Impedance Matching Analysis of an Optical Nanocircuit Fed by a Gaussian Beam

K. Q. Costa, V. Dmitriev, J. L. Souza, and G. L. Silvano

***Abstract*— In this paper, we analyze the impedance matching characteristic of an optical nanocircuit fed by Gaussian beam. The circuit is composed by a receiving and an emitting dipole connected by a two-wire optical transmission line. The receiving dipole is illuminated by a focused Gaussian beam with λ=830nm and beam waist of 340nm. The numerical analysis of this circuit is performed by the linear method of moments. The results show the current distributions, near field diagrams and the voltage reflection coefficient for circuits with different geometrical parameters. We present some conclusions about the conditions where one can have a better impedance matching.**

Keywords—Nanoantennas, optical nanocircuit, impedance matching, Gaussian beam, method of moments (MoM).

I. INTRODUCTION

Aperture probes are optical sensors composed by a tapered optical fiber coated by a metallic layer, and a tiny aperture localized in its apex. The spatial resolution of these optical sensors is better than that of uncoated fiber probes where the resolution is defined by the diffraction limit of the aperture. However, this kind of sensors possesses small transmission efficiency which reduces the field enhancement in front of the aperture [1]. One way to increase this field enhancement and improve the spatial resolution is to use optical antennas near the aperture [2]-[3].

A potential application of optical antennas is to provide a good matching between guided plasmonic waves and radiated fields, and vice versa. One example of enhancement of the reception of propagating surface plasmons using nanodipoles placed in front the waveguide's end aperture is given in [4]. Another example is enhancement of the near field of an aperture optical fiber probe by using a monopole near this aperture [5]. Also, a nanodipole can provide a better enhancement and confinement of the radiated fields of a semiconductor laser diode [6]. In all these examples, the dimensions and resonances of the antennas were optimized to improve the energy transfer between the guided and radiated fields. These problems can be viewed as a classical input impedance matching design, in analogy with the theory of RF- and microwave antennas [7]-[8].

In this paper, we present an impedance matching analysis of an optical nanocircuit fed by an aperture probe, where the circuit is composed by a receiving and an emitting nanoantenna connected by a two-wire optical transmission line (OTL). We model the radiated field from the aperture probe by a focused Gaussian beam [9], which illuminates the receiving dipole. Then we use the classical antenna theory to make the input impedance matching to optimize the energy transfer in the nanocircuit. The numerical analysis of this circuit is performed by a simple and efficient computational method based on method of moments (MoM) [8]. In all the analysis we use a Gaussian beam with λ=830nm and beam waist of 340nm. The results present the variation of the induced current, near field distributions, and reflection coefficient for different length and radius of the transmitting dipole, and different radius and gap of the OTL. We present some conclusions about the conditions where we have better impedance matching.

II. THEORY

This section presents the geometry of the problem, the linear method of moments (MoM) model of the circuit and the Gaussian beam used to feed the circuit.

A. *Description of the Problem*

The optical circuit considered is shown in Fig. 1. In this figure, an aperture probe is electromagnetically coupled to a nanocircuit, which is composed by a receiving dipole, two wire optical transmission line, and an emitting dipole. We model the coupling between the aperture probe and the receiving antenna by an equivalent focused Gaussian beam radiated from the aperture. The polarization of this field is considered parallel to the axis of the receiving dipole to maximum coupling efficiency.

In this system, we focus on the impedance matching problem between the optical transmission line and the emitting dipole, i.e. we are interested in search of the parameters of the nanocircuit which provide maximum energy transferred to the emitting dipole and minimum reflected energy.

