BER and PAPR Reduction Method for Nonlinearly Amplified OFDM Signals

Roberto Câmara Gentil Porto; Robson França de Moraes; Ernesto Leite Pinto.

***Abstract*— This paper presents a new technique to improve the performance of OFDM systems with nonlinear amplifiers. This method aims to achieve good balances between PAPR and BER at low computer cost. A performance and complexity comparison with known techniques for BER reduction is presented. The results show that the proposed method has lower computational complexity (about 33% fewer multiplications), similar BER performance and better PAPR performance than the other techniques here evaluated.**

***Keywords*— OFDM, PAPR, BER Reduction, PTS, nonlinear amplifiers.**

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has been widely used in communication systems that demand high data rate transmissions due to its inherent characteristics of high spectral efficiency, inter symbol interference (ISI) resilience and immunity to frequency-selective fading.

One important issue in OFDM systems is the large peak-to-average power ratio (PAPR), generated by a constructive combination of in-phase sub-carrier signals, which can cause nonlinear distortions in power amplifiers and degrade the overall performance. Several techniques have been proposed to handle PAPR reduction, and a brief of them is discussed in [1].

A number of methods have been proposed in literature to mitigate PAPR by searching among several modifications of an OFDM symbol, the best one in the sense of minimum PAPR. This method to select the modified OFDM symbol is referred to as CONV (from conventional) in present work.

Other approaches to improving OFDM systems performance in nonlinear channels target bit-error-rate (BER) reduction, instead of focusing on PAPR mitigation. These methods use prior knowledge of a nonlinear amplifier model to predict its output and choose among the modified OFDM symbols the one corresponding to the minimum mean square error (MSE) [2] or maximum cross correlation (CORR) [3] between output and input. Both methods improve BER reduction with considerable lower computational complexity in comparison to previous works that follow the same approach.

This paper introduces a new criterion to select the modified OFDM symbol, taking into account not only the PAPR but also its most powerful samples. Numerical results show that the proposed technique achieves BER reduction comparable to the ones of the techniques proposed [2] and [3], reducing the overall PAPR of transmitted OFDM symbols. Furthermore, the proposed method has significantly lower computational complexity and provides flexibility for achieving goal balances between PAPR and BER reduction.

To demonstrate the potential of the proposed technique, simulations were performed using the Partial Transmit Sequence (PTS) approach [4] to generate the modified OFDM symbols and the results were compared with the ones produced by other methods, such as CONV, MSE [2] and CORR [3].

The rest of this paper is organized as follows. Section II summarizes the basic concepts of OFDM systems subject to nonlinear amplifiers. Section III introduces the proposed method for BER and PAPR reduction. Section IV presents simulation results. Section V shows a computational complexity comparison between the proposed and existing methods. Finally, some concluding remarks are given in Section VI.

II. OFDM SYSTEM MODEL

Consider an OFDM system with N sub-carriers as shown in Figure 1. Let $\mathbf{d} = [d_0, d_1, ..., d_{N-1}]^T$ be the symbol vector, with d_i being the data symbol, selected from a M-ary quadrature amplitude modulation constellation. The OFDM signal is generated using N size Inverse Fast Fourier Transformation (IFFT), as:

$$\mathbf{x} = \mathbf{F}^H \mathbf{d} \tag{1}$$

where $\mathbf{F}^H$ is the IFFT matrix and $\mathbf{x}$ the time domain signal vector. It is also convenient to express element of $\mathbf{x}$ as:

$$x_n = \frac{1}{N} \sum_{k=1}^{N-1} d_k e^{j\frac{2\pi kn}{N}}, n = 0, 1, 2, ..., N-1. \tag{2}$$

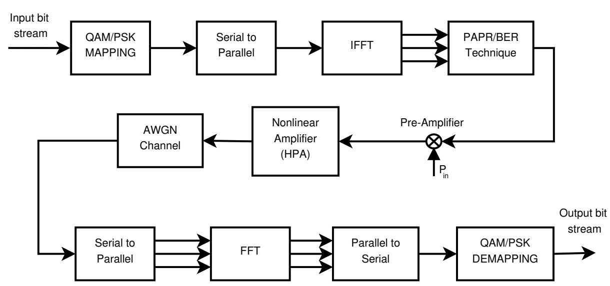

Fig. 1: Block diagram of OFDM System.

