A Message-passing Approach to Precoder Selection in Wireless Communication Networks

Igor M. Guerreiro, Dennis Hui, and Charles C. Cavalcante

***Abstract*— This paper addresses distributed techniques to the problem of precoder selection in a multi-cell scenario. This proposes an iterative method based on a message-passing procedure in factor graphs. A particular example of discrete precoder codebook is considered for transmission of a single data stream on two transmit antennas. For more realistic results, the wireless channel is modeled based on measured data. Evaluations on the potential of such an approach in a wireless communication network are provided and its performance and convergence properties are compared with the greedy approach. Also, the output of the first iteration of the graph-based method is compared with greedy solution. Simulation results for the precoder selection example are presented and discussed, which show that the graph-based technique generally obtains gain in sum rate over the other approaches at the cost of a larger message size. Besides, the proposed method usually reaches the global optima in a efficient manner in terms of computational complexity and signaling load.**

***Keywords*— distributed optimization; message-passing algorithm; factor graphs; precoder selection.**

I. Introduction

In a cellular network, there are many occasions in which each cell needs to set a parameter value, such as reference signal, transmit power, beam direction, or scheduled user, in such a way that the setting is, preferably in a compatible way with the settings of the neighboring cells, in order to achieve a certain notion of optimality, such as maximizing the average system or user throughput, of the entire network [1], [2]. The choice made by one cell on a local parameter often affects the interference level experienced by its immediate neighbors and hence their respective choices made on their local parameters, which in turn would influence the choices made by their neighbors' neighbors.

In some cases, such as the transmit power control or precoder selection problems, the parameter is dynamic and requires coordination to be continually performed. Therefore, a systematic methodology for coordinating the choices of any parameters across the network is desired. Moreover, in order to facilitate flexible, dense deployment of small base-stations in future cellular networks, there is also an increased interest in methods of performing the coordination of parameters among neighboring cells in an autonomous and distributed fashion without a central controller, as any unplanned addition (or removal) of base-stations can substantially alter the system topology and thus the preferred settings.

Factor graph and the associated sum-product algorithm have been widely used in probabilistic modeling of the relationship among interdependent (random) variables or parameters. There are numerous successful applications [3] including, most notably, various fast-converging algorithms for decoding low-density parity check (LDPC) codes and turbo codes, generalized Kalman filtering, fast Fourier transform (FFT), etc. Similar (but different) applications of factor graphs have also been recently proposed for the problem of fast beam coordination among base-stations in [4], [5], [6], [7]. The basic idea in those works is to model the relationship between the local parameters to be coordinated among different communication nodes of a network and their respective performance metrics or costs using a factor graph [3]. In [4], [5], the belief propagation algorithm is adopted to solve the downlink transmit beamforming problem in a multi-cell multiple-input-multiple-output (MIMO) system considering a one-dimensional cellular model. Moreover, in [6], [7] some message-passing algorithms (including the sum-product algorithm) are deployed to coordinate parameters of downlink beamforming in a distributed manner in a multi-cell single-input-single-output (SISO) system.

In this work, we propose a method founded on the min-sum algorithm on factor graphs for the application of precoder selection in a distributed manner. Different from our work in [8], where the underlying method was applied to the problems of transmit antenna selection (TAS) and fixed-beam selection, in this work precoding matrices are considered to be coordinated. Based on factor graphs, a variant of the sum-product algorithm [3], namely the min-sum algorithm [9], can then be applied in order for all nodes, through iterative message passing with their respective neighbor nodes, to decide upon the best set of local parameters that can collectively maximize a global performance metric across the network. The algorithm allows each communication node to be indecisive of its own decision until sufficient information about how its decision would affect the overall network performance is accumulated. The performance of such a graph-based method along with other distributed methods, e.g. game-theoretic approach [10], for coordination of discrete parameters in a wireless communication network are evaluated.

