Resource Allocation for OFDMA Systems and Energy Harvesting Communications in Multi-User Offline Scenarios

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***Abstract*— We formulate the resource allocation problem of maximizing throughput in an OFDMA (Orthogonal Frequency Division Multiple Access) downlink network where the BS (Base Station) adapts the power allocation according to non-causal (offline) knowledge of the harvested energy and channel state. The offline case is important from a theoretical point of view since it provides a bound on the performance of the online problem (causal). Differently from previous work, that consider a continuous relation between SNR (Signal-to-Noise Ratio) and transmit data rate, we employ a discrete mapping that depends on the required MCSs (Modulation and Code Schemes). Also, we propose a heuristic algorithm that provides near-optimal results and achieves a good complexity/performance trade-off.**

***Keywords*— Resource Allocation, Rate Maximization, OFDMA, MCS, Energy Harvesting, Heuristic.**

I. INTRODUCTION

Energy harvesting communications are powered by renewable energy sources and experiment an unlimited lifetime [1]. However, such energy source presents a stochastic nature since it depends on environmental conditions that determine the amount of power available to perform transmissions. Differently from communication devices that work with fixed power supplies and have assurance of power availability, EH (Energy Harvesting) devices save any unused energy in a storage component such as a rechargeable battery. Then, because batteries have finite storage capacity, we subject our problem to the battery capacity constraints, that limit the available power to the maximum value supported by the battery. Moreover, we apply the energy consumption causality constraints that limit the used energy to the quantity available at the moment, despite the knowledge of future energy arrivals. Both restrictions are known as energy harvesting constraints, and are present in several works [1]–[4].

Many options of EH sources are available, for example: solar radiation, natural wind, radio frequency waves, vibration and thermal energy [1], [2]. Hence, in our study we considered a BS powered by a photovoltaic system similar to [12]. Also, our EH model follows a first-order stationary Markov chain, supported by studies on solar energy in [12]–[14]. The main advantages of EH systems include long term operation without stable power supplies, decreased need of cabling and component replacements, and consequently smaller cost per project. All these features are present in EH communications, and motivated the authors of this paper to investigate EH scenarios related to wireless communications.

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More specifically, our problem is in an offline setting, that means previous knowledge of the EH profile and full CSI (Channel State Information) at the BS. Even though previous knowledge of channel gain and harvested energy is very hard to obtain in practice, solutions for offline scenarios serve as performance benchmarks to more realistic situations and offer important insights on the development of solutions to the online scenario, where energy arrivals and channel gains are not known beforehand. Furthermore, RRA (Radio Resource Allocation) appears as a fundamental functionality in order to improve the use of the scarce radio resources when network nodes are powered by energy harvesters [2], since the main purpose of RRA consists in intelligently distribute the available resources on the goal to satisfy requirements or to optimize services. In this article we consider the RRA problem of maximizing the system throughput, that is a typical and important problem studied in the literature [1]–[4].

II. STATE OF THE ART AND CONTRIBUTIONS

Generally, cooperative communications and relay networks have been studied in literature, and specially in EH scenarios [1], [8], [9]. The authors in [1] analyze a source, relay, destination network in offline (previous knowledge of EH profile) and online (only casual information) situations, and propose a solution for the online case through convex optimization. The studies in [8], [9] present, respectively, optimal and heuristic solutions for power allocation problems with relay selection. Moreover, OFDMA systems and EH technology are the main topics in [5], where the authors develop resource allocation algorithms for a hybrid EH base station, powered by fixed and renewable sources. Similarly, [6] introduces a self-sustainable approach for OFDM receivers in the context of green networks. Long-established studies in [2], [4] give the foundations of rate maximization in EH systems, and more recent work in [10], [11] consider cognitive radio networks (CRNs) powered by energy harvesters, where [10] optimize spectral efficiency and [11] focus on secrecy performance. Finally, the paper in [7] investigates a promising trend in wireless communications, that includes RF (Radio Frequency) EH for mobile devices in cloud-based networks.

Though the aforementioned works provide relevant contributions, all these studies support the unrealistic assumption of continuous mapping between SNR and transmit data rate, but, in real systems, the set of possible throughput values is discrete and finite. Therefore, we implement a discrete mapping through MCSs, where the achievable transmit rates are determined by modulation levels and channel coding. Also, unlike the authors in [1]–[11], who employ a continuous mapping due to the ease of using convex optimization methods, our problem becomes entirely combinatorial with the use of discrete MCSs, which increases the computational complexity of the optimal solution. On the other hand, the papers in [1]–[3] assume an EH profile with discrete values from a finite set. However, empirical measurements in [12]–[14] demonstrate that the harvested energy assume continuous values in practice.

