LR-Aided Detection for Large-MIMO System via Ant Colony Optimisation

José Carlos Marinello & Taufik Abrão

***Abstract*— In this work heuristic ant colony optimisation (ACO) procedure is deployed in conjunction with lattice reduction (LR) technique aiming to improve the performance-complexity tradeoff of detection schemes in MIMO communication. A hybrid LR-ACO MIMO detector using the linear minimum mean squared error (MMSE) criterion as initial guess is proposed and compared with other traditional (non)linear MIMO detectors, as well as heuristic MIMO detection approaches from the literature, in terms of both performance and complexity. Numerical results show that the proposed LR-ACO outperforms the conventional ACO MIMO detector, as well as the proposed ACO detector with the MMSE solution as initial guess, with a significant complexity reduction. The LR-ACO-based MIMO detector achieves a substantial improvement in terms of BER performance while expending lower computational resources than others ACO-based MIMO detectors considered. The performance-complexity results suggest that the proposed LR-ACO-MIMO is a promising solution for large number of antennas and/or modulation order, or even correlated MIMO channel scenarios.**

***Keywords*— Large MIMO systems; ACO; lattice reduction; maximum-likelihood estimation; MMSE.**

I. Introduction

Multiple-input-multiple-output (MIMO) systems are known by providing a significant spectral efficiency improvement on wireless communication systems [1]. Therefore, in the last decade many researches in this field have been done due to the new wireless system requirement of high data rates, imposed by the recent telecommunications services and technologies, allied with the large interest in saving resources, like spectrum and power. It is known that very high data rates can be achieved when using a large number of antennas and/or modulations orders (Large MIMO), and that on these cases the detection task becomes challenging, since there is a considerably enhancement on the inter layer/antenna interference (ILI), and/or a lower noise robustness, requiring thus more sophisticated and efficient detection techniques.

Among the linear detectors widely known the zero forcing (ZF) and the minimum mean squared error (MMSE) based techniques present low complexity and the ability to operate under ill-conditioned channel matrices; however both linear detection techniques for MIMO is clearly inferior to the performance achieved by the maximum likelihood (ML) detector. Recently, the sphere decoder (SD) approach [2] has becoming an alternative to the ML, presenting a near-optimum performance; however SD approach results in a prohibitive complexity for implementation under low or medium signal-to-noise ratio (SNR) operation regions on real communication systems; in fact, under low SNR operation region, the SD complexity becomes of the same order of ML complexity.

An appealing procedure to improve the MIMO linear detectors performance under correlated channels and simultaneously resulting in a feasible complexity consists in deploying lattice reduction (LR) technique [3], [4]. This technique transforms the partial correlated channel into an equivalent one, with a better conditioned channel matrix. Having a near-orthogonal near-uncorrelated channel transformation implemented, at the receiver side the detection can be carried out easily deploying a low-complexity linear detector scheme.

The ant colony optimisation (ACO) is a technique inspired on the behavior of ants in nature that was originally proposed for combinatorial optimisation problems, such as the traveling salesman problem [5]. Many works have been disseminated applying this technique to several combinatorial (or discrete) and continuous optimisation problems that arise in telecommunications, such as detection in code division multiple access systems (CDMA) [6] [7], detection in MIMO systems [8] [9], resource allocation on wireless networks [10], among others. A simple ACO procedure applied to MIMO detection is presented in [8], in which the heuristic is combined with ZF and V-BLAST MIMO detectors. However, numerical results show that the proposed detectors were not able to achieve ML performance for medium and high SNR regions. In [9], a different detection scheme based on ACO has been proposed; but also this technique results in a remarkable performance loss compared with ML detector.

Hence, as an alternative to the ACO detector of [8] and [9], we propose in this contribution a near-optimum MIMO detector based on ant colony optimisation heuristic approach, in which part of the search complexity has been shifted for the initial detection stage, by deploying a low complexity linear detector as a start point in the search solution carried out in a second stage. This way, it will be shown that performing the search on the LR reduced domain, it is possible to obtain a significant improvement in the performance $\times$ complexity tradeoff with the LR-ACO detector even when applied to a MIMO system under large number of antennas.