II. System Model

Consider a communication network with N communication nodes. A communication node described here represents a pair of base-station (BS) and its associated user equipment (UE)[1] in a multi-cell MIMO system. Particularly, only downlink transmissions are considered. Each BS has N_t available transmit antennas and each UE has N_r receive antennas. Let p_i denote a discrete parameter of the ith communication node, or simply node i, whose value is drawn from a finite set $\mathcal{P}_i$ of $|\mathcal{P}_i|$ possible parameter values for that node, where $|\mathcal{P}_i|$ denotes the cardinality of $\mathcal{P}_i$, and let

$$\mathbf{p} \equiv \begin{bmatrix} p_1 & p_2 & \cdots & p_N \end{bmatrix}^T$$

be a vector collecting all the parameters in the network, where

$$p_i \in \mathcal{P}_i,\ i = 1, 2, \ldots, N.$$

Each node i is associated with a list $\mathcal{N}_i$ of proper neighbor nodes (i.e. excluding node i) whose choices of parameter values can affect the local performance of node i. For convenience, also let

$$\mathcal{A}_i \equiv \mathcal{N}_i \cup \{i\}$$

denote the "inclusive" neighbor list or just the neighbor list of node i. Let $\mathbf{p}_{\mathcal{A}_i}$ denote the vector of those parameters of nodes in $\mathcal{A}_i$,

Igor M. Guerreiro and Charles C. Cavalcante, Wireless Telecommunications Research Group (GTEL), Federal University of Ceará (UFC), Fortaleza-CE, Brazil, E-mails:{ igor,charles}@gtel.ufc.br, Dennis Hui, Ericsson Research, San José, California, USA, E-mail: dennis.hui@ericsson.com, This work was partially supported by the Innovation Center, Ericsson Telecomunicações S.A., Brazil, and CNPq (306774/2011-9).

[1]For convenience, each item of UE is simply referred to as a UE.

with its ordering of parameters determined by the sorted indices in $\mathcal{A}_i$. Associated with each node i is a performance metric or cost, denoted by $M_i\left(\mathbf{p}_{\mathcal{A}_i}\right)$, which is a function of those parameters in the neighbor list $\mathcal{A}_i$ of node i. Each node i is assumed to be capable of communicating with all nodes in $\mathcal{A}_i$.

In this work, each parameter p_i represent a precoding matrix index (PMI) for BS i indicating which precoder from a predetermined set $\mathcal{P}_i$ of precoders that BS i should use at a certain radio resource block to transmit signals. In practical systems, different UEs may be scheduled, and thus different precoders may be used, at different radio resource blocks. In this case, the coordination of precoders may be performed independently for each individual radio resource block.

The BS i transmits precoded and spatially multiplexed vector $\mathbf{x}_i$ to its associated UE i. The vector $\mathbf{x}_i$ is defined as

$$\mathbf{x}_i = \sqrt{\frac{1}{N_s}}\mathbf{W}_i\mathbf{s}_i,$$

where N_s is the number of data streams, $\mathbf{s}_i$ is the $N_s \times 1$ spatially multiplexed (SM) symbol vector and $\mathbf{W}_i \in \mathcal{W}$ is the $N_t \times N_s$ precoding matrix specified by the parameter p_i. Here, $\mathcal{W}$ is the finite set of all precoding matrices available for every communication node in the network. In order to index the elements of $\mathcal{W}$, assume an index set $\mathcal{I}$, which is equivalent to $\mathcal{P}_i$ for all the communication nodes. Then, a bijective function $f\colon \mathcal{P}_i \leftrightarrow \mathcal{W}$ maps the elements of $\mathcal{P}_i$ onto the elements of $\mathcal{W}$ properly. This work focuses on the particular case of precoding matrices $\mathbf{W}$ for the $N_t = 2$ and $N_s = 1$, considering complex weighting, case. That is,

$$\mathcal{W} = \left\{ \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\end{bmatrix}, \begin{bmatrix}1\\j\end{bmatrix}, \begin{bmatrix}1\\-j\end{bmatrix} \right\}, \tag{1}$$