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A. PAPR

The peak to average power ratio is a measure commonly used to investigate non-linear distortion effects in OFDM systems and can be expressed as [5]:

$$PAPR = \frac{\max[|x_n|^2]}{E[|x_n|^2]}, n = 0, 1, ... \tag{3}$$

Assuming that the data vector elements d_i are independent and identically distributed random variables and that the number of sub-carriers is sufficiently large, the time domain samples x_n can be modeled by a Gaussian distribution of zero mean. If statical independence between those samples is assumed, the signal power will be central chi-square distributed. Thus, the complementary cumulative distribution function (CCDF) of PAPR can be expressed as:

$$P(PAPR \geq \alpha_0) = 1 - (1 - e^{-\alpha_0})^N. \tag{4}$$

The CCDF is the most frequently used statistic for comparing the performance of PAPR reduction techniques [5].

B. Non-Linear Amplifier Model

Assuming a memoryless non-linear amplifier model, its output is given by:

$$y_n = A(\rho_n)e^{j[\theta_n + \phi(\rho_n)]}, \rho_n \triangleq |x_n|, \theta_n \triangleq \arg(x_n), \tag{5}$$

where operator $A(.)$ denotes the amplitude to amplitude (AM/AM) conversion and $\phi(.)$ the amplitude to phase (AM/PM) conversion.

A practical high power amplifier, used for WiMAX, is the Solid State Power Amplifier (SSPA) described in [6] whose characteristics can be approximated by the Rapp and polynomial models seen in [3]. In both models, only the AM/AM conversion is considered, so implies $\phi(\rho) = 0$.

For the Rapp model, the AM/AM conversion operator is given by [7]:

$$A_{rapp}(\rho_n) = \rho_n[1 + \frac{\rho_n}{A_0}^{2p}]^{\frac{-1}{2p}} \tag{6}$$

where A_0 is the saturated output and the parameter p controls the smoothness of the transition from linear to saturation region.

The amplifier operation point can be set by selecting the desired input back-off (IBO), defined as:

$$IBO = 10\log_{10}\frac{P_{sat}}{P_{avg}} \tag{7}$$

with P_{sat} being the amplifier saturation power and P_{avg} the average power of the input of signal.

To reduce the computational complexity of metric calculation, the CORR and MSE techniques approximate the AM/AM conversion operator by an odd third order non-linearity, so the amplifier output is approximately expressed as:

$$y_n \approx \alpha_1 x_n + \alpha_3 x_n |x_n|^2. \tag{8}$$

The α_1 and α_3 coefficients can be obtained by curve fitting as shown in [3]. Using the SSPA amplifier from [6] as an example, both Rapp and polynomial models can be compared as shown in Fig. 2.

A well-known model for Traveling Wave Tube Amplifier (TWTA) was presented by Saleh in [8]. This model includes AM/AM and AM/PM conversions respectively given by:

$$A_{Saleh}(\rho_n) = \frac{\alpha_1 \rho_n}{1 + \beta_1 \rho_n^2} \tag{9}$$

$$\phi_{Saleh}(\rho_n) = \frac{\alpha_2 \rho_n^2}{1 + \beta_2 \rho_n^2} \tag{10}$$

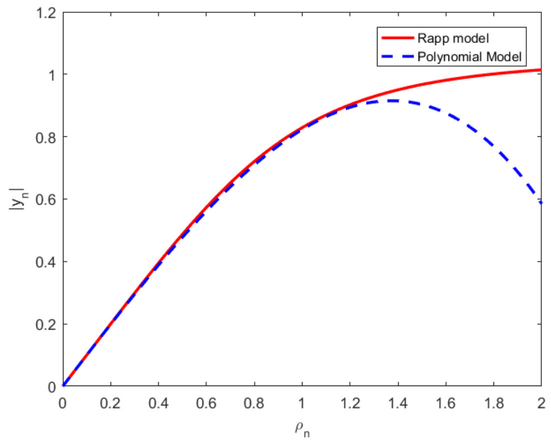

Fig. 2: AM/AM conversion for Rapp and Polynomial models with $A_0 = 1$, $2p = 3.286$, $\alpha_1 = 1$, and $\alpha_3 = -0.1769$.