II. MIMO System Model

In the adopted MIMO system, it is assumed that there are n_T transmit antennas at the transmitter side, and n_R receive antennas at the receiver. Besides, the information transmitted symbol vector is $\mathfrak{x} = [x_1\, x_2\, \ldots\, x_{n_T}]^T$, where x_i denotes the transmitted symbol at the ith antenna and

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assumes a value of the squared quadrature amplitude modulation (M-QAM) alphabet; so, the complex-valued symbol (finite) set is given by $\mathcal{S} = \left\{\mathcal{A} + \sqrt{-1} \cdot \mathcal{A}\right\}$, where $\mathcal{A} = \left\{\pm\frac{1}{2}a;\ \pm\frac{3}{2}a;\ \ldots;\ \pm\frac{\sqrt{M}-1}{2}a\right\}$ is the real-valued finite set. The parameter $a = \sqrt{6/(M-1)}$ is used for normalizing the power of the complex valued transmit signals to 1. Using matrix notation, the received complex-valued signal over a MIMO channel is written as:

$$\mathfrak{r} = \boldsymbol{\mathcal{H}}\mathfrak{x} + \mathfrak{n} \tag{1}$$

where the corresponding receive signal vector $\mathfrak{r}$ has dimension $n_R \times 1$; besides, $\mathfrak{n} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$ represents the additive white Gaussian noise (AWGN) with variance $\sigma_n^2 = N_0$, which is observed at the n_R receive antennas, thus $E_b/N_0 = n_R/(\log_2(M)\sigma_n^2)$ holds. Furthermore, the $n_R \times n_T$ complex-valued channel matrix $\boldsymbol{\mathcal{H}}$ was assumed uncorrelated complex Gaussian fading gains with unit variance.

In order to facilitate the numerical analysis, real and imaginary part of (1) are treated separately; so, the system model can be rewritten as:

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{2}$$

with the real-valued channel matrix

$$\mathbf{H} = \begin{bmatrix} \Re\{\boldsymbol{\mathcal{H}}\} & -\Im\{\boldsymbol{\mathcal{H}}\} \\ \Im\{\boldsymbol{\mathcal{H}}\} & \Re\{\boldsymbol{\mathcal{H}}\} \end{bmatrix} \in \mathbb{R}^{n \times m} \tag{3}$$

and the real-valued vectors

$$\mathbf{r} = \begin{bmatrix} \Re\{\mathfrak{r}\} \\ \Im\{\mathfrak{r}\} \end{bmatrix};\quad \mathbf{x} = \begin{bmatrix} \Re\{\mathfrak{x}\} \\ \Im\{\mathfrak{x}\} \end{bmatrix};\quad \mathbf{n} = \begin{bmatrix} \Re\{\mathfrak{n}\} \\ \Im\{\mathfrak{n}\} \end{bmatrix} \in \mathbb{R}^n \tag{4}$$

where $\mathbf{r}, \mathbf{n} \in \mathbb{R}^n$, $\mathbf{x} \in \mathcal{A}^m$, $m = 2n_T$ and $n = 2n_R$. Note that now, the information vector assumes values only over the finite set of real-valued: $\mathbf{x} \in \mathcal{A}^{2n_T}$.

The *maximum likelihood* (ML) detector (MLD) searches the symbol in the set $\mathcal{A}^{2n_T}$ that minimises the distance between the receive signal $\mathbf{r}$ and the reconstructed signal $\mathbf{Hx}$, i.e

$$\widehat{\mathbf{x}}_{\text{ML}} = \arg \min_{\mathbf{x} \in \mathcal{A}^{2n_T}} ||\mathbf{r} - \mathbf{H}\mathbf{x}||^2 \tag{5}$$

Hence, the MLD in MIMO systems is equivalent to an exhaustive search of a combinatorial optimisation problem, which becomes prohibitive when the constellation order and number of antennas increase substantially.

