Spectrum Sensing of M-PSK Signals Subject to Impulsive Noise in Generalized Fading Channels

José V. de M. Cardoso, Raphael T. Alencar, Wamberto J. L. de Queiroz, and Marcelo S. Alencar

***Abstract*— Novel approximated expressions are derived for the probability of miss of M-PSK signals at the output of an energy detector subject to impulsive noise in generalized fading channels. Monte Carlo simulation results corroborate the proposed theorectical formulae.**

***Keywords*— Spectrum Sensing, Energy Detector, Impulsive Noise, Fading Channels**

I. Introduction

Spectrum sensing is the primary technique for enabling spectrum sharing among primary users (PUs) and secondary users (SUs), such that SUs might exploit the spectrum opportunistically, i.e. without causing interference on PUs' transmissions. Thus, SUs must be able to distinguish (decide) whether or not the spectrum is occupied. Among several techniques for spectrum sensing, the energy detector has been widely investigated, primarily because of its low-complexity and reasonable performance even for severe fading channels [1].

This paper presents novel approximated expressions for the probability of miss for M-PSK signals in generalized fading channels, namely, $\kappa - \mu$ [2], $\eta - \mu$ [2], and $\alpha - \mu$ [3], at the output of the energy detector subject to impulsive noise.

The impulsive noise is modeled according a Bernoulli-Gaussian (BG) channel. This channel has been investigated due to its practical importance especially in multi-carrier transmission systems based on orthogonal frequency division multiplexing (OFDM) [4].

The probability of miss is approximated using the generalized Gauss-Laguerre quadrature method, which approximates a class of integrals to a finite sum [5]. This approximation presents an attractive alternative due to its low computational cost and high accuracy. Moreover, differently from the assumption of the Central Limit Theorem, which has been commonly used to approximate the expressions for the probabilities of detection and false alarm [6], the proposed approximation does not require a large number of samples in order to become accurate, and therefore it does not compromise the sensing time.

II. The Energy Detector

Consider the following hypothesis testing

$$\mathcal{H}_0 : X_n = W_n + C \cdot U_n, \qquad n = 1, 2, ..., N \quad (1)$$

$$\mathcal{H}_1 : X_n = H \cdot S_n + W_n + C \cdot U_n, \qquad n = 1, 2, ..., N \quad (2)$$

José V. de M. Cardoso, Raphael T. Alencar, Wamberto J. L. de Queiroz, and Marcelo S. Alencar are with the Department of Electrical Engineering, Federal University of Campina Grande, Campina Grande-PB, Brazil, E-mails: {josevinicius, raphael, wamberto, malencar}@iecom.org.br.

in which $\{X_n\}_{n=1}^N$ is the received signal, $\{W_n\}_{n=1}^N$ is an i.i.d. circularly symmetric complex Gaussian process, i.e., $\{W_n\}_{n=1}^N \sim \mathcal{CN}\left(0, \sigma_W^2 \mathbf{I}_N\right)$, $\mathbf{I}_N$ is the identity matrix of order N, and $H = Re^{j\Theta}$ is the complex channel gain, in which R is the envelope and Θ is the phase.

The impulsive noise is modeled according the Bernoulli-Gaussian model, i.e., $\mathbb{P}(C = 1) = 1 - \mathbb{P}(C = 0) = p$ and $\{U_n\}_{n=1}^N \sim \mathcal{CN}\left(0, \sigma_U^2 \mathbf{I}_N\right)$. Finally, S_n is a symbol taken from an M-PSK constellation, which is assumed to be uniformly distributed, i.e., $\mathbb{P}(S_n = s) = \frac{1}{M},\ s \in \mathcal{S}$, in which $\mathcal{S}$ is the alphabet of S_n.

The well-known energy detection rule, used to decide between the two aforementioned hypotheses, is defined as follows

$$\mathrm{d}_\lambda(Y_N) = I\left(Y_N \geq \lambda\right), \quad (3)$$

in which $Y_N \triangleq \sum_{n=1}^N |X_n|^2$, λ is a strictly positive real number, $I(\cdot)$ is the indicator function, and $\mathrm{d}_\lambda(Y_N) = j,\ j \in \{0, 1\}$, means that the detector has decided in favor of the hypothesis $\mathcal{H}_j$.

A. Hypothesis $\mathcal{H}_0$: Absence of transmitted symbol

Considering the hypothesis $\mathcal{H}_0$, one may show that the cumulative distribution function of Y_N, for any $y > 0$, is

$$P_{Y_N}(y) = (1-p)\gamma\left(N, \frac{y}{\sigma_W^2}\right) + p\gamma\left(N, \frac{y}{\sigma_W^2 + \sigma_U^2}\right), \quad (4)$$

in which $\gamma(\cdot, \cdot)$ is the lower incomplete gamma function defined as $\gamma(a, z) \triangleq \frac{1}{\Gamma(a)} \int\limits_0^z t^{a-1} e^{-t}\ \mathrm{d}t$.