and

$$\mathcal{P}_i \equiv \mathcal{I} = \{1, 2, 3, 4\}. \tag{2}$$

The sampled incoming signal vector at the UE i is given as being

$$\mathbf{y}_i = \sqrt{g_{ii}}\mathbf{H}_{ii}\mathbf{x}_i + \sum_{j \in \mathcal{N}_i} \sqrt{g_{ji}}\mathbf{H}_{ji}\mathbf{x}_j + \mathbf{v}_i\,, \tag{3}$$

where $\mathbf{H}_{ji}$ denotes the MIMO channel response from BS j to the UE served by BS i in the downlink, *quasi*-static over a data block, and $\mathbf{v}_i$ is a zero-mean circularly symmetric complex Gaussian (ZMCSCG) noise vector with covariance matrix $N_o\mathbf{I}$. The constant g_{ji} is a gain that corresponds to the path loss of each signal, here modeled in a simplified way as being

$$g_{ji} = \left(\frac{1}{d_{ji}}\right)^{\alpha}, \tag{4}$$

where the constant α refers to the path loss exponent and d_{ji} is the distance between the transmitter j and the receiver i. The second term on the right-hand side refers to the interference caused by the neighboring communication nodes. For each transmitter, the average transmit power is constant and given by

$$\mathbb{E}\left\{\|\mathbf{x}_i\|^2\right\} = \frac{1}{N_s}\mathrm{tr}\left(\mathbf{W}\mathbf{W}^H\right) = P_T, \tag{5}$$

where P_T is the average transmitted power in units of energy per signaling period. Also, the symbols are assumed to be uncorrelated, which means that $\mathbb{E}\left\{\mathbf{s}_i\mathbf{s}_i^H\right\} = \mathbf{I}_{N_s}$.

The channel response includes both a fast fading component and a path loss component, the latter determined by the distance between the corresponding BS and UE, according to (4). A basic system model is therefore needed to compute the relative distances between BS and UEs for each random drop of UEs in the cell grid considering fixed BSs' positioning. For convenience, the log-normal shadowing has not been modeled in this work. To obtain more realistic results, each MIMO channel response was drawn from a data set $\mathcal{D}$ of measured channel matrices acquired by Ericsson Research during measurement campaigns made in Kista neighborhood, in Stockholm, Sweden. The measurement campaigns were performed using a single BS placed on the roof of a building and a UE mounted inside a van at a convenient driving speed (see more details in [11], [12]). A total of 324,000 samples of 1×2 channel matrices measured along a particular route of Kista compounds the set $\mathcal{D}$. For the sake of removing any "original" large-scale fading effect, each entry of the channel matrices was previously transformed into a zero-mean and unity-variance variable, such that

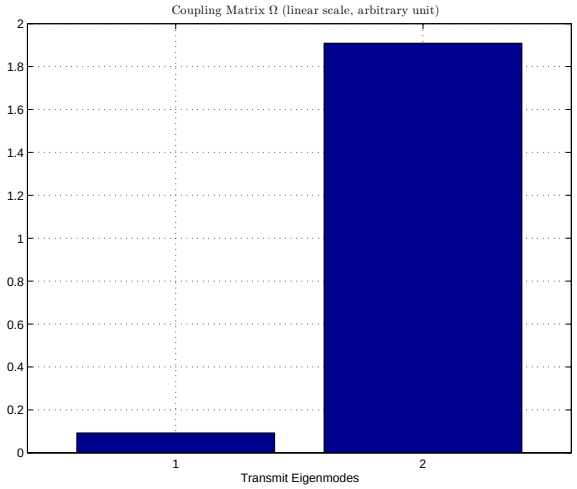

Fig. 1. Estimated power coupling matrix of 1x2 MIMO channel matrices.