The *conventional minimum mean squared error* (MMSE) detector for MIMO system is another linear detector, whose preprocessor output is given by:

$$\widehat{\mathbf{x}}_{\text{MMSE}} = \mathbf{H}\left(\frac{N_0}{E_s}\mathbf{I} + \mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{r} \tag{6}$$

where $E_s = \log_2(M)E_b$ is the energy per symbol. Note that, by defining the real-valued *extended channel matrix* and the *extended receive signal vector* as follows:

$$\mathbf{H}_{\text{EXT}} = \begin{bmatrix} \mathbf{H} \\ \kappa\mathbf{I}_{2n_T} \end{bmatrix};\quad \mathbf{r}_{\text{EXT}} = \begin{bmatrix} \mathbf{r} \\ \mathbf{0}_{2n_T} \end{bmatrix} \tag{7}$$

where $\kappa = \sqrt{\frac{N_0}{E_s}}$ and $\mathbf{0}_{2n_T}$ is a zero-valued column vector of length $2n_T$, we can redefine (6) as: $\widehat{\mathbf{x}}_{\text{MMSE}} = \mathbf{H}_{\text{EXT}}^{\dagger}\mathbf{r}_{\text{EXT}}$.

III. Lattice Reduction Aided MIMO Detectors

The aim of lattice-reduction is to transform a given basis $\mathbf{H}$ into a new basis $\widetilde{\mathbf{H}}$ with vectors of shortest length or, equivalently, into a basis consisting of roughly orthogonal basis vectors. Usually, $\widetilde{\mathbf{H}}$ is much better conditioned than $\mathbf{H}$.

Hence, let us suppose that the lattices generated by the column vectors of MIMO channel $\mathbf{H}$ and LR-reduced channel $\widetilde{\mathbf{H}}$ matrices are the same. This implies there exists an integer unimodular matrix $\mathbf{T}$ that satisfies $\widetilde{\mathbf{H}} = \mathbf{HT}$. Then, the received signal in (2) can be rewritten as:

$$\mathbf{r} = \mathbf{H}\mathbf{T}\mathbf{T}^{-1}\mathbf{x} + \mathbf{n} = \widetilde{\mathbf{H}}\mathbf{z} + \mathbf{n}, \qquad \text{where} \quad \mathbf{z} = \mathbf{T}^{-1}\mathbf{x} \tag{8}$$

LR-based Minimum Mean Squared Error MIMO detection. The extended received signal vector $\mathbf{r}_{\text{EXT}}$ is multiplied by the pseudo-inverse of the reduced extended channel matrix $\widetilde{\mathbf{H}}_{\text{EXT}}^{\dagger}$, resulting: $\widetilde{\mathbf{z}}_{\text{MMSE}} = \widetilde{\mathbf{H}}_{\text{EXT}}^{\dagger}\mathbf{r}_{\text{EXT}}$.

Quantisation operation and the demapping onto the original base. Since in the lattice reduction procedure the original symbols of the QAM constellation are shifted and scaled by the matrix $\mathbf{T}$, before re-mapping the vector $\mathbf{z}_{\text{MMSE}}$ to the original constellation, this shifting-scaling operations must be reverted [11], resulting in:

$$\widehat{\mathbf{z}}_{\text{MMSE}} = 2 \cdot \left\lceil \frac{\mathbf{z}_{\text{MMSE}} - \beta'\mathbf{T}^{-1}\mathbf{1}}{2} \right\rfloor + \beta'\mathbf{T}^{-1}\mathbf{1} \tag{9}$$

where $\mathbf{1}$ is unitary column vector, and $\lceil\cdot\rfloor$ is the rounding function for the near integer. As shown in [3], this process is equivalent to the quantisation operation (over the vector $\mathbf{z}_{\text{MMSE}}$) for the nearest point of the constellation $\mathbf{T}^{-1}\mathbf{x}$, as described by (9). After quantisation operation, the vector $\widehat{\mathbf{z}}_{\text{MMSE}}$ is demapping onto the original base by multiplying to the unimodular matrix: $\widehat{\mathbf{x}}_{\text{MMSE}} = \mathbf{T}\widehat{\mathbf{z}}_{\text{MMSE}}$. Hence, $\widehat{\mathbf{x}}_{\text{MMSE}}$ is detected as the symbol associated to the constellation point with the minimal distance.