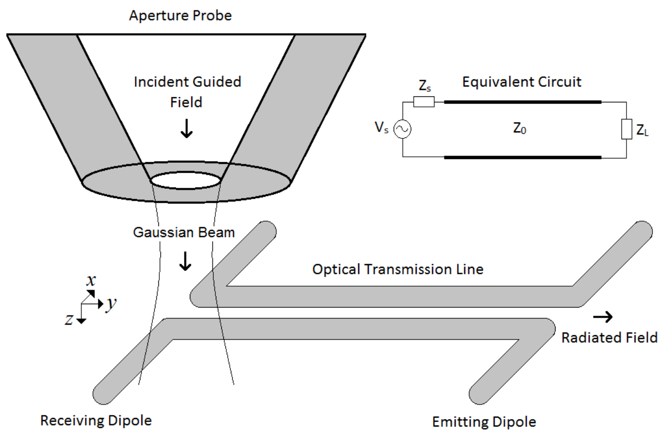

Fig. 1. Aperture probe coupled to optical nanocircuit and equivalent circuit model. Nanocircuit is formed by receiving dipole, optical transmission line and emitting dipole. Material of nanocircuit is gold.

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B. *Method of Moments Model*

To analyze the equivalent nanocircuit of the system presented in Fig. 1, we use the linear method of moments (MoM) approximation with sinusoidal basis functions and equivalent surface impedance [8]. Fig. 2 shows the geometry of the original problem, and the MoM equivalent model. In Fig. 2a, we have receiving dipole 1 (left), an optical transmission line (OTL), and an emitting dipole 2 (right). The dimensions of this nanocircuit are: h_1, a_{h1} and h_2, a_{h2} the arm length and radius of the nanodipoles 1 and 2, respectively, L and a_L the length and radius of the OTL, respectively, d the distance between the axis of the OTL, $D=d-2a_L$ the gap distance between the surfaces of the OTL. The total length of the nanodipoles 1 and 2 are $H_1=2h_1+d$ and $H_2=2h_2+d$, respectively.

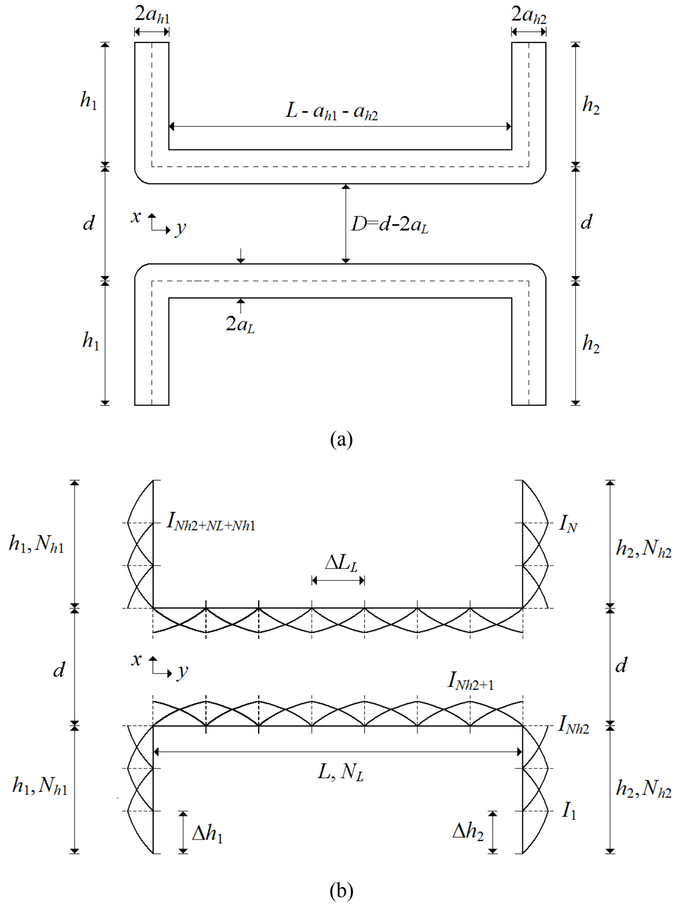


Fig. 2. Cylindrical plasmonic nanocicuit composed by receiving (left) and emitting (right) nanodipoles, and optical transmission line (middle). (a) Geometry of original problem. (b) Discretization used in MoM model.