C. Outline of PAPR/BER Redution Methods

The method here addressed for PAPR and/or BER reduction, including the one proposed in this work, have in common the fact that they select a modified OFDM symbol (the best one, in a specific sense) among several candidates. This is illustrated in Fig. 3.

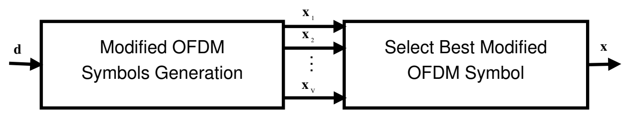

Fig. 3: Block diagram of the proposed method.

Among the techniques that use multiple variations of the OFDM symbol, in this paper, we will use the conventional partial transmit sequence (PTS) scheme [4].

In the PTS scheme, the data vector $\mathbf{d}$ is divided into V partitions, as $\mathbf{d} = [d^{(0)}, d^{(1)} \dots d^{(V-1)}]$, with $d^{(v)} = [d_0^{(v)}, d_1^{(v)}, \dots, d_{N-1}^{(v)}]$ and $v \in \{0, 1, \dots, V-1\}$, where:

$$d_i^{(v)} = \begin{cases} \mathbf{d}_i, \text{ if } i = \frac{vN}{V}, \frac{vN}{V} + 1, \dots, \frac{(v)N}{V} + (\frac{N}{V} - 1), \\ 0, \text{ otherwise.} \end{cases} \tag{11}$$

The application of N-Point IFFT to each partition leads to:

$$\mathbf{x}_v = \mathbf{F}^H \mathbf{d}_v. \tag{12}$$

.

Next, the output of each IFFT block is multiplied by a factor $b_v = e^{j\phi_v}$ and summed to produce a modified OFDM symbol given by:

$$\mathbf{x} = \sum_{v=0}^{V-1} b_v^t \mathbf{x}_v. \quad (13)$$

In this scheme, the phase sequence $\mathbf{b} = [b_0, b_1, ..., b_{V-1}]$, which generates **x** with the lowest PAPR, is selected. The number of modified OFDM symbols generated depends on the number of different sequences **b**.

In the next subsection, some metric criteria are presented for selecting the modified OFDM symbol (right block, Fig. 3). It is noteworthy that the proposed metrics can be easily adapted to others OFDM symbols modifiers schemes (left block, Fig. 3).

D. Some Current Symbol Selection Approaches

The CONV method consists in selecting, among the modified OFDM symbols in a set $\{x\}$, the one with lowest PAPR which can be expressed as:

$$\dot{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}}(\text{PAPR}_{(\mathbf{x})}). \quad (14)$$

The MSE and CORR methods use the polynomial model from Eq. (8) to calculate their respective optimization metrics. The MSE method minimizes the mean square error between the input and the output of the polynomial amplifier model. Given by [2]:

$$MSE_{(xy)} = \sum_{n=0}^{N-1} |x_n - y_n|^2 = \alpha_3^2 \sum_{n=0}^{N-1} |x_n|^6 \quad (15)$$

The optimal modified OFDM symbol for this method is expressed as:

$$\dot{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}}(MSE_{(xy)}). \quad (16)$$

The CORR method indirectly reduces the signal distortion by maximizing the similarity between the input and the output of the polynomial amplifier model, which is measured by the cross correlation expression as [3]:

$$R_{xy}^{(0)} = \alpha_1 \sum_{n=0}^{N-1} |x_n|^2 + \alpha_3 \sum_{n=0}^{N-1} |x_n|^4. \quad (17)$$

The optimal modified OFDM symbol, in this case, is given by:

$$\dot{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmax}}(R_{xy}^{(0)}). \quad (18)$$

III. Proposed Method

The proposed method selects one modified OFDM symbol aiming to achieve a good balance between BER and PAPR performances at low computational cost. A block diagram of the proposed selector is shown in Fig. 4.

The first block on the left is responsible for selecting the T modified OFDM symbols that present lowest PAPR. These selected OFDM symbols will be denoted as $\mathbf{z}_t$, where $t \in \{1, 2, \ldots, T\}$.

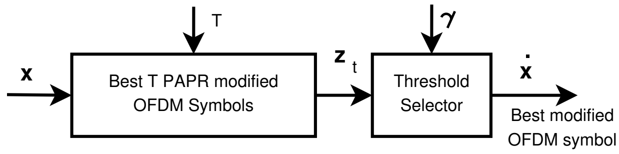

Fig. 4: Proposed OFDM Symbol Selector Block Diagram.