IV. ACO MIMO Detection

From (5), the MIMO detection problem can be seen as a combinatorial optimisation problem. Therefore, the ant colony optimisation is a proper technique to be applied. In the ACO-MIMO detector proposed in [9], N_{ants} ants seek iteratively for better solutions accordingly to a cost function. For MIMO communications systems, the optimal solution is given by (5), so the following cost function can be adopted in the ant colony optimisation context:

$$F(\mathbf{s}) = ||\mathbf{r} - \mathbf{H}\mathbf{s}||^{\mathbf{2}} = ||\widehat{\mathbf{r}} - \mathbf{R}\mathbf{s}||^{\mathbf{2}} \tag{10}$$

where $\widehat{\mathbf{r}} = \mathbf{Q}^H\mathbf{r}$, and $\mathbf{H} = \mathbf{QR}$ is the QR decomposition of $\mathbf{H}$, being $\mathbf{Q}$ a orthogonal matrix with dimension $n \times m$ and $\mathbf{R}$ an upper triangular matrix, dimension $m \times m$.

Hence, the d_{ij} distance related to the "s_{ij}"path, i.e., assumed that the jth symbol of the set $\mathcal{A}$ has been transmitted at the ith antenna is calculated into the ACO algorithm deploying the recursive relation:

$$d_{ij} = |\widehat{r}_i - \sum_{l=i+1}^{m} R_{il}\widetilde{s}_l - R_{ii}s_j|, \qquad \widetilde{s}_i = \arg \min_{s_{ij} \in \mathcal{A}} d_{ij}. \tag{11}$$

where i should progressively decrease from m to 1, $s_j \in \mathcal{A}$, and $\widetilde{s}_l$ are the hard decision of the transmitted symbol s_l that have been tentatively made decision [9].

Besides, the distances d_{ij} are then converted into the heuristic values η_{ij} using a log-sigmoid function: $\eta_{ij} = (1+e^{d_{ij}})^{-1}$. Such values have influence in the probability calculation of the paths traced by the active ants across the iterations of the algorithm; this ant paths following probability is computed as:

$$p_{ij} = \frac{[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{j\in M}[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}} \tag{12}$$

where τ_{ij} is the pheromone level over the path s_{ij}, and the constants α and β weight the importance of τ_{ij} and η_{ij}, respectively. These levels are a way to implement evolution mechanism in the algorithm along the iterations, and are analog to the substances that real ants deposit on the best paths in searching food process. They are set up with an initial value, and as soon as the N_{ants} complete their paths $\widetilde{\mathbf{s}}_k^{(n)}$, they are updated at the $n+1$ iteration according to

$$\tau_{ij}^{(n+1)} = (1+\rho)\tau_{ij}^{(n)} + \sum_{k=1}^{N_{\text{ants}}} \Delta\tau_{ij}^{k} \tag{13}$$

respectively, where ρ is the pheromone evaporation rate (ER) and $\Delta\tau_{ij}^{k}$ is given by:

$$\Delta\tau_{ij}^{k} = \begin{cases} F(\widetilde{\mathbf{s}}_k^{(n)}) & \text{if} \quad (i,j) \in \widetilde{\mathbf{s}}_k^{(n)}, \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

The lower cost function found so far and the associated path are assigned, respectively, to the variables F_{best} and $\mathbf{s}_{best}$, which represents the solution given by the algorithm.

A. Proposed Detection Schemes

Firstly, some modifications in the ACO-MIMO detector described above is introduced in such a way that the search can be readily started from an initial point, which in fact would be the solution offered by any low-complexity linear MIMO detector; besides, the pheromone update is done on a closest way of that shown in [5]. Finally, adapting this heuristic search algorithm taking into account the improved channel conditions provided by the LR technique, we can considerably improve the overall MIMO system performance at cost of an affordable increasing in complexity, as discussed in Section V.