In the scattering problem of Fig. 1, the gold material of the cylindrical conductors of the circuit is represented by the Lorentz-Drude model for the complex permittivity $\varepsilon_1=\varepsilon_0\varepsilon_{r1}$ [1], where

$$\varepsilon_{r1}=\varepsilon_\infty-\frac{\omega_{p1}^2}{\omega^2-j\Gamma\omega}+\frac{\omega_{p2}^2}{\omega_0^2-\omega^2+j\gamma\omega} \tag{1}$$

being: $\varepsilon_\infty=8$, $\omega_{p1}=13.8\times10^{15}\text{s}^{-1}$, $\Gamma=1.075\times10^{14}\text{s}^{-1}$, $\omega_0=2\pi c/\lambda_0$, $\lambda_0=450\text{nm}$, $\omega_{p2}=45\times10^{14}\text{s}^{-1}$, and $\gamma=9\times10^{14}\text{s}^{-1}$. This model is a good approximation for the wavelengths $\lambda>500\text{nm}$. The losses in metal are described by surface impedance Z_s. This surface impedance can be obtained approximately by considering cylindrical waveguide with the mode TM_{01}. In this case, the surface impedance is given by

$$Z_s=\frac{TJ_0(Ta)}{2\pi aj\omega\varepsilon_1 J_1(Ta)},\quad T=k_0\sqrt{\varepsilon_{r1}},\quad k_0=\omega\sqrt{\mu_0\varepsilon_0}\,. \tag{2}$$

The boundary condition for the electric field satisfied at the surface's conductor is $(\bar{E}_s+\bar{E}_i)\cdot\bar{a}_l=Z_sI$, where a_l is a unitary vector tangential to the surface of the metal, E_s is the scattered electric field due the induced linear current I on the conductor, E_i the incident electric field of the Gaussian beam source (Fig. 1), and I is the longitudinal current in a given point of the nanodipole.

The integral equation of the scattered field along the length l of the nanodipole is given by

$$\bar{E}_s(\bar{r})=\frac{1}{j\omega\varepsilon_0}\left[k_0^2\int_l \bar{I}g(R)dl'+\int_l\frac{dI}{dl'}\nabla g(R)dl'\right],$$
$$g(R)=\frac{e^{-jk_0R}}{4\pi R},\quad R=|\bar{r}-\bar{r}'|,\quad \bar{I}=I\,\bar{a}_l\,. \tag{3}$$

The numerical solution of the problem formulated by (1)-(3), is performed by linear MoM as follows. Firstly, we discretize the linear circuit as shown in Fig. 2b, where N_L, N_{h1} and N_{h2} are the number of straight segments in L, h_1 and h_2, respectively. In this case, we have $N_L=7$, $N_{h1}=N_{h2}=3$. The discretization is uniform in L, h_1 and h_2, but the length of the discretization can be different, i.e. $\Delta L_L=L/N_L$, $\Delta h_1=h_1/N_{h1}$ and $\Delta h_2=h_2/N_{h2}$. With this discretization, the total number of straight segments of the nanocircuit is $N_t=2N_{h2}+2N_L+2N_{h1}$. For stability of the method, we use the convergence conditions $\Delta h_2>2a_{h2}$, $\Delta h_1>2a_{h1}$, and $\Delta L_L>2a_L$. Later, the current in each segment is approximated by sinusoidal basis functions. The expansion constants I_n are shown in Fig. 2b where each constant defines one triangular sinusoidal current. To determine these constants, we use $N=N_t-2$ rectangular pulse test functions with unitary amplitude and perform the conventional testing procedure. The following linear system of equations is obtained