Then, the study in [13] proposes the design of a quantizer in order to work with discrete values of harvested energy. Such quantization process is acceptable because some harvesters collect an approximately constant amount of energy throughout the time [13], what justifies the use of low order quantizers. Since our problem is combinatorial, we prefer to enhance diversity by adopting an EH model that accepts continuous values over a finite range, thus, providing a more precise model because other approaches are prone to quantization error. Lastly, as far we know, the problem of rate maximization for multi-carrier and multi-user systems in EH communications with discrete MCSs has not been studied in literature, so, we present a heuristic algorithm to tackle the problem in offline scenarios and evaluate performance of the proposed solution. Hence, our contributions can be summarized to:

- Mathematical formulation of resource allocation problem with realistic model for link adaptation (discrete MCSs);
- More precise EH model based on Markov processes with states represented by continuous intervals;
- Heuristic solution (offline case) that provides close-to-optimal results at low computational cost.

III. System Modeling

Our scenario consists of a BS located in the center of a circular cell of radius R that transmits to J users. Moreover, the system has a total of N OFDM subcarriers, and allows M possible MCS levels per subcarrier. In our problem, information is transfered through T transmission time intervals (TTIs), and for each TTI the BS harvest H_i units of energy, where i represents the i-th TTI. Assuming that γ_m is the needed SNR to achieve the m-th MCS level, and that $\alpha_{i,n,j}$ denotes the channel gain between subcarrier n and user j for the i-th TTI, we define

$$p^{\mathrm{r}}_{i,n,j,m} = \frac{\sigma^2 \gamma_m}{\alpha_{i,n,j}}, \tag{1}$$

as the required power to user j transmit in the subcarrier n at the TTI i with the MCS m. Also, note that σ^2 is the noise power at the receiver in the bandwidth of a subcarrier. Other important variables are the maximum energy storage capacity of the battery, $B_{\max}$, and the transmit data rate of the m-th MCS level, r_m, necessary to define the discrete mapping $\gamma_m \longleftrightarrow r_m$ for the link adaptation, where $\gamma_m < \gamma_{m+1}$ and $r_m < r_{m+1}$.

On the matter of harvest dynamics we consider a Markov model for solar radiation, and according to [13], [14], we can assume a first-order stationary Markov chain. Basically, stationarity means that the states and transition probabilities do not vary over time. By first-order process we mean that the current state is the only and direct cause of the next one. These assumptions simplify the model, since we have a memoryless system with only one matrix of transition probabilities that works for any instant in time. This is not the case for wind energy [13], that needs a generalized Markov model.

Firstly, let S be the number of states in our model, and $\mathbf{P}$ the square matrix of transition probabilities of order S. Then, we define $P_{l,k}$ as the probability of transition from state l to state k. We also define the vector $v = [v_1, \ldots, v_S]$ where v_k represents the probability of finding the system in state k after a large number of transitions, which is called marginal probability. Finally, following a model similar to [14], the harvested energy H_i is a random number from an uniform distribution belonging to a continuous interval $[(k-1)(H_{\max}/S),\ k(H_{\max}/S)]$ for $k = 1, \ldots, S$, where $H_{\max}$ is the maximum energy that can be harvested, and each interval represents the S possible states of the Markov chain. This representation of states as continuous intervals simulates the actual behavior of the harvested energy, that presents a continuous nature in practice, thus, being an improvement over the studies in [1]–[3] that consider discrete values of energy.

IV. Problem Formulation and Optimal Solution

Before showing the problem formulation, we define $x_{i,n,j,m}$ as the binary decision variable that assumes 1 when subcarrier n with MCS m is assigned to user j at the i-th TTI. Furthermore, our goal is to determine $p^{\mathrm{a}}_{i,n}$, that is the allocated power to subcarrier n at interval i, and the subcarrier assignment $n \longleftrightarrow j$, with $\mathcal{T} = \{1, \ldots, T\}$, $\mathcal{N} = \{1, \ldots, N\}$, $\mathcal{J} = \{1, \ldots, J\}$, and $\mathcal{M} = \{1, \ldots, M\}$ being the set of all TTIs, subcarriers, users, and MCS levels, respectively, where $i \in \mathcal{T}$, $n \in \mathcal{N}$, $j \in \mathcal{J}$ and $m \in \mathcal{M}$. The total data rate from T transmission time intervals is given by

$$r_{\mathrm{total}} = \sum_{i=1}^{T} \sum_{n=1}^{N} \sum_{j=1}^{J} \sum_{m=1}^{M} r_m x_{i,n,j,m}. \tag{2}$$