1) ACO MIMO Detection with an Initial Guess: Initially, if in the eq. (11) we use $\widetilde{s}$ as the initial solution given by any low-complexity linear MIMO detector, such as ZF or MMSE, with or without LR aiding, the heuristic information will be obtained taking advantage of the information provided by this detector. This way, as much better the information provided by the initial detector is (i.e., the bit-error-rate (BER) performance), the more reliable the final achieved heuristic information, and less processing (time-consuming) is needed for the ants to search and find a reliable high-quality solution. Hence, in this work we use the initial guess for the ACO input as $\widetilde{s} = \widehat{\mathbf{x}}_{\text{MMSE}}$, deployed in the calculation of (11).

The other modification introduced herein is related to the pheromone updating. Since the cost function (10) should be minimised, eq. (13) and (14) may be not completely appropriate, since these calculations possibly introduce an excessive pheromone accumulation on the paths controlled by ρ, which can erroneously provide a higher pheromone deposition level over the paths under worse evaluation (i.e., high cost function). To correct this effect, herein (14) is re-written as:

$$\Delta\tau_{ij}^{k} = \begin{cases} \frac{\varrho}{F(\widetilde{\mathbf{s}}_k^{(n)})} & \text{if} \quad (i,j) \in \widetilde{\mathbf{s}}_k^{(n)}, \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

where ϱ is a constant to be adjusted experimentally. In a similar way suggested in [5], herein we also implemented a second pheromone deposition rule, which takes into account the best solution found so far by the ACO algorithm:

$$\Delta\tau_{\text{elitist}}^{(n)} = \begin{cases} \frac{\delta}{F(\mathbf{s}_{\text{best}}^{(n)})} & \text{if} \quad (i,j) \in \mathbf{s}_{\text{best}}^{(n)}, \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

where δ is a constant to be adjusted experimentally. As a consequence, eq. (13) becomes:

$$\tau_{ij}^{(n+1)} = (1-\rho)\tau_{ij}^{(n)} + \sum_{k=1}^{N_{\text{ants}}} \Delta\tau_{ij}^{k} + \Delta\tau_{\text{elitist}}^{(n)} \tag{17}$$

2) LR-based ACO MIMO detection: One of the main issues to be solved in the ACO MIMO detector adaptation to the reduced domain, with better conditioned channel gain matrix, is the fact on the LR domain, there is no fixed constellation, as discussed in [12]. Since $\mathbf{z} = \mathbf{T}^{-1}\mathbf{x}$, each channel matrix remains on a different reduced transformed constellation. Obviously, to calculate all these constellation points for each channel matrix gain realisations is practically unfeasible.

In order to solve this problem, the ants' search for the optimal solution initially takes place on the neighborhood of a initial solution, namely reduced domain neighborhood (RDN) procedure [12]. In this work, the initial solution is fed by the LR-MMSE MIMO detector.

Since every constellation can be seen as a shifted and scaled version of the integer constellation $\mathbb{Z}^n$ [11], the procedure starts taking the shifted and scaled version of the vector $\mathbf{z}_{\text{MMSE}}$, expressed by the first term of (9): $\widetilde{\mathbf{z}}_{\text{MMSE}} = \frac{\mathbf{z}_{\text{MMSE}} - \beta' \mathbf{T}^{-1}\mathbf{1}}{2}$. Hence, the neighborhood is obtained as the combinations of the adjacent integers on each element of $\widetilde{\mathbf{z}}_{\text{MMSE}}$, after quantisation, being N the number of elements calculated per dimension. $N = 5$ was adopted in [12]; however, the problem was treated in a complex-values format. Since in this work we are adopting equivalent real-values format, we have adopted $N = 3$. Also eq. (11) should be adapted to the reduced domain, becoming [12]:

$$\widetilde{d}_{ij} = \left\|\widetilde{\mathbf{R}}(\widetilde{\mathbf{z}}_{\text{MMSE}} - \widetilde{\mathbf{z}})\right\|^2 \tag{18}$$

in which i should decrease from m to 1, $\widetilde{\mathbf{z}}$ is a vector formed by the elements of $\lceil \widetilde{\mathbf{z}}_{\text{MMSE}} \rfloor$ on the positions $i+1 \ldots m$, i.e., the j-th neighbor of $\lceil \widetilde{z}_{\text{MMSE}_i} \rfloor$ at the i-th position, and zeros at the positions $i-1 \ldots 1$, and $\widetilde{\mathbf{R}}$ is given by the QR decomposition of the reduced channel matrix $\widetilde{\mathbf{H}}$. Finally, the cost function deploying the reduced LR domain description becomes:

$$\widetilde{F}(\mathbf{z}) = ||\widetilde{\mathbf{r}} - \widetilde{\mathbf{R}}\mathbf{z}||^2, \qquad \text{where} \quad \widetilde{\mathbf{r}} = \widetilde{\mathbf{Q}}\mathbf{r}. \tag{19}$$