B. Hypothesis $\mathcal{H}_1$: Presence of transmitted symbol

Given $R = r$, $r > 0$, one may show that the cumulative distribution function of Y_N, for any $y > 0$, is

$$P_{Y_N|R}(y|r) = 1 - \left[(1-p)Q_N\left(\sqrt{\frac{2Nr^2E_s}{\sigma_W^2}}, \sqrt{\frac{2y}{\sigma_W^2}}\right) + pQ_N\left(\sqrt{\frac{2Nr^2E_s}{\sigma_W^2 + \sigma_U^2}}, \sqrt{\frac{2y}{\sigma_W^2 + \sigma_U^2}}\right)\right], \quad (5)$$

in which $Q_a(\cdot, \cdot)$ is the Marcum-Q function and E_s is the energy of an M-PSK symbol. Henceforth, define the signal-to-noise ratio (SNR) as $\frac{E_s}{\sigma_W^2}$ and the signal-to-impulsive-noise ratio (SIR) as $\frac{E_s}{\sigma_U^2}$. In addition, it is assumed, without loss of generality, that the energy of the PSK constellation is equal to unity, so that $E_s = \frac{1}{M}$, allowing a fair comparison of the performance of the spectrum sensing system for different sizes of PSK constellations.

Given an arbitrarily chosen threshold $\lambda > 0$, the probability of false alarm P_F and the probability of a miss P_M may be written as

$$P_F \triangleq \mathbb{P}\left(\mathrm{d}_\lambda(Y_N) = 1 | \mathcal{H}_0\right) = 1 - P_{Y_N}(\lambda), \quad (6)$$

$$P_M \triangleq \mathbb{P}\left(\mathrm{d}_\lambda(Y_N) = 0 | \mathcal{H}_1\right) = \int_0^\infty P_{Y_N|R}(\lambda|r) p_R(r)\, \mathrm{d}r, \quad (7)$$

in which p_R is the probability density function (pdf) of the fading envelope R.

The threshold λ is selected based on the Neyman-Pearson criterion, i.e., λ is the solution of (6) for a given probability of false alarm. Unfortunately, it cannot be solved analytically, however, since (6) is a strictly decreasing function of λ, it may be inverted numerically.

The pdfs of the fading envelope considered for the derivations are those of $\eta - \mu$ (Format 2), $\kappa - \mu$, and $\alpha - \mu$ random variables in their normalized forms.

III. Theorectical Results and Numerical Analysis

The performance of the energy detector is determined once the integral (7) is solved. However, to the best of the authors knowledge, there is no closed form solution for (7), even in special cases, for instance, when R is Nakagami distributed. Hence, the generalized Gauss-Laguerre quadrature method is applied to approximate the probability of miss (7).

Thus, (7) may be approximated as (8), (9), and (10), for $\eta - \mu$, $\kappa - \mu$, and $\alpha - \mu$ fading channels, respectively,

$$P_M^{\eta,\mu} \approx \frac{\sqrt{\pi}\left(1-\eta^2\right)^{\mu}\Gamma(K+\mu+\frac{1}{2})}{(2\eta)^{\mu-\frac{1}{2}}\Gamma(\mu)K!(K+1)^2}\times$$
$$\sum_{k=1}^{K}\left\{\frac{v_k I_{\mu-\frac{1}{2}}(\eta v_k)}{\left[L_{K+1}^{\mu-1/2}(v_k)\right]^2} P_{Y_N|R}\left(\lambda\middle|\sqrt{\frac{(1-\eta^2)v_k}{2\mu}}\right)\right\}, \quad (8)$$

$$P_M^{\kappa,\mu} \approx \frac{(\kappa\mu)^{\frac{1-\mu}{2}}\Gamma\left(K+\frac{\mu+1}{2}\right)}{K!(K+1)^2\exp\left(\kappa\mu\right)}\times$$
$$\sum_{k=1}^{K}\left\{\frac{v_k I_{\mu-1}\left(2\sqrt{\kappa\mu v_k}\right)}{\left[L_{K+1}^{\frac{\mu-1}{2}}(v_k)\right]^2} P_{Y_N|R}\left(\lambda\middle|\sqrt{\frac{v_k}{\mu(1+\kappa)}}\right)\right\}, \quad (9)$$