$$[\mathbf{H}_{ji}]_{1,l} = \frac{[\mathbf{D}_{ji}]_{1,l} - \mu_D}{\sigma_D}, \tag{6}$$

where $\mathbf{D}_{ji} \in \mathcal{D}$, for $l = 1, 2$, randomly picked up from set $\mathcal{D}$, is associated with receiver j and transmitter i, μ_D and σ_D^2 are the mean and the variance of the entries of the matrices in $\mathcal{D}$, and $\mathbf{H}_{ji}$ is the transformed MIMO channel matrix also associated with receiver j and transmitter i. The index l indexes the element $(1, l)$ of both matrices $\mathbf{D}_{ji}$ and $\mathbf{H}_{ji}$. Then, the path loss modeled by the parameters g_{ji} is in turn inserted to the matrix $\mathbf{H}_{ji}$ according to (3). It is worth noting that each element of $\mathcal{D}$ is randomly chosen only once so that each pair of receiver and transmitter has a different channel matrix.

Particularly, the resulting channel matrices are characterized by the presence of only one eigenmode. Such a feature is observed in the estimated power coupling matrix Ω [13] of resulting MIMO channel matrices, which is shown in Figure 1. The matrix Ω shows the spatial arrangement of scattering objects between the transmitter and the receiver, where its columns refer to the transmit eigenmodes, the rows the receive eigenmodes. This matrix characterizes the entire data set $\mathcal{D}$. Consequently, as all the pairs of receiver and transmitter draw their channel matrices from $\mathcal{D}$, they observe the same spatial correlation. The motivation to use such a channel model is to provide a suitable scenario for the beam selection technique, which usually benefits from the characteristics of spatially-correlated channel matrices.

III. Parameter Coordination Problem

Our goal is for each node i to find, in a distributed fashion, its own optimal parameter p_i^*, which is the corresponding component of the optimal global parameter vector $\mathbf{p}^*$ that minimize the total (global) performance metric given by

$$M\left(\mathbf{p}\right) \equiv \sum_{i=1}^{N} M_i\left(\mathbf{p}_{\mathcal{A}_i}\right). \tag{7}$$

where $M_i(\mathbf{p}_{\mathcal{A}_i})$ that represents the negative of the data throughput [14], [15] of the cell corresponding to BS i. The local performance metric of node i, i.e. $M_i(\mathbf{p}_{\mathcal{A}_i})$, is measured by

$$M_i(\mathbf{p}_{\mathcal{A}_i}) = -\log\det\left(\mathbf{I} + |g_{ii}|\mathbf{R}_i^{-1}\mathbf{H}_{ii}\mathbf{W}_i\mathbf{W}_i^H\mathbf{H}_{ii}^H\right), \quad (8)$$

where $\mathbf{R}_i$, defined herein as

$$\mathbf{R}_i \triangleq \mathbf{R}_{\mathbf{v}_i} + \sum_{j\in\mathcal{N}_i}|g_{ji}|\mathbf{H}_{ji}\mathbf{W}_j\mathbf{W}_j^H\mathbf{H}_{ji}^H, \quad (9)$$

denotes the covariance matrix of the noise-plus-interference experienced by the UE served by BS i in the downlink given that $\mathbf{R}_{\mathbf{v}_i}$ is the covariance matrix of the noise vector $\mathbf{v}_i$.

Hence, the goal here is to employ a distributed algorithm for the BS to negotiate their choices of downlink precoding matrices with their respective neighbors so that the total data throughput in the network is maximized. This problem of coordinating parameters can be solved adopting basically two types of solutions: *1*) centralized approach, which yields the optimal global parameter vector; and *2*) distributed approaches, which on one hand often provide sub-optimal solutions through greedy techniques such as non-cooperative games, but on the other hand can provide near-optimal solutions by using message pass in factor graph.

A. Centralized Solution

Conceptually, the simplest approach to the optimization problem described above is to solve it jointly at a central location by direct computing

$$\mathbf{p}_C \equiv \begin{bmatrix} p_{C,1} \\ p_{C,2} \\ \cdots \\ p_{C,N} \end{bmatrix} \equiv \arg\min_{\mathbf{p}} \sum_{i=1}^{N} M_i(\mathbf{p}_{\mathcal{A}_i}), \quad (10)$$

which is an optimal solution to the problem by definition. A major issue of this approach is its huge computational complexity for large network size N, as the complexity grows exponentially as the number of communication nodes increases, along with the inherent high signaling load (backhaul traffic) between the communication nodes and a central processing unit.