Since, for a certain TTI i, a subcarrier n can assume only one MCS m and be assigned to a single user j, we write this restriction as

$$\sum_{j=1}^{J} \sum_{m=1}^{M} x_{i,n,j,m} = 1,\ \forall i,\ \forall n. \tag{3}$$

Once we determined a feasible configuration for $x_{i,n,j,m}$, the allocated power to subcarrier n at TTI i can be written as

$$p^{\mathrm{a}}_{i,n} = \sum_{j=1}^{J} \sum_{m=1}^{M} p^{\mathrm{r}}_{i,n,j,m} x_{i,n,j,m},\ \forall i,\ \forall n, \tag{4}$$

and we can finally define the energy consumption causality constraints given by

$$\sum_{i=1}^{t} \sum_{n=1}^{N} p^{\mathrm{a}}_{i,n} \leq \frac{1}{\tau} \sum_{i=1}^{t} H_i,\ t = 1, \ldots, T. \tag{5}$$

The constraints in equation (5) mean that the BS cannot use energy packets which are yet to arrive, and that the used transmit power at TTI t cannot exceed the harvested power until that TTI. The constant τ consists in a factor to convert energy in/from power, i.e., the corresponding energy is obtained by multiplying τ by the power. Lastly, we need to consider that the harvested energy is being stored in the battery, and that a remaining amount of energy is always left over. In many situations, a considerable quantity of energy will be saved for next transmissions, and the next energy packet added to this saving can exceed the battery capacity. In order to assure that energy will not be wasted we define

$$\sum_{i=1}^{t+1} H_i - \tau \sum_{i=1}^{t} \sum_{n=1}^{N} p^{\mathrm{a}}_{i,n} \leq B_{\max},\ t = 1, \ldots, T-1, \quad (6)$$

as the battery capacity constraints. Now, we have to maximize the system throughput by solving the optimization problem:

$$\max_{\boldsymbol{x}} \sum_{i=1}^{T} \sum_{n=1}^{N} \sum_{j=1}^{J} \sum_{m=1}^{M} r_m x_{i,n,j,m}, \quad (7)$$

subject to

Equations (3), (4), (5) and (6), (8a)

$$x_{i,n,j,m} \in \{0, 1\},\ \forall i,\ \forall n,\ \forall j,\ \forall m. \quad (8b)$$

The problem described above is an ILP (Integer Linear Programming) mathematical optimization, that in general is a NP-hard problem. Algorithms based on BB (Branch and Bound) methods provide optimal results at the cost of high computational complexity, specially when the number of constraints and variables is increased. Hence, we obtained the optimal solution through the IBM ILOG CPLEX solver [15]. Due to the high computational complexity to obtain the optimal solution, we propose in the next section a low-complexity algorithm to solve the problem.

V. Heuristic Solution

A classical and optimal bit loading algorithm for point-to-point links with deterministic power supply is the HH (Hughes Hartogs) solution [16]. The main idea of this algorithm is to raise the MCS level of the subcarriers that need less power to reach the next level. This procedure is repeated in iterative manner while there is available transmit power or until all subcarriers achieve the maximum MCS level. Naturally, the HH algorithm could not be applied in a EH system since using all the available power on the battery at TTI i, represented by b_i, is not the best option. In this section, we propose a solution that limits to l_i the maximum used power per TTI i. And instead of starting to load the subcarriers that require the least power, the heuristic begins loading the subcarriers with smaller cost per bit, defined by $c_{i,n,m}$. Firstly, we calculate $p^{\mathrm{r}}_{i,n,j,m}$ using its definition in equation (1), and in the next step we determine an initial power allocation $q^{\mathrm{a}}_{i,n}$ by taking the minimum required power of the last MCS level among all users. Consequently, we obtain an initial subcarrier assignment $u_{i,n}$, and define $p^{\mathrm{s}}_{i,n,m}$ as the required power for the users selected in this assignment.

