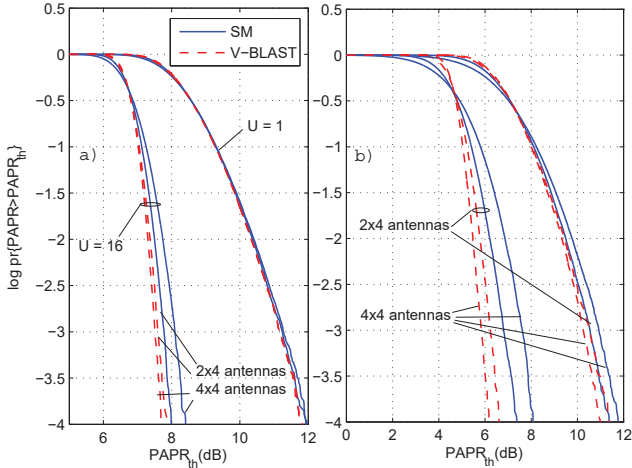


Fig. 2. CCDF of PAPR for PSO-based PTS technique: a) $N = 512$; b) $N = 64$.

TABLE I

PAPR AT 0.1% CCDF AND $U = 16$ TO PSO-BASED PTS TECHNIQUE.

System	Antennas	No Reduction		PSO-based PTS		Variation	
		64	512	64	512	64	512
SM	2×4	10.5dB	11.2dB	6.9dB	7.6dB	3.6dB	3.6dB
	4×4	10.9dB	11.2dB	7.5dB	8.0dB	3.4dB	3.2dB
V-BLAST	2×4	10.5dB	11.2dB	6.4dB	7.4dB	4.1dB	3.8dB
	4×4	10.2dB	11.2dB	5.9dB	7.4dB	4.3dB	3.8dB

B. PAPR Reduction Effect on Performance of SM-OFDM and V-BLAST-OFDM Systems

The BER performance of the SM and V-BLAST-OFDM systems operating in the linear region (non clipping HPA), as well as under HPA saturation (IBO = 3dB and 5dB) equipped with $N_t = 2$ or $N_t = 4$ Tx antennas and $N_r = 4$ Rx antennas without the PAPR reduction, as well as with application of the PSO-based PTS PAPR reduction method are analyzed herein. For simplicity, we have adopted an ideal SI estimation.

In Fig. 3 and 4, the BER to PSO-based PTS technique in the linear region have the same performance of the systems with no PAPR reduction technique. However, under SM and V-BLAST OFDM systems with PSO-based PTS technique for 2×4 antennas, 512 subcarriers and $\mathcal{S} = 6$ bits/subcarrier, Fig. 4, we observe a little improvement in each system for both saturation parameters, i.e., IBO = 3 and 5dB. The SM-OFDM system achieves the same performance of the V-BLAST OFDM system to IBO = 5dB when we use PSO-based PTS reduction technique. For IBO = 3dB, we note that

the signal remains degraded even after reduction technique for both systems.

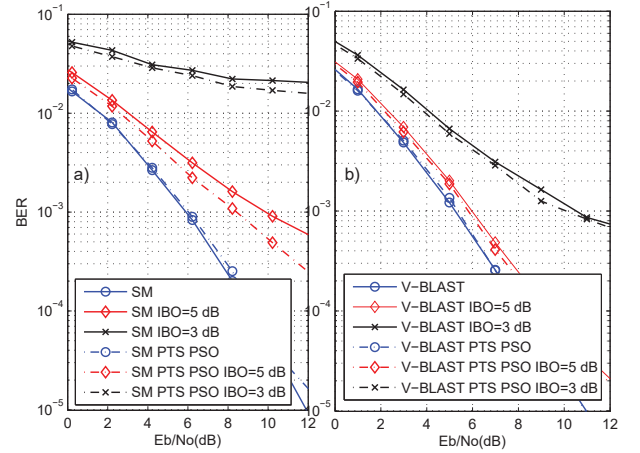


Fig. 3. Performance of systems to 4×4 antennas $N = 512$ subcarriers, $\mathcal{S} = 8$ bits/subcarrier, multipath Rayleigh fading channel and ML detection; (a) SM OFDM 64-QAM; (b) V-BLAST OFDM 4-QAM.

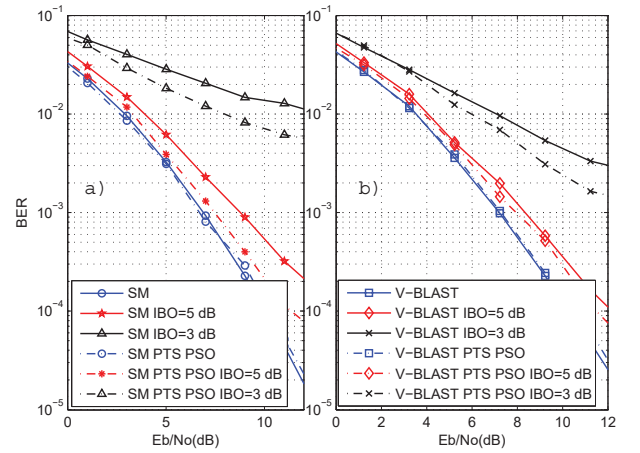


Fig. 4. Performance of systems with 2×4 antennas, $N = 512$ subcarriers, $\mathcal{S} = 6$ bits/subcarrier, multipath Rayleigh fading channel and ML detection; (a) SM OFDM 32-QAM; (b) V-BLAST OFDM 8-QAM.

The SM-OFDM and V-BLAST OFDM systems performance with PSO-based PTS reduction technique have a bit improvement for non linear region of HPA. One reason for this situation can be the system transmit just one sequence of phases with the lowest PAPR medium among the antennas. The ideal would be to transmit the best phases-sequence for each antenna but it has the disadvantage of increasing receiver complexity and overhead.