The remainder of the algorithm is constructed in the same way as the ACO MIMO detector on the original channel gain matrix domain.

V. Numerical Results

It is well-known that the ACO algorithm performance depends on the appropriate choice for its parameters, ensuring this way a desirable acceleration in algorithm's convergence [7] while simultaneously guaranteeing a reduction in the computational complexity. Hence, as a first step in this section, a procedure aiming to obtain a ACO-MIMO detector input parameter optimisation is carried out.

A. *Input Parameter Optimisation*

The parameters ϱ, δ and evaporation rate (ER) ρ present little influence on the ACO algorithm performance [5]. [7]. This way, after non-exhaustive tests, these parameters were chosen empirically, being adopted the following values: $\varrho = 1$, $\delta = 3$ and $\rho = 0.3$. On the other hand, α and β parameters drastically affect the ACO algorithm performance depending on the optimisation problem type. As discussed in [5], [7], α is related to the importance given to the pheromone levels in the probability calculations, eq. (12), in such a way to determine the convergence speed of the algorithm, while β is related to the importance given to the *"a priori"* information in (12).

Varying the parameters $\alpha \in [0, 1]$ and $\beta \in [0.2, 2]$, with steps of 0.2, all possible parameter combinations on these intervals were performed for three set of antennas and order modulation combinations: a) 4×4 and 64-QAM; b) 8×8 and 16-QAM; c) 20×20 4-QAM. Besides, for each system configuration analysed, the minimum bit-error-rate ($\mathrm{BER}_{\min}$) achieved was saved. Fig. 1 shows the achieved BER performance as a function of both ACO input parameter variations, for system configuration c) 20×20 and 4-QAM. Similar results were obtained considering system configuration a) and b).

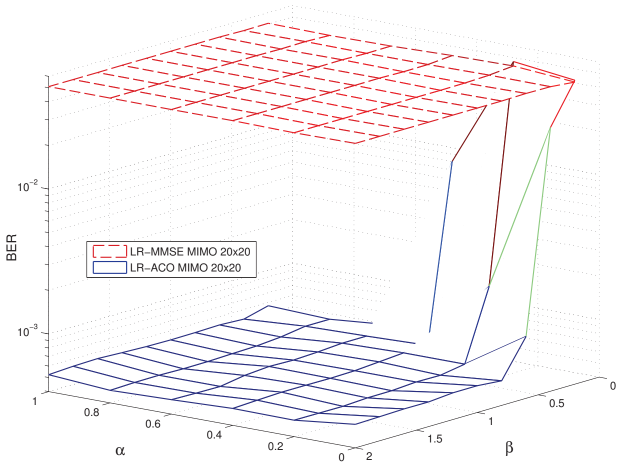

Fig. 1. LR-ACO input α and β parameters variation for 20×20, 4-QAM and $SNR = 16$ dB.

In order to obtain the best α, β-parameter combination, the ratios between the bit error rate of each combination and the minimum BER obtained over that specific MIMO configuration, $\frac{\mathrm{BER}_{\alpha,\beta}}{\mathrm{BER}_{\min}}$, were calculated. Also, for each parameter combination, the mean among the ratio considering the three configurations were obtained. Finally, the optimum α, β-parameter combination was obtained as that one with the lower mean. As a result, the best combination obtained was $\alpha = 0.8$ and $\beta = 0.8$ for three system configurations. Hereafter, these values are adopted for both proposed LR-ACO MIMO detectors.