Algorithm 1 HH-Based Heuristic Power Allocation

1: Calculate $p^{\mathrm{r}}_{i,n,j,m}$ using equation (1)
2: $q^{\mathrm{a}}_{i,n} = \mathbf{min}(p^{\mathrm{r}}_{i,n,j,M} : \forall j \in \mathcal{J}),\ \forall i,\ \forall n$
3: $u_{i,n} = \mathbf{argmin}(p^{\mathrm{r}}_{i,n,j,M} : \forall j \in \mathcal{J}),\ \forall i,\ \forall n$
4: $p^{\mathrm{s}}_{i,n,m} = p^{\mathrm{r}}_{i,n,j,m},\ \forall i,\ \forall n,\ \forall m,$ for $j = u_{i,n}$
5: **for** $i \leftarrow 1$ **to** T **do**
6:     **for** $n \leftarrow 1$ **to** N **do**
7:         $m = M$
8:         **while** $q^{\mathrm{a}}_{i,n} > \frac{1}{N} \sum_{k=1}^{N} q^{\mathrm{a}}_{i,k}$ **do**
9:             $m = m - 1$
10:             $q^{\mathrm{a}}_{i,n} = p^{\mathrm{s}}_{i,n,m}$
11:         **end while**
12:         $\lambda_{i,n} = m$
13:     **end for**
14: **end for**
15: $q_i = \sum_{n=1}^{N} q^{\mathrm{a}}_{i,n},\ \forall i$
16: *AdjustReq*$(\boldsymbol{q}, \boldsymbol{\lambda}, \boldsymbol{p}^{\mathbf{s}}, \boldsymbol{q}^{\mathbf{a}})$
17: $\Delta r_{m-1} = r_m - r_{m-1},$ for $m = 2, \ldots, M$
18: $\Delta p^{\mathrm{s}}_{i,n,m-1} = p^{\mathrm{s}}_{i,n,m} - p^{\mathrm{s}}_{i,n,m-1},\ \forall i,\ \forall n,$ for $m = 2, \ldots, M$
19: $c_{i,n,m} = \Delta p^{\mathrm{s}}_{i,n,m} / \Delta r_m,\ \forall i,\ \forall n,$ for $m = 1, \ldots, M-1$
20: $c_{i,n,M} = \infty,\ \forall i,\ \forall n \quad \Delta p^{\mathrm{s}}_{i,n,M} = \infty,\ \forall i,\ \forall n$
21: $b_1 = h_1 + b_0 \quad p^{\mathrm{a}}_{i,n} = 0,\ \forall i,\ \forall n$
22: $\lambda_{i,n} = 1,\ \forall i,\ \forall n \quad \lambda_{i,1} = 0,\ \forall i$
23: **for** $i \leftarrow 1$ **to** T **do**
24:     $d_i = 0$
25:     **if** $i < T$ **then**
26:         $d_i = (b_i \sum_{t=i+1}^{T} q_t - q_i \sum_{t=i+1}^{T} h_t) / \sum_{t=i}^{T} q_t$
27:         **if** $d_i < 0$ **then** $d_i = 0$
28:         **end if**
29:         **if** $h_{i+1} + d_i > b_{\max}$ **then** $d_i = b_{\max} - h_{i+1}$
30:         **end if**
31:     **end if**
32:     $l_i = b_i - d_i \quad s_{\mathrm{pow}} = 0 \quad p = 0 \quad n = 1$
33:     **while** $s_{\mathrm{pow}} < l_i$ **do**
34:         $p^{\mathrm{a}}_{i,n} = p^{\mathrm{a}}_{i,n} + p$
35:         $\lambda_{i,n} = \lambda_{i,n} + 1$
36:         $n = \mathbf{argmin}(c_{i,k,1} : \forall k \in \mathcal{N})$
37:         $p = \Delta p^{\mathrm{s}}_{i,n,1}$
38:         $c_{i,n,m} = c_{i,n,m+1},$ for $m = 1, \ldots, M-1$
39:         $\Delta p^{\mathrm{s}}_{i,n,m} = \Delta p^{\mathrm{s}}_{i,n,m+1},$ for $m = 1, \ldots, M-1$
40:         $s_{\mathrm{pow}} = s_{\mathrm{pow}} + p$
41:     **end while**
42:     **if** $i < T$ **then**
43:         $f = (h_{i+1} + b_i - \sum_{k=1}^{N} p^{\mathrm{a}}_{i,k}) - b_{\max}$
44:         **if** $f > 0$ **and** $s_{\mathrm{pow}} < b_i$ **then** $p^{\mathrm{a}}_{i,n} = p^{\mathrm{a}}_{i,n} + p$
45:             $\lambda_{i,n} = \lambda_{i,n} + 1$
46:         **else if** $f > 0$ **then** $aux = h_{i+1} + b_i - b_{\max}$
47:             *AdjustPA*$(\boldsymbol{\lambda}, \boldsymbol{u}, \boldsymbol{p}^{\mathbf{a}}, \boldsymbol{p}^{\mathbf{r}}, i, aux)$
48:         **end if**
49:         $b_{i+1} = h_{i+1} + b_i - \sum_{n=1}^{N} p^{\mathrm{a}}_{i,n}$
50:     **end if**
51: **end for**
52: $r_{\mathrm{total}} = \sum_{i=1}^{T} \sum_{n=1}^{N} r_m,$ for $m = \lambda_{i,n}$