The second block initially compares the instantaneous power of the $\mathbf{z}_t$ modified OFDM symbols with a threshold γ, resulting in:

$$\dot{\mathbf{z}}(\gamma, t) = [\dot{z}_0(\gamma, t), \dot{z}_1(\gamma, t) \ldots \dot{z}_k(\gamma, t)], \quad (19)$$

where $\dot{z}_i(\gamma, t)$ are the values of the sequence $\mathbf{z}_t$ for which $|z_{n,t}|^2 > \gamma$, $n \in \{0, 1, \ldots, N-1\}$ and $t \in \{1, 2, \ldots, T\}$. The best modified symbol is obtained as

$$\dot{\mathbf{x}} = \underset{\mathbf{t}}{\operatorname{argmin}}\left(\sum_{i=0}^{k} (|\dot{z}_i(\gamma, t)|^2 - \gamma)\right). \quad (20)$$

Initial tests showed that γ can be empirically obtained for a given nonlinear amplifier model and suggested that the complete knowledge of the amplifier model is not required to use this method.

It is worthy to note that the selection of the parameter T is directly linked to the PAPR/BER trade-off. If T is equal to the number of generated sequences, the BER performance will be optimized and the proposed method in Fig. 4 will be composed only by the Threshold Selector. On the other hand, if $T = 1$ is chosen, the proposed method becomes equivalent to the CONV technique.

IV. Simulation Results

Computer simulation was used to compare the PAPR and BER reduction performances for the investigated methods. A set of 10^5 randomly generated OFDM symbols was produced using $N = 128$ sub-carriers and 16-QAM symbol modulation.

The PTS approach was used to generate the modified OFDM symbols with 4 and 16 data vector partitions V. The optimal vector **b** for $V = 4$ was chosen among all eight possibilities with possible elements in the set $\{1, -1\}$ and the parameter T was set at 3. For the simulation with 16 partitions, instead of searching for the optimum vector **b** among the 2^{V-1} possibilities, we performed the search in 64 randomly chosen weight vectors, with $T = 16$. These vectors were used in all data sets of OFDM symbols.

In order to measure the BER performance, an additive white gaussian noise (AWGN) was assumed in all simulations and the signal to noise ratio (SNR) was given in terms of $\frac{P_{avg}}{N_0}$, where N_0 is the noise power spectral density.

The Rapp model from Eq. (6) was used in the initial simulations. The model parameters were calculated by curve fitting using the SSPA WiMAX amplifier data presented in [6]. The resulting parameters were $A_0 = 1$, $2p = 3.286$, $\alpha_1 = 1$ and $\alpha_3 = -0.1769$. The operation point of the amplifier was calculated with $IBO = 2.8$ dB and $P_{sat} = 1$ whereas the gain for the amplifier is unitary. We assumed perfect knowledge of the vector **b** by the receptor.

The chosen value of $\gamma = 0.209$ for the Threshold Selector was empirically obtained calculating BER for different γ values, as shown in Fig. 5. It is possible to conclude that the proposed method can achieve similar BER compared to CORR and MSE techniques for a considerable range of threshold values. As noticed before, this suggests that the complete knowledge of the amplifier model is not needed in the proposed method.

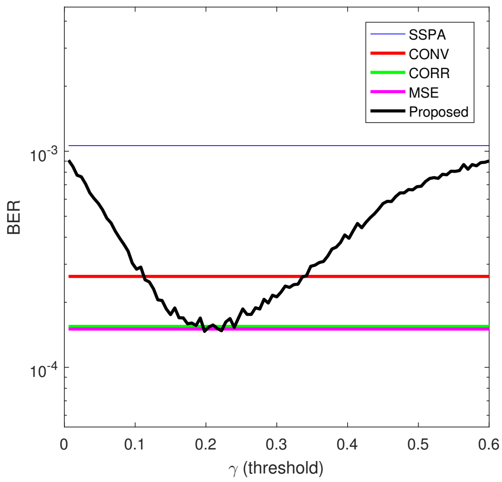

Fig. 5: BER vs threshold value in a AWGN with SNR = 35 dB.