The computational complexity of the centralized solution is indeed very high. The minimum (or maximum) value of a cost (utility) function is usually found throughout all the combinations of the discrete parameters. The total number of combinations of parameters c is given by

$$c = \prod_{i=1}^{N} |\mathcal{P}_i|.$$

For instance, if the network has $N = 61$ nodes and $|\mathcal{P}_i| = 3$ for $i = 1, 2, \ldots, N$, a number of $c \approx 10^{29}$ computations must be done to find the optimal value, which might be computationally prohibitive. Alternatively, the centralized technique may be replaced with the standard alternating-coordinate optimization technique for dense networks. Such an approach starts with an arbitrary choice of $\mathbf{p}$ and iteratively optimizes each element (or a particular set of elements) of $\mathbf{p}$ one at a time while holding others fixed. Its convergence to the globally optimum result can be guaranteed under under some conditions, e.g. convex utility/cost function.

B. Greedy Solution

Another approach to the optimization problem above is for each communication node to selfishly set its own parameter to optimize its own local performance based on the most recent choices made and given by its neighbors. In this approach, the complexity of each node grows only linearly with the cardinality of the set of parameters $\mathcal{P}_i$ since only the parameters of the node itself is considered in the optimization at the node. More precisely, in this approach, the local parameter p_i at each communication node is iteratively chosen as

$$p_{S,i}^{(n+1)} \equiv \arg\min_{p_i} M_i(\mathbf{p}_{\mathcal{A}_i})\Big|_{\mathbf{p}_{\mathcal{N}_i}=\mathbf{p}_{S,\mathcal{N}_i}^{(n)}} \quad (11)$$

where n is the iteration index, $p_{S,i}^{(n+1)}$ denotes the choice of p_i made at iteration $n+1$ using this selfish/greedy approach, and $\mathbf{p}_{S,\mathcal{N}_i}^{(n)}$ denotes the vector of those parameter choices made by nodes in $\mathcal{N}_i$ at the nth iteration. In turn, the parameter $p_{S,i}^{(n)}$ at each node is exchanged to its neighboring nodes so that every node obtains its parameter vector $\mathbf{p}_{S,\mathcal{N}_i}^{(n)}$ to compute its next parameter $p_{S,i}^{(n+1)}$. From a game-theoretic perspective, the greedy solution may be seen as a non-cooperative game defined as a pure-strategy Nash equilibrium (NE) [16].

Remark 1: Considering some simplifications, the solution for precoder selection described in [17] can be seen as a single iteration of the greedy solution, particularly the first iteration. That is, such a solution is the first best response reaction in (11), which may be formulated as

$$p_{L,i} \equiv \arg\min_{p_i} M_i(\mathbf{p}_{\mathcal{A}_i})\Big|_{\mathbf{p}_{\mathcal{N}_i}=\mathbf{p}_{S,\mathcal{N}_i}^{(1)}} \quad (12)$$

where $p_{L,i}$ denotes the choice of p_i based on $\mathbf{p}_{S,\mathcal{N}_i}^{(1)}$, which corresponds to the vector of those parameter choices made by nodes in $\mathcal{N}_i$ at $n = 1$ in the greedy approach.

In the following we describe another approach to the problem of minimizing the global metric in (7) by modeling the communication nodes and the associated local performance metrics using a factor graph.

IV. Message Pass in Factor Graphs

A factor graph is a bipartite graph consisting of a set of variable nodes and a set of factor nodes. Each variable node represents a variable and can only be connected to a factor node (but not another variable node) through an edge, while each factor node represents a factor which is a function of some of the variables. A factor node is connected to a variable node if and only if the corresponding function represented by the factor node depends on that variable.