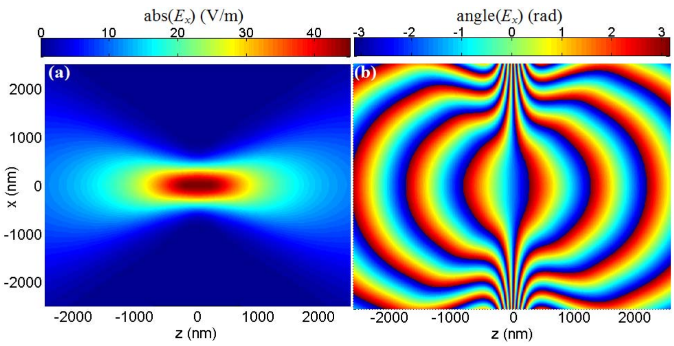
$$V_m=Z_sI_m\Delta_m-\sum_{n=1}^{N}Z_{mn}I_n,\quad m=1,2,3,\ldots,N \tag{4}$$

where Z_{mn} is the mutual impedance between sinusoidal current elements m and n, $\Delta_m=1/2[\Delta L_m+\Delta L_{m+1}]$. The solution of (4) produces the current along the nanocircuit. With this solution, it is possible to calculate the near and far field distributions of the electric field and other parameters.

C. *Gaussian Beam Source*

In all the analysis presented in this work we use a Gaussian beam with wavelength $\lambda=830\text{nm}$ and beam waist $w_0=340\text{nm}$. We use these values for some comparisons with the reference paper [7], but the analysis presented here can be applied for others parameters. This Gaussian beam is focused on the receiving dipole 1 (Fig. 2), where its polarization is along the dipole axis (x), the direction of propagation is $+z$, the beam axis is along the axis z, and the minimum waist (w_0) is localized at $z=0$, which is the plane of the nanocircuit (Fig. 2).

The field distribution of this Gaussian beam is showed in Fig. 3, where the electric field amplitude abs(E_x) and the phase distribution angle(E_x) at the plane xz are presented in Figs. 3a and 3b, respectively.



(a) (b)

Fig. 3. Field distribution of Gaussian beam with λ=830nm and beam waist w_0=340nm at plane *xz:* (a) field amplitude abs(E_x), (b) phase angle(E_x).

III. NUMERICAL RESULTS

Based in theory presented in the precious section, we developed a MoM code in Matlab to analyze the nanocircuit shown in Fig. 2. The following two sections present examples of simulation, showing the impedance matching characteristics of a particular circuit and a parametric analysis of the impedance matching for different values of h_2, a_{h2}, D and a_L for fixed values of h_1, a_{h1} and L.

A. Numerical Example

This section presents an example of impedance matching analysis of the nanocircuit shown in Fig. 2a fed by the Gaussian beam of Fig. 3. Fig. 4 shows the geometry and the discretizatrion parameters used in this simulation. The current and near field distribution obtained for this example are presented in Figs. 5 and 6, respectively. These results show the stationary behavior in the OTL is because of the mismatching in the impedances of the nanodipole 2 with the OTL.

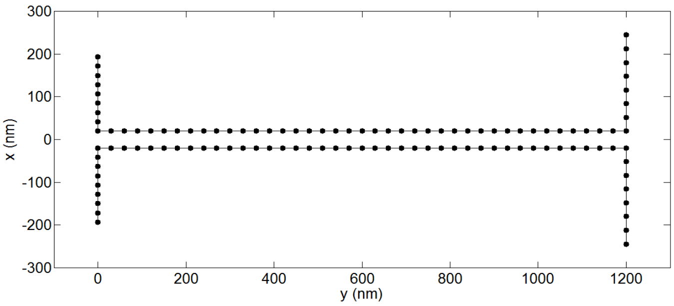


Fig. 4. Geometry and discretization of the example nanocircuit with H_1=386nm, H_2=488nm, L=1200nm, a_{h1}=10nm, a_L=a_{h2}=15nm, D=10nm, N_{h1}=8, N_{h2}=7, N_L=40, ΔL_L=30nm, Δh_1=21.6nm, Δh_2=32nm and N=108.