The *for* loop from lines 5 to 14 decreases the MCS level of a subcarrier n until its power becomes smaller than the average over all subcarriers in TTI i. Then, the resulting MCS is stored in $\lambda_{i,n}$, and after the loop ends we calculate q_i, that is the total power requirement for each TTI. In line 16 we call the procedure *AdjustReq*, that reduces the MCS level in overloaded carriers until the power requirement q_i gets smaller than the average over all transmissions. This procedure is shown in Algorithm 2. These adjustments give a more accurate estimation of the power to be used in practice, because it is not worthy to spend resources in subcarriers that require much power to achieve a higher modulation level. Alternatively, we prefer to save power for subcarriers that, in future transmissions, will experience more favorable channel conditions. Continuing Algorithm 1, we compute the data rate increase Δr_m and the incremental power matrix $\Delta p^{\mathrm{s}}_{i,n,m}$ in

Algorithm 2 Adjust Requirement

procedure *AdjustReq*($\boldsymbol{q}, \boldsymbol{\lambda}, \boldsymbol{p}^{\mathbf{s}}, \boldsymbol{q}^{\mathbf{a}}$)
    for $i \leftarrow 1$ **to** T **do**
        while $q_i > \frac{1}{T}\sum_{t=1}^{T} q_t$ **do**
            $n = \textbf{argmax}(q^{\mathrm{a}}_{i,k} : \forall k \in \mathcal{N})$
            $m = \lambda_{i,n} - 1$
            $q^{\mathrm{a}}_{i,n} = p^{\mathrm{s}}_{i,n,m}$
            $\lambda_{i,n} = m$
            $q_i = \sum_{n=1}^{N} q^{\mathrm{a}}_{i,n}$
        end while
    end for
end procedure

order to determine the efficiency ratio $c_{i,n,m}$, that measures the cost per bit for each MCS leap. Since the rate increase Δr_m is not uniform as m grows, the first subcarriers to be loaded are chosen by evaluating $c_{i,n,m}$, which is an improvement over performing a selection through $\Delta p^{\mathrm{s}}_{i,n,m}$ as the HH algorithm proposes [16]. Moreover, we set the last increment in $c_{i,n,M}$ and $\Delta p^{\mathrm{s}}_{i,n,M}$ to infinity for indicating that the final MCS level has been reached. Then, in line 21 we initialize the power allocation $p^{\mathrm{a}}_{i,n}$ to zero, and define the value of b_1 (available power at TTI 1) as the sum between h_1 (available power at TTI 1 resulting from the conversion to power of the harvested energy H_1) and b_0 (the initial power at the battery). Next, the MCSs $\lambda_{i,n}$ are set to one except for $\lambda_{i,1}$, that is set to zero because it always starts incremented by 1 in line 35.

Furthermore, we finally run the *for* loop from lines 23 to 51, that calculates for every $i < T$ the power decrease d_i applied to b_i in order to compute the power limit l_i to be spent on the i-th TTI. The calculation of d_i in line 26 is an intuitive result, since the greater the available power b_i and the requirements for other transmissions (q_t for $t > i$) are, the greater is the decrease applied to b_i and lower will be l_i. On the other hand, the bigger the requirement q_i and the available power for future transmissions (h_t for $t > i$) are, the smaller is the value of d_i and higher will be l_i. In fact, the expression in line 26 is the solution of equation $(b_i - d_i)/q_i = (h_{i+1} + d_i - d_{i+1})/q_{i+1} = \cdots = (h_T + d_{T-1})/q_T$ for d_i. This equation means that the ratio between available power and required power must be the same for each TTI, thus, applying the principle of reserving more power to the TTI that needs it the most. The condition in line 27 is necessary to enforce the energy consumption causality constraints, because when $d_i < 0$, this would mean to take power from energy packets still not accessible. And the extra condition in line 29 adjusts d_i to prevent the violation of the battery capacity constraints.