Given a multivariate function, a factor graph expresses the mathematical structure of the factorization of such a multivariate function into several local functions. In our problem at hand, the global performance metric $M(\mathbf{p})$ is factorized into a sum of N local performance metrics $M_i(\mathbf{p}_{\mathcal{A}_i})$, which is described in (7) [8]. Specifically, for the problem formulated above, we associate each variable node with the parameter p_i of a communication node and each factor node with its local performance metric $M_i(\mathbf{p}_{\mathcal{A}_i})$. Accordingly, we label a variable node corresponding to p_i as $v(p_i)$ and a factor node corresponding to $M_i(\mathbf{p}_{\mathcal{A}_i})$ as $v(M_i)$. An edge connecting a factor node $v(M_i)$ with a variable node $v(p_k)$ exists if and only if $k \in \mathcal{A}_i$.

The graph for the problem of a communication network, for the case of $N = 7$ communication nodes, can be organized as in Figure 2, which clearly shows the bipartite property of the graph with factor nodes connected only to variable nodes through the respective edges [8].

A message-passing algorithm, namely the min-sum algorithm[2], can then be executed on such a graph [8]. Each message depends only on the variable whose associated variable node is a vertex of the edge over which the message is passed along.

[2]The min-sum algorithm is the variant of sum-product algorithm that is based on the min-sum commutative semi-ring [9].

More precisely, each message is simply a table of values with each entry corresponding to one of the possible values of the variable. Figure 3 shows the two kind of messages passing on in a fragment of a factor graph. The min-sum algorithm, when applied to our problem at hand, simply iterates between the following two kinds of message computations and exchanges [8]:

1) *Factor node to Variable node*:

$$\mu_{M_i \to p_k}(p_k) = \min_{\mathbf{p}_{\mathcal{A}_i \setminus \{k\}}} \left\{ M_i\left(\mathbf{p}_{\mathcal{A}_i}\right) + \sum_{j \in \mathcal{A}_i \setminus \{k\}} \mu_{p_j \to M_i}\left(p_j\right) \right\}, \tag{13}$$

where the notation $\setminus\{k\}$ means the underlying operator is performed over all associated variables except to variable k. To prevent messages from increasing endlessly, the messages are normalized to have zero mean.

2) *Variable node to Factor node*:

$$\mu_{p_k \to M_i}\left(p_k\right) = \sum_{j \in \mathcal{A}_k \setminus \{i\}} \mu_{M_j \to p_k}\left(p_k\right), \tag{14}$$

which aggregates all the incoming messages at variable node $v\left(p_k\right)$ except to the one from factor node $v\left(M_i\right)$.

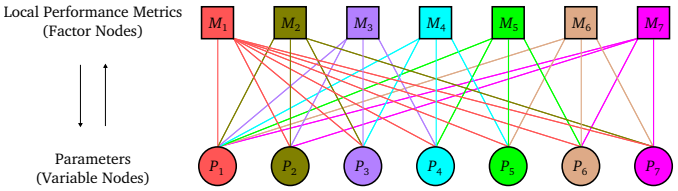

Fig. 2. Organized factor graph for a communication network of 7 communication nodes with local parameters and local performance measures.

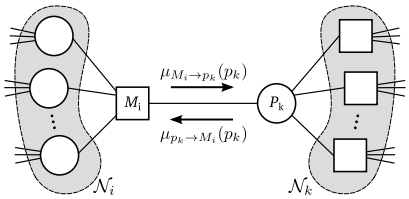

Fig. 3. A factor-graph fragment, showing the message pass between factor node M_i and variable node p_k.