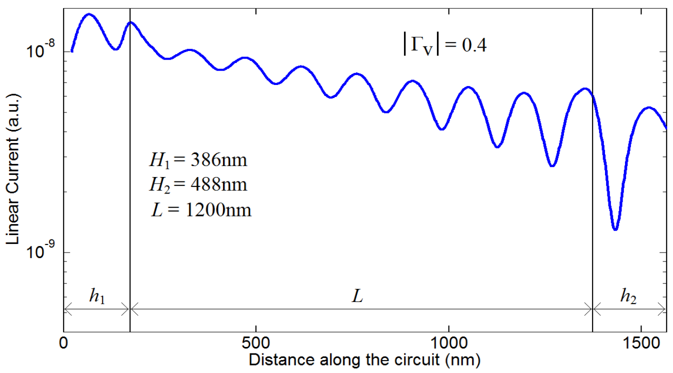


Fig. 5. Linear current distribution along the nanocircuit. The voltage reflection coefficient of this circuit is 0.4.

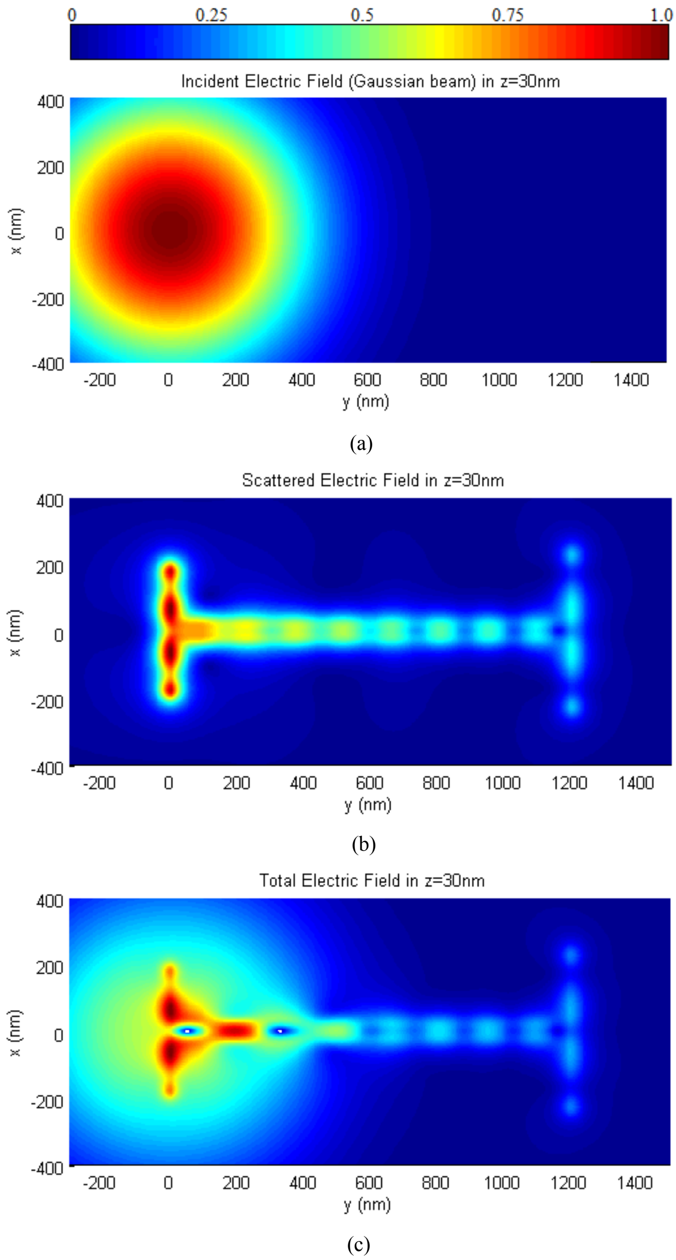


Fig. 6. Normalized electric field distribution at plane z=30nm: (a) incident field of the Gaussian beam. (b) scattered field. (c) total field.

To make a quantitative measure of the impedance matching, we calculate approximately the voltage stationary wave ratio (*VSWR*) near the nanodipole 2 as $VSWR=I_{max}/I_{min}$, where I_{max} and I_{min} are, respectively, the maximum and minimum current magnitude nearest the dipole 2. With this parameter, we calculate approximately the magnitude of voltage reflection coefficient as $|\Gamma_V|=(VSWR-1)/(VSWR+1)$. In this numerical example we obtained $|\Gamma_V|$=0.4.