B. *LR-ACO MIMO Performance under Optimised Input Parameters*

Fig. 2 shows the performance for ACO detectors: a) after $\mathcal{I} = 40$ iterations; and b) correspondent convergence at a SNR of 26 dB, considering the ACO-MIMO detectors for 4×4 antennas and 64-QAM modulation. For a notation issue, the ACO proposed in [9] is called "ACO_1", while the ACO proposed herein which performs the search starting from the LR-MMSE solution in the original domain as "ACO_2", and the proposed ACO herein performing the search in a reduced domain is denominated "LR-ACO". Analysing the convergence velocity in Fig. 2.b, one can see that the LR-ACO needs only $\mathcal{I} = 10$ iterations to achieve total convergence. However, for the other ACO-based detectors either the search remains evolving slowly along the $\mathcal{I} = 40$ iterations, or converge result is very poor (high BER).

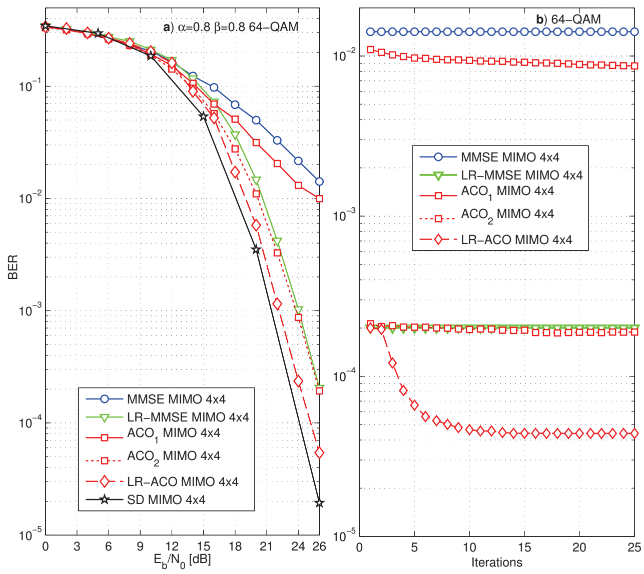

Fig. 2. a) BER performance after $\mathcal{I} = 40$ iterations for ACO-based detectors; b) Convergence under SNR=26dB, considering 4×4 antennas, 64-QAM and optimised ACO input parameters.

Fig. 3 shows the performance of other both configurations: a) 8×8, 16-QAM and b) 20×20, 4-QAM. All MIMO detectors convergence behavior and speed are similar to those obtained in 2.b, with LR-ACO total convergence achieved after a small number of iterations when the constellation order decreases:

LR-ACO Convergence	4×4, 64-QAM	8×8, 16-QAM	20×20, 4-QAM
$\mathcal{I}$	10	5	3

C. *Complexity Analysis*

The algorithm complexities can be evaluated in terms of the total number of floating-point operations (*flops*). A *flop* is defined as an addition, subtraction, multiplication or division between two floating point numbers [13]. Table I shows the number of *flops*, obtained by manually counting them, for the considered ACO-MIMO detectors, assuming equal numbers of antennas ($n_{\mathrm{T}} = n_{\mathrm{R}} = n$) and M-QAM modulation format. The complexity for all four detectors is of order $\mathcal{O}(n^3)$. Alternatively, the MIMO detector complexities are depicted on Fig. 4 when both the number of antennas $n_{\mathrm{T}} = n_{\mathrm{R}}$ and the modulation order M increase.

In the same way as [12], another advantage of the LR-ACO is that its complexity does not depend on the modulation order (M), since the neighborhood size (N) remains the same. One can see that the LR-ACO detector complexity remains always below the complexity of ACO_1 and ACO_2, whereas it presented a substantial performance improvement, for the Large-MIMO conditions. It should be noted from Table I that the complexities of all ACO MIMO detectors do not include the parameters optimisation stage, since it is done just only a single time for adjusting the algorithm, and from then on, the ACO-based algorithms are able to operate under any MIMO modulation order and number of antennas.