Here, an ideal error-free message pass is considered. It is worth mentioning that the initialization with unit messages may lead nodes to compute and propagate messages with equal entries. In such situation, nodes are not capable of iteratively finding the best parameters as all the entries return the same cost. To circumvent this, the initial incoming messages $\mu_{p_k \to M_i}\left(p_k\right)$ can be initialized to random values close to zero. Upon receipt of the message $\mu_{M_i \to p_k}\left(p_k\right)$, each variable node $v\left(p_k\right)$ then compute outgoing message $\mu_{p_k \to M_i}\left(p_k\right)$ to $v\left(M_i\right)$ for each $i \in \mathcal{A}_k$. Those new messages $\mu_{p_k \to M_i}\left(p_k\right)$ can be conveniently normalized to avoid messages increasing endlessly. The parameter for communication node i is determined at its variable node $v\left(p_i\right)$ by

$$p_i^* = \arg\min_{p_i} \left\{ \sum_{j \in \mathcal{A}_i} \mu_{M_j \to p_i}\left(p_i\right) \right\}. \tag{15}$$

The algorithm then iterates until a stopping criterion is reached, either a pre-determined maximum number of iteration λ or when the set of parameters computed in (15) converges to a fixed state, that is, the updated messages are equal to the previous computed messages, or equivalently,

$$p_i^{(n+1)} = p_i^{(n)}, \quad \forall i = 1, 2, \ldots, N, \tag{16}$$

where n is an iteration index such that $n \leq \lambda$.

Note that both messages computed in (13) and (14) depend only on the value of p_k. Since $p_k \in \mathcal{P}_k$ and $\mathcal{P}_k$ is assumed to be discrete and finite, each of the messages can be represented by a table of $|\mathcal{P}_k|$ entries. In particular, the computation in (14) is just adding up the corresponding entries of multiple tables of the same size together.

When the factor graph contains no cycle (i.e. a closed path in the graph), it can be shown [3] that the message passing algorithm described above will yield the exact optimal solution that optimizes (7) in a single iteration. However, when it contains cycles the algorithm has no natural termination and the messages pass multiple times on the edges of the factor graph in an iterative manner. In this case, the algorithm typically yields good approximations to the true optimal solution [3].

Remark 2: For cyclic graphs, aspects of message-passing scheduling must be considered in order to ensure convergence. In this work we assume the same strategy used in [8].

V. Simulation Results

The global performance metric of the precoder selection problem presented in (7) is investigated in order to evaluate how it behaves statistically in terms of cumulative distribution functions (CDFs) curves. The graph-based technique is compared with the centralized solution, which is optimal by definition and the greedy solution, which is expected to provide a sub-optimal result. Additionally, the non-iterative solution, described in Section III-B as the output of the first iteration of the greedy technique, as compared with the output of the first iteration of the graph-based method. Moreover, the 50th CDF percentile of the sum rate is evaluated to realize how much gain each distributed technique obtains over the iterations. The MIMO setup of the $N = 7$ communication nodes is such that each transmitter has $N_t = 2$ available transmit antennas, $N_s = 1$ data streams to be transmitted and each receiver has $N_r = 1$ receive antennas, which provides the set of precoding matrices considered. Thus, the parameters to be coordinated are 4 PMIs for all the cells. The signal-to-noise ratio (SNR) is set to 20dB. The parameter initialization is at random, i.e. nodes pick one of the PMIs randomly following a uniform distribution at the beginning of each simulation run in the greedy technique. In the graph-based approach, the initial messages defined in (14) are equal to zero. The maximum number of iterations λ in each simulation run is 100. Finally, a total of 1000 runs were conducted for statistical purposes. These simulation parameters above were adopted for the simultaneous message-passing scheduler in the graph-based technique and other simulation parameters were taken from [8].

The graph-based technique appears to reach the optimal solution provided by the centralized approach in terms of sum rate. In Figure 4, the graph-based technique approaches the optimal solution in all the simulation runs, reaching the global optimum in 98% of the cases. That is, the proposed method provides a near-optimal solution. The maximum achievable sum rate is about 38 bits per channel use, reached by both the centralized and the graph-based techniques. The greedy technique reaches about 32 bits per channel use at the most. Figure 4 also shows two additional curves, one regarding the output of the first iteration of the greedy technique, and another considering the output of the first iteration the graph-based method. The former shows that the greedy technique reaches its best response at the very first iteration. The latter indicates the graph-based method can still obtain some gain in sum rate through the iterative process.