B. Parametric Analysis

This section presents a parametric analysis of the impedance matching of the nanocircuit for different values of h_2, a_{h2}, a_L and D. In this analysis, we fixed the dimensions h_1=173nm, a_{h1}=10nm and L=1200nm. Figs. 7-10 present the voltage reflection coefficient $|\Gamma_V|$ versus the total length of the nanodipole 2 $H_2=2h_2+d$, for different values of D=10, 15 and 20nm for the four cases (a_L=10nm, a_{h2}=10nm), (a_L=10nm, a_{h2}=15nm), (a_L=15nm, a_{h2}=15nm), and (a_L=15nm, a_{h2}=20nm), respectively.

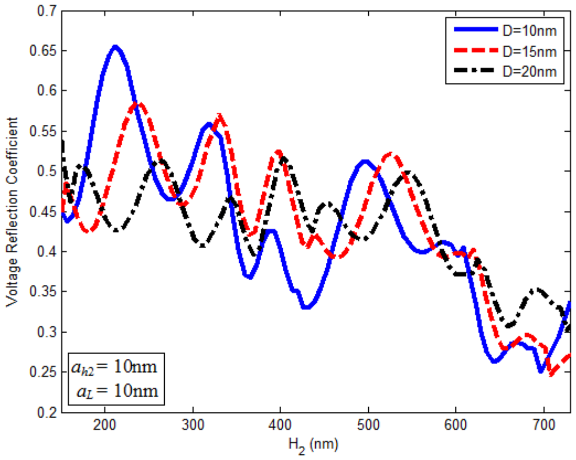


Fig. 7. Voltage reflection versus H_2=2h_2+d for different values of D=10, 15 and 20nm, a_L=10nm and a_{h2}=10nm.

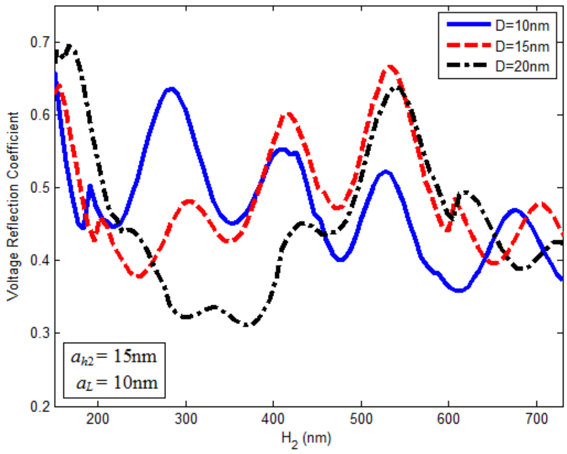


Fig. 8. Voltage reflection versus H_2=2h_2+d for different values of D=10, 15 and 20nm, a_L=10nm and a_{h2}=15nm.

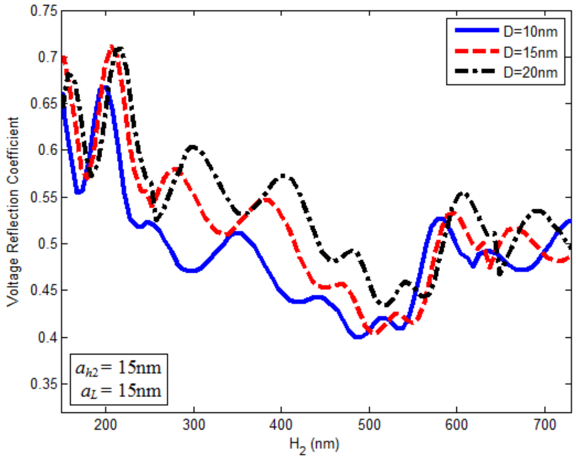


Fig. 9. Voltage reflection versus H_2=2h_2+d for different values of D=10, 15 and 20nm, a_L=15nm and a_{h2}=15nm.