As seen in Fig. 6, the BER performance of the proposed method is comparable with the produced by CORR and MSE techniques with the advantage of requiring lower computational effort.

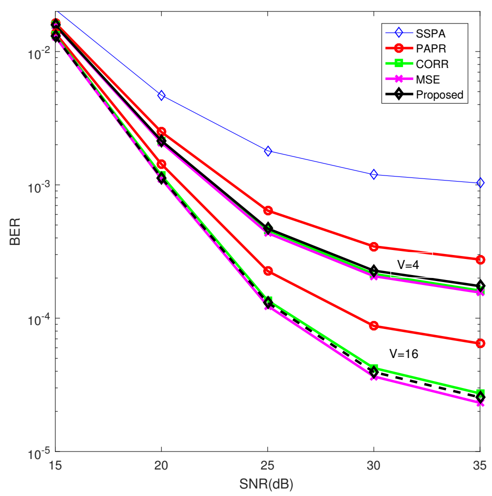

Fig. 6: BER for SSPA amplifier over AWGN channel using $V = 4$ and 16, N = 128 and $16QAM$ modulation.

The PAPR results for $T = 16$ are shown in Fig. 7, indicating that the proposed method performs better in this respect than by CORR and MSE.

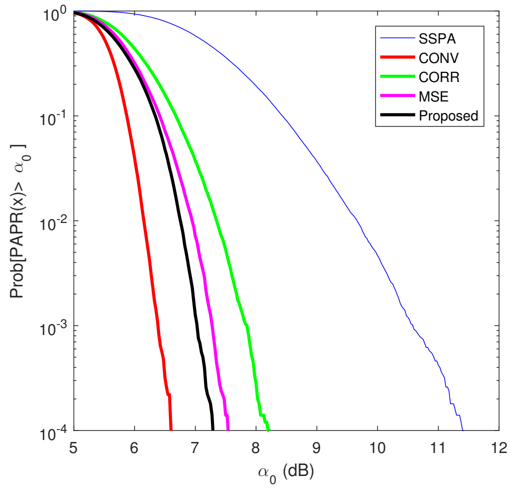

Fig. 7: CCDF of PAPR for different methods with $V = 16$, $N = 128$ and $16QAM$ modulation.

To evaluate the effect of choosing a specific subset of weight vectors (64 vectors out of 2^{15} possibilities), the BER was estimated for 10 samples of 10^4 OFDM symbols, each one with different values for **b**. The result can be seen in Fig. 8, which shows that for a large data set the BER estimates will not vary significantly, independently of the randomly chosen sequences subsets.

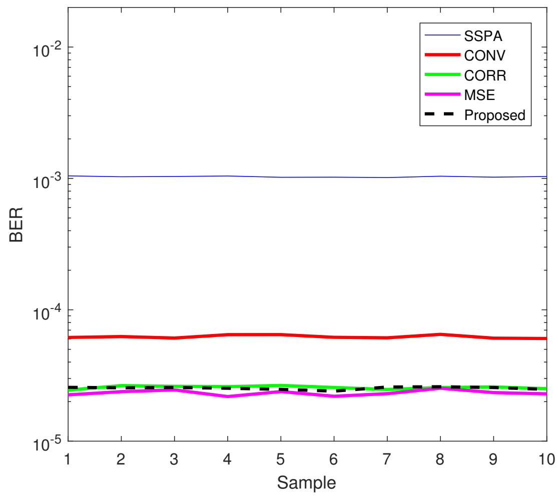

Fig. 8: BER analysis for different subsets of $b_{16\times64}$ for AWGN channel with SNR = 35 dB, V = 16.

The BER and PAPR performance of the proposed and the other techniques were also evaluated by simulations with the use of Selective Mapping (SLM) [1]. The modified OFDM symbols were generated by eight randomly chosen phase vectors whose elements belong to the set $\{\pm 1, \pm j\}$. The results are not reported here for conciseness. They demonstrated the same trend observed in Fig. 6 and 7. These results highlight the effectiveness of the proposed selection method independently of the technique used to produce the modified OFDM symbols.

The BER curves obtained with the investigated methods considering a non-linear amplifier with both AM/AM and AM/PM conversion distortions are presented in Fig. 9. The Saleh amplifier model from Eq. (9) and Eq. (10) was used with parameters $\alpha_1 = 2.16$, $\alpha_2 = 4$, $\beta_1 = 1.15$ and $\beta_2 = 9.1$, fitted from experimental TWTA data [8].