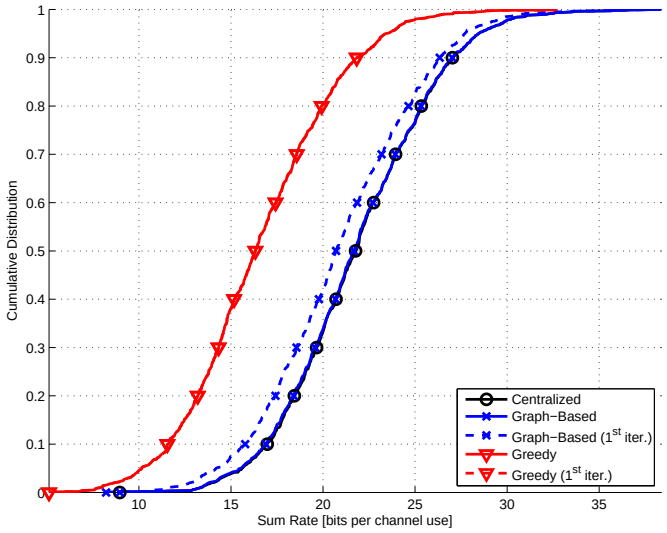

Fig. 4. Performance analysis of graph-based technique for the precoder selection problem in terms of sum rate in 7-node network.

As expected, the graph-based technique outperforms the greedy method with a large gain in sum rate. Figure 5 shows the 50th CDF percentile of the curves in Figure 4. The graph-based technique reaches the globally optimum within the first 10 iterations and the percentage gain obtained over the greedy technique is approximately 33% in sum rate at the fifth iteration. Considering the output at the first iteration, the graph-based approach outperforms the greedy with a percentage gain of about 26% in sum rate. Thus, one may think that the graph-based approach can provide a better performance in terms of sum rate than the non-iterative solution.

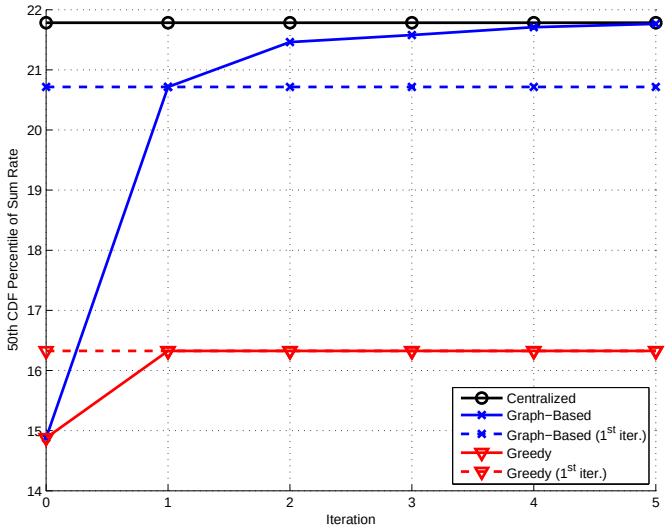

Fig. 5. 50th CDF percentile of sum rate against iteration in 7-node network showing how much gain the graph-based technique obtains over greedy solution.

VI. Conclusions

The graph-based method for distributed parameter coordination considers the impact of nodes decisions on their neighboring nodes. The information (message) exchange is only among neighbors. Such a technique reaches the (near) optimal solution at cost of larger message size compared with the greedy solution. As for the numerical results, the graph-based technique provides good gains in the global cost over the greedy solution and, consequently, the non-iterative solution which is referred to as being similar to the output of the first iteration of the greedy technique. It is worthwhile to note that the graph-based approach is totally adaptable to any discrete problem of parameter coordination and any network size. As future studies, one may think of working on message-passing scheduling with faster convergence and message exchange with reduced message size.

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