C. Performance under Not Ideal SI estimation

In the Fig. 5.a), we have the performance of the SM-SC and V-BLAST-SC just for the SI phases. We use the MMSE detection to 2×4 and 4×4 antennas. The ML detection has a prohibitive execution time due to its high complexity in the single carrier system since the symbols need to be detected simultaneously. Both systems have better performance to 2×4 antennas; consequently, this configuration to transmit the SI was chosen.

Ensuring the SI recovering with no error we need to find the best E_b/N_0 ratio to achieve this goal. The performance in Fig. 6 demonstrates the effectiveness of the PSO-based PTS technique in V-BLAST OFDM and SM-OFDM systems equipped

with 2×4 antennas, 512 subcarriers, 6 bits/subcarrier, ML detection, transmission of the 16 phases of SI, 2^6 quantization levels and MMSE detection. Notice that for the V-BLAST OFDM system in the Fig. 6.b) the best performance is obtained under $\text{SNR} = 21\text{dB}$, while for the SM-OFDM system in the Fig. 6.a) the best performance is for $\text{SNR} = 27\text{dB}$.

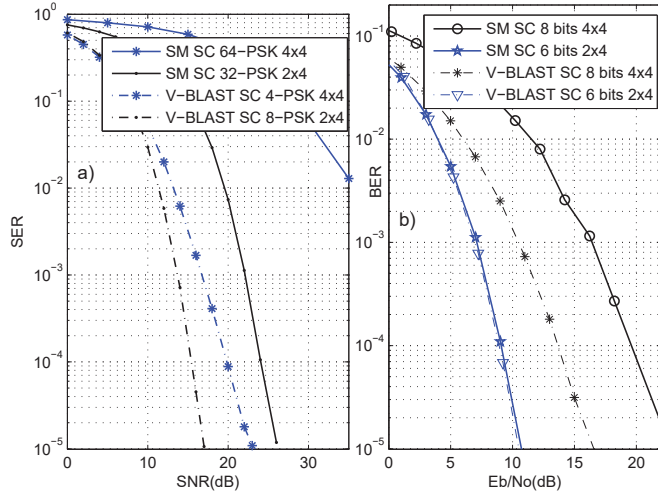


Fig. 5. Performance for the SM-SC and V-BLAST-SC systems, multipath Rayleigh channel and MMSE detection: a) SER for transmission of 16 phases of SI; b) BER

We can observe that the overhead to V-BLAST OFDM and SM OFDM system with the transmission of SI is 3.1% to 512 subcarriers and 25% to 64 subcarriers. For $N < 64$, we conclude that it is impracticable.

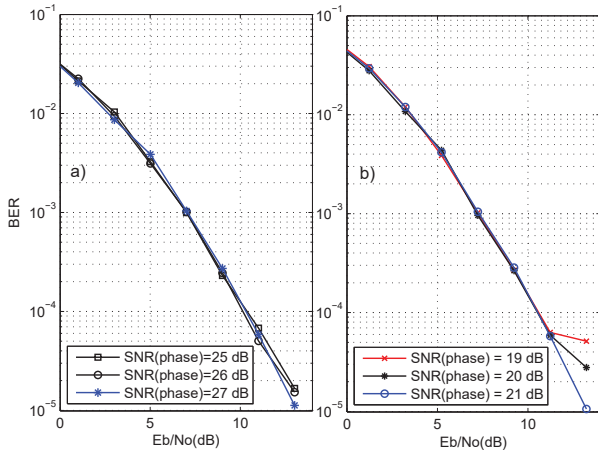


Fig. 6. BER performance with PSO-based PTS technique, 2×4 , $N = 512$ subcarrier, $S = 6$ bits/carrier, Rayleigh channel, ML detection. SC transmission of 16 phases of SI, 2^6 quantization levels and MMSE detection: (a) SM OFDM; (b) V-BLAST OFDM.

D. SM-SC and V-BLAST-SC Performance

Avoiding the problem of PAPR we can use the V-BLAST-SC and SM-SC as shown in the Fig. 5.b). To 2×4 antennas both the systems with SC technique are about 2dB better than both the systems with OFDM technique in the linear region (no clipping HPA). On the other hand, to 4×4 antennas, the V-BLAST-OFDM system outperform the V-BLAST SC system even if the system is saturated. While the SM-OFDM are better than SM-SC just to low saturation. Despite that the SM-OFDM and V-BLAST-OFDM can be used to high subcarriers number to keep down the overhead.

V. CONCLUSION

In this paper we apply the PAPR reduction with ideal and not ideal SI estimation. We analyze the V-BLAST OFDM and SM OFDM systems with specific PAPR reduction technique with SI recover and compare with V-BLAST SC and SM SC systems. For a given system configuration the SC demonstrated superior performance. The disadvantage of PSO-based PTS technique for PAPR reduction is the increased transmission rate, since at the receiver side it must know the SI aiming to retrieve the signal appropriately. To simplify reception and given a specific configuration, the best sequence of phases is obtained by averaging the PAPR of the antennas in SM and V-BLAST systems. Some clear differences between the OFDM and SC techniques are that we do not need the SI while avoiding the PAPR problem in the SM-SC and V-BLAST-SC systems. Comparing the system performance, one can conclude that both 2×4 SM-SC and V-BLAST-SC systems are better than the SM-OFDM and V-BLAST OFDM systems operating in linear region of the amplifier. Equipped with 4×4 antennas the V-BLAST OFDM system outperforms the V-BLAST SC even under saturated system with the drawback of loss on the throughput. To reduce the overhead, the SM-OFDM and V-BLAST-OFDM need to be used just under high number of sub-carriers ($N > 64$).

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