The simulation were performed using 4-PSK symbol modulation, IBO = 3.34 dB, V = 4 and γ = 0.016. The results of Fig. 9 suggest that the BER performance of the proposed method with a TWTA amplifier is better than the CONV technique, being similar to the ones CORR and MSE techniques.

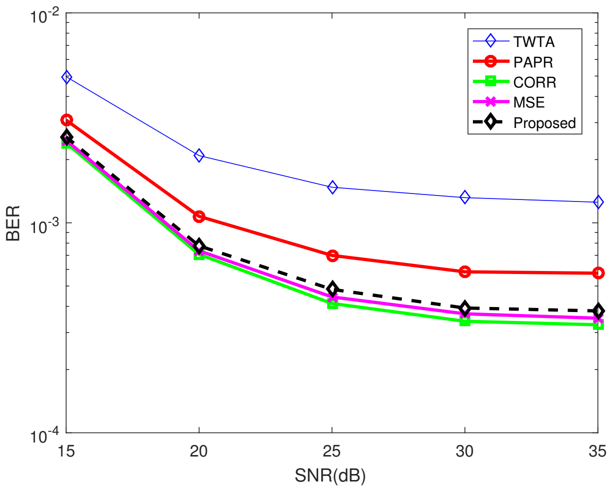

Fig. 9: BER for TWTA amplifier over AWGN channel using V = 4, N = 128 and $PSK4$ modulation.

V. Computational Complexity

Computational cost is evaluated by the number of real multiplications M_{method} and additions A_{method} required by the method calculations. The total number of possible modifications of the OFDM symbol will be denoted by C_x. Since all the investigated models use this same approach, the computational complexity required by them is identical, except for the optimization metric calculations, which are described as follows:

1) CONV: metric computed with Eq. (3). The computational complexity can be calculated observing only the numerator since the denominator is equal to P_{avg}. The computation of this numerator requires $M_{CONV} = 2N.C_x$ and $A_{CONV} = N.C_x$;
2) MSE: metric computed with Eq. (15), requiring $M_{MSE} = (4N+1).C_x$ and $A_{MSE} = (2N-1).C_x$;
3) CORR: metric computed with Eq. (17), requiring $M_{CORR} = (3N+1).C_x$ and $A_{CORR} = (3N-1).C_x$;
4) Proposed: metric calculated in two steps with Eq. (3) and (20) as illustrated in Fig. 4. Since all the $|x_n|^2$ have already been calculated in the first step, the number of multiplications is evaluated as by $M_{proposed} = 2N.C_x$ (the same as in CONV method). The number of additions depends on the size k of the vector $\dot{\mathbf{z}}$ and is calculated as $A_{proposed} \approx (N).C_x + (2k-1)T$, where the first and second terms refer to Eq. (3) and (20) respectively.

It can be concluded that the total number of multiplications required by the proposed method is approximately 33% lower than by CORR and 50% lower than by MSE. Analyzing the number of additions demanded by the proposed technique, simulations have shown that for $\gamma = 0.209$ the $(2k-1)T \approx \frac{NT}{7}$. For the simulated parameters $\gamma = 0.209$, $T = 3$ and $C_x = 8$, this values leads approximately to a reduction in the number of additions of 65% and 47%, when compared with CORR and MSE, respectively.

It is important to note that for complete computational complexity requirements, the following should be considered:

- CONV and proposed methods require the evaluation of the maximum value in a set of N elements for each iteration;
- Time spent in the selection of optimal OFDM symbol based on the metric results;
- Time spent in the comparison of the $|\mathbf{z}_t|^2$ with the threshold γ as in Eq. (19).

VI. Conclusions

A new method with low computational complexity to reduce PAPR and improve BER in OFDM systems with non-linear amplifiers was proposed and tested. The complexity and performance investigation here reported demonstrated that the proposed technique outperforms the MSE and CORR techniques in terms of PAPR reduction and provides similar BER results, while reducing the overall computational complexity.

Additional advantages of the proposed method reside in the ability to establish a trade-off between PAPR and BER performances, and the possibility of being applied without complete knowledge of the power amplifier characteristics.

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