$$P_M^{\alpha,\mu} \approx \frac{\mu^K}{K!(K+1)^2}\times$$
$$\sum_{k=1}^{K}\left\{\frac{v_k}{\left[L_{K+1}^{\mu-1}(v_k)\right]^2} P_{Y_N|R}\left(\lambda\middle|\left(\frac{v_k}{\mu}\right)^{1/\alpha}\right)\right\}, \quad (10)$$

in which $L_a^b(\cdot)$ denotes the generalized Laguerre polynomial of order a and parameter b (for appropriate values of a and b), v_k is the $k-$th root of $L_K^b(\cdot)$, and $I_\nu(\cdot)$ is the modified Bessel function of first kind and order ν. Note also that the parameter K accounts for the order of the Laguerre polynomial and the number of terms in the sum.

As an example, consider the complementary receiver operating characteric (ROC) depicted in Fig. 1.

For the 8-PSK curves as well as for the 64-PSK, it can be noted that, for $P_F < p$, the pair of curves ($p = 0.0$ and $p = 0.1$) decay roughly with the same rate as P_F increases, and the difference between them remains approximately constant. This fact is due to the influence of the impulsive noise on the receiver. But, it may also be observed that, for $P_F > p$, the performace of the energy detector is significantly improved and the effect of the impulsive noise is strongly reduced as P_F increases.

Therefore, while the $\eta - \mu$ fading impairs the spectrum sensing performance of the energy detector as a whole, i.e, for all values of P_F, the impulsive noise effect turns out to be negligible for $P_F > p$.

The aforementioned observations are not particular of this example, in fact, they were verified in several other simulated scenarios of fading, impulsive noise, and signal to noise ratios.

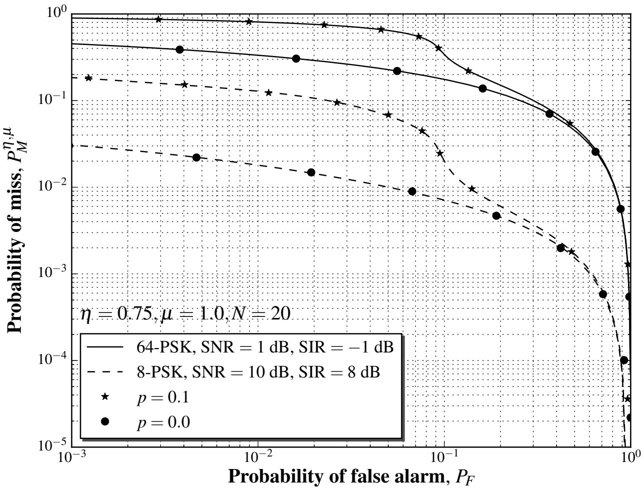

Fig. 1. Complementary ROC. Markers represent Monte Carlo simulation with 10^6 realizations, while lines represent results from (8). Note that the SNR and SIR were chosen such that the variances of the noises, σ_W^2 and σ_U^2, remained the same for each scenario.

References

[1] P. Sofotasios, E. Rebeiz, L. Zhang, T. Tsiftsis, D. Cabric, and S. Freear, "Energy detection based spectrum sensing over $\kappa-\mu$ and $\kappa-\mu$ extreme fading channels," *IEEE Trans. Veh. Technol.*, vol. 62, no. 3, pp. 1031–1040, March 2013.

[2] M. Yacoub, "The $\kappa - \mu$ distribution and the $\eta - \mu$ distribution," *IEEE Antennas Propag. Mag.*, vol. 49, no. 1, pp. 68–81, Feb 2007.

[3] ——, "The $\alpha - \mu$ distribution: A physical fading model for the Stacy distribution," *IEEE Trans. Veh. Technol.*, vol. 56, no. 1, pp. 27–34, Jan 2007.

[4] R. Pighi, M. Franceschini, G. Ferrari, and R. Raheli, "Fundamental performance limits of communications systems impaired by impulse noise," *IEEE Trans. Commun.*, vol. 57, no. 1, pp. 171–182, January 2009.

[5] P. Concus, D. Cassatt, G. Jaehnig, and E. Melby, "Tables for the evaluation of $\int_0^\infty x^\beta e^{-x} f(x)\, \mathrm{d}x$ by gauss-laguerre quadrature," *Math. Comp.*, vol. 17, pp. 245–256, 1963.

[6] D. Horgan and C. Murphy, "On the convergence of the chi square and noncentral chi square distributions to the normal distribution," *IEEE Commun. Lett.*, vol. 17, no. 12, pp. 2233–2236, December 2013.

Acknowlegment

Thanks to Institute of Advanced Studies in Communications (Iecom), Universidade Federal de Campina Grande (UFCG), and Copele/DEE/UFCG, for funding.