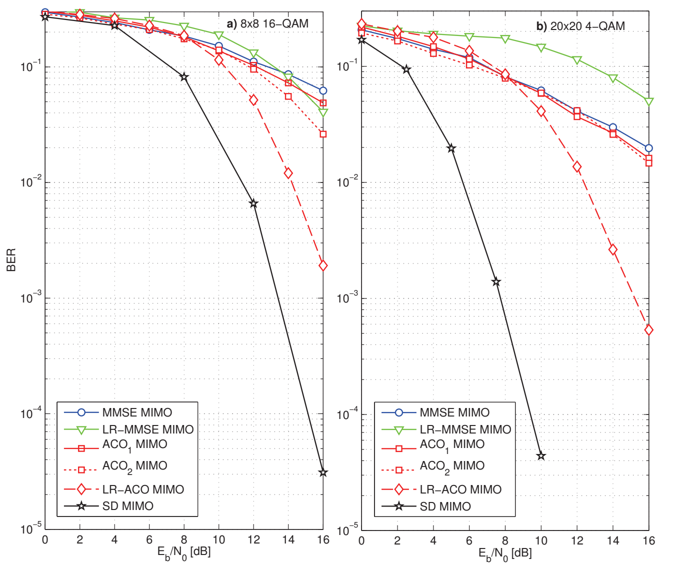

Fig. 3. BER Performance for a) 8×8, 16-QAM; b) 20×20, 4-QAM. $\mathcal{I} = 40$ iterations for ACO detectors.

TABLE I

NUMBER OF OPERATIONS FOR EACH MIMO DETECTOR.

Detector	Number of Operations
LR-MMSE	$(258/3)n^3 + 33n^2 - 1$
ACO_1	$(208/3)n^3 + 8n^2 - 4n + 4n^2\sqrt{M} + 8n\sqrt{M} + \mathcal{I}\left[10n\sqrt{M} + (4n^2 + 6n)N_{\mathrm{ants}} + 1\right]$
ACO_2	$(466/3)n^3 + 41n^2 - 4n + 4n^2\sqrt{M} + 8n\sqrt{M} + \mathcal{I}\left[10n\sqrt{M} + (4n^2 + 6n)N_{\mathrm{ants}} + 2n + 1\right]$
LR-ACO	$(466/3)n^3 + 41n^2 - 4n + 4n^2N + 8nN + \mathcal{I}\left[10nN + (4n^2 + 6n)N_{\mathrm{ants}} + 2n + 1\right]$

N_{ants}: # ants; $\mathcal{I}$: # iterations; N: LR-ACO neighborhood size.

VI. CONCLUSION

A low-complexity heuristic-based detector for Dense-MIMO communications systems combining ACO with LR techniques has been proposed; the LR-ACO-MIMO achieves a substantial improvement in terms of BER performance

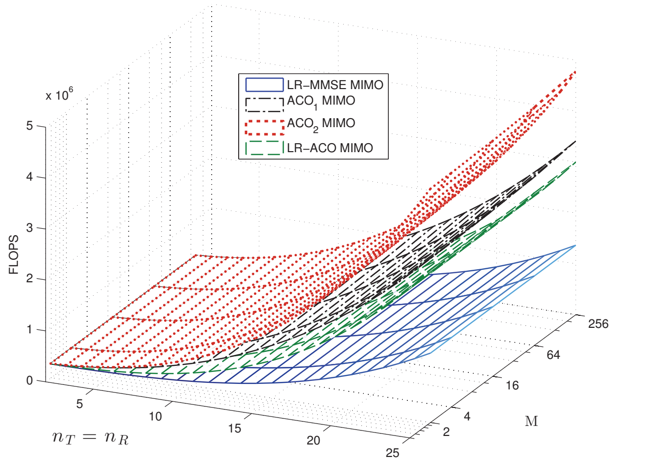

Fig. 4. Detector Complexities. For ACO_1 and ACO_2: $N_{\mathrm{ants}} = 20$, $\mathcal{I} = 40$; LR-ACO: $N_{\mathrm{ants}} = 20$, $\mathcal{I} = 10$.

while expending lower computational resources than others ACO-based MIMO detectors considered. The performance and complexity results suggest that the proposed LR-ACO-MIMO is a promising solution for large number of antennas and/or modulation order, or even correlated MIMO channel scenarios.

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