Analyzing these curves we come to some conclusions. We note that the nanocircuits possess in general smaller degree of input impedance matching (higher $|\Gamma_V|$) when the gap of the OTL D is increased. The exception are the cases of Figs. 7 and 8 in the range H_2<400nm, where we have a better matching for higher D.

We also observe that in general the impedance matching is better when a_{h2}>a_L. This means that the values of $|\Gamma_V|$ in Fig.10

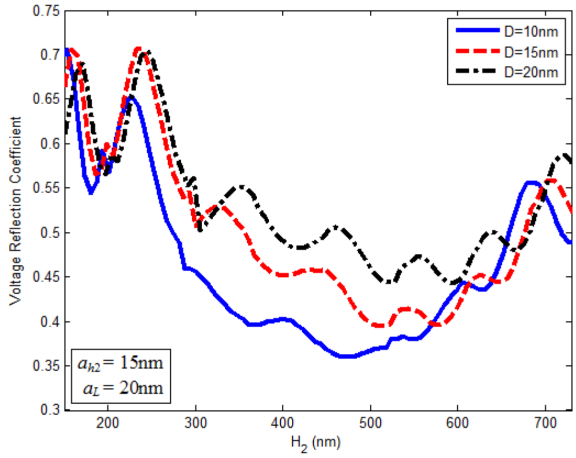


Fig. 10. Voltage reflection versus H_2=2h_2+d for different values of D=10, 15 and 20nm, a_L=15nm and a_{h2}=20nm.

are smaller than $|\Gamma_V|$ of Fig. 9, and the values of $|\Gamma_V|$ in Fig. 8 is smaller than $|\Gamma_V|$ of Fig. 7. This later case is only true in the range of H_2<400nm (Figs. 7and 8).

All these results show that we have many situations of good matching for different values of D, a_L, a_{h2} and H_2. Some optimal results (min$|\Gamma_V|$) occur for larger H_2, for examples the cases D=10nm and H_2=640nm in Fig. 7, where $|\Gamma_V|\approx 0.26$ and D=10nm and H_2=610nm in Fig. 8, where $|\Gamma_V|\approx 0.35$. Other good results occur for smaller H_2, for example the case D=20nm and H_2=300nm in Fig. 8, where $|\Gamma_V|\approx 0.31$.

One way to choose the best geometric parameters of the circuit is to consider the case with better efficiency. The efficiency of the circuit depends mainly on the attenuation of the current along the OTL, i.e., depends on the loss constant α of the OTL. This parameter is constant for the principal mode that propagates on the OTL, and can be obtained approximately by the average inclination of the current versus distance along the OTL. In equation form we have $\alpha = (I_{h1} - I_{h2})/L$, where I_{h1} and I_{h2} are the average amplitude of the input current in the nanodipoles 1 and 2 in decibel (dB), respectively, and L given in nm. With this definition the unit of this parameter is dB/nm. In the numerical example presented in previous section we have $\alpha \approx 0.0081$dB/nm. This result is close to that obtained in [7], where the OTL is similar to our OTL in Fig. 4 (a_L=15nm and D=10nm).

To understand better the behavior of the impedance matching and efficiency characteristic of the results presented in Figs. 7-10 we plot in Figs. 11 and 12 current distributions for different geometric parameters. Fig. 11 shows the currents for two cases with same a_L=10nm but different voltage reflection coefficient $|\Gamma_V|$=0.31 and 0.67. In these results we observe more stationary wave for the case $|\Gamma_V|$=0.31 than the case $|\Gamma_V|$=0.67. This shows that our approximate method to calculate $|\Gamma_V|$ presents a good measure of the degree of impedance matching. We also observe that the two cases present the same attenuation along the circuit, i.e., the two cases possess the same loss constant α= 0.0111dB/nm.

Fig. 12 presents the current distribution for two cases with good impedance matching $|\Gamma_V|$=0.26 and 0.36, but with different loss constant of α=0.0123dB/nm and 0.008dB/nm, respectively. The difference in the values of α in this figure is mainly due to the difference of the radius a_L of the OTL. For lower values of a_L the attenuation is higher on OTL and this result is similar to that observed in RF-microwave regimes. This can be explained by the surface impedance model of (2), where smaller radius produces higher Z_s and, consequently, higher loss in the conductors.

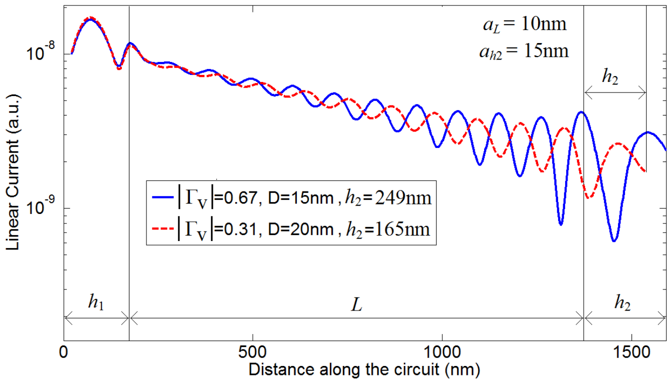


Fig. 11. Current distributions of two circuits with same a_L=10nm but different voltage reflection coefficient (|Γ_V|=0.31 and 0.67).

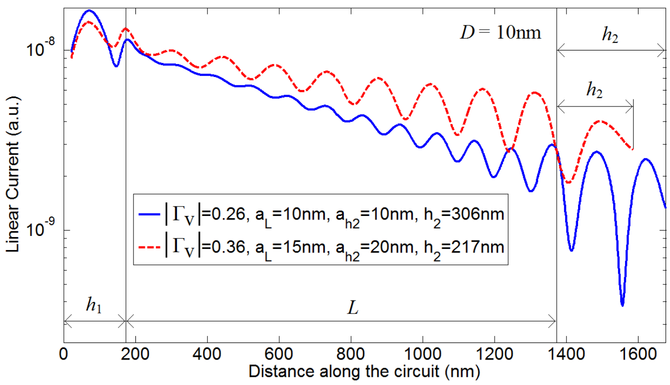


Fig. 12. Current distributions of two nanocircuits with good voltage reflection corfficient (|Γ_V|=0.26 and 0.36) but different a_L.

IV. CONCLUSION

In this paper we presented a theoretical analysis of the impedance matching characteristics of a plasmonic optical nanocircuit fed by a Gaussian beam. The method of moments model with equivalent surface impedance of the conductors was used for numerical calculations. In the numerical examples, we fixed the following Gaussian beam parameters: wavelength λ=830nm and beam waist w_0=340nm. However, such analysis can be performed for beams with other parameters.

In the analysis, we first presented a numerical example and we observed that in general plasmonic optical nanocircuits composed by OTL have properties analogous to those of lossy transmission lines operating in RF-microwave regimes, i.e., similar behaviors of stationary wave and attenuation are observed for both cases (optical and RF-microwave). This suggests applying the same techniques of impedance matching known in RF-microwave regimes for the analysis of OTL.

We presented also a parametric analysis of the reference circuit for different geometrical dimensions such as line and dipole radius, gap distance of the line and length of transmitting dipole. We observed that better impedance matching is obtained when the radius of the transmitting dipole is larger than the radius of the OTL. We also noted that the impedance matching can be optimized varying gap distance of the line and length of transmitting dipole, and that there are many possible solutions for good impedance matching.

With relation to the attenuation loos in the line, we observed that good impedance matching does not necessarily mean good efficiency on the circuit. In other words, we can have circuits with the same degree of impedance matching but with different efficiency. This occurs because the efficiency is mainly a function of the radius of the OTL, where for smaller radius the conductors present larger equivalent surface impedance, and consequently possess higher conduction loss.

The analysis presented here can be useful as guidelines to design efficient plasmonic optical nanocircuits for applications in nanophotonics and nanoelectronics, for example, in efficient aperture probes based on nanoantennas. In future works, we shall investigate the overall efficiency of the nanocircuit and consider some other geometries of nanoantennas and optimize the energy transfer in the circuit.

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