Thereafter, the *while* loop from lines 33 to 41 applies the HH-based method described previously in the beginning of this section, and obtains the power allocation $p^{\mathrm{a}}_{i,n}$ for TTI i and the corresponding MCSs $\lambda_{i,n}$. Nevertheless, we need to calculate f, that represents the battery overflow in case the current power allocation is enforced. This becomes necessary because the power increase p is discrete, and the sum of the allocated power never equals l_i. If the overflow $f > 0$ and the power sum s_{pow} does not exceed what is available in the battery, we employ the last power increment p chosen previously. This ensures that $f \leq 0$ because of the condition in line 29, as s_{pow} surpasses l_i guarantees that the battery power limit $b_{\max}$ will be observed. Our final option

Algorithm 3 Adjust Power Allocation

procedure *AdjustPA*($\boldsymbol{\lambda}, \boldsymbol{u}, \boldsymbol{p}^{\mathbf{a}}, \boldsymbol{p}^{\mathbf{r}}, i, aux$)
    $\boldsymbol{g}_n = \textbf{sortdesc}(\alpha_{i,n,j} : \forall j \in \mathcal{J}),\ \forall n,$ where $\boldsymbol{g}_n = \{g_{n,j} : \forall j\}$
    $\boldsymbol{w}_n = \textbf{argsortdesc}(\alpha_{i,n,j} : \forall j \in \mathcal{J}),\ \forall n, \quad \boldsymbol{w}_n = \{w_{n,j} : \forall j\}$
    $f = 1$
    while $f > 0$ **do**
        $n = \textbf{argmax}(g_{k,2} : \forall k \in \mathcal{N})$
        $m = \lambda_{i,n}$
        $j = w_{n,2}$
        $u_{i,n} = j$
        $p^{\mathrm{a}}_{i,n} = p^{\mathrm{r}}_{i,n,j,m}$
        $w_{n,j} = w_{n,j+1}$, for $j = 2, \ldots, J-1$
        $g_{n,j} = g_{n,j+1}$, for $j = 2, \ldots, J-1$
        $g_{n,J} = 0$
        $f = aux - \sum_{n=1}^{N} p^{\mathrm{a}}_{i,n}$
    end while
end procedure

to eliminate the overflow is to call the procedure *AdjustPA*, that gradually increases the power consumption by changing the user to subcarrier assignment stored in $u_{i,n}$. Since the achieved MCS is kept the same when changing users, this procedure has the disadvantage of raising the consumed power without improving the data rate. However, this is necessary to always comply with the restrictions in equation (6). This raise in power is gradual because we choose the users with greater gains in the second column of $g_{n,j}$ (the first column contains values already selected). Given large enough values for N and J the loop in procedure *AdjustPA* is guaranteed to finish and the problem constraints are imposed. This procedure is presented in Algorithm 3, where *sortdesc* is a function that returns a vector sorted in descending order and *argsortdesc* returns the indices corresponding to the sorted values. Lastly, the battery level is updated in line 49 by adding the remaining amount to the next harvested value, and after the *for* loop ends the total data rate is computed in line 52.

VI. Simulation Results

On the goal to compare optimal and heuristic solutions, we performed simulations of the proposed model with M = 16, N = 15, J = 10, and T = 3 to 12. Our choice to vary T is justified by the fact that decisions related to power saving become more difficult as T increases. The results were obtained by running 3,000 instances for each set $\mathcal{A}_T = \{T, N, J, M\}$, counting to a total of 30,000 instances analyzed. The mapping through MCSs is $\mathcal{D}$ = {(γ,r): (0,0), (0.3,25), (0.4,39), (0.6,63), (0.9,101), (1.4,147), (2.1,197), (3.5,248), (5.1,321), (7.8,404), (12.3,458), (19.1,558), (28.8,655), (42.7,759), (79.4,859), (109.6,933)}, with r in bps. Other important constants were defined as $h_{\max}$ = 112 mW, $b_{\max}$ = 208 mW, σ^2 = 4.74 $\times$ 10^{-16} W, with $\alpha_{i,n,j}$ ranging from 10^{-8} to 10^{-10}. Also, the number of states of our Markov model is S = 7, with each state being an interval of length 16 mW. Initially, the simulation distributes uniformly J = 10 users inside a circular area of radius R = 467 m, and establishes a minimal distance of 70 m between user and BS in order to avoid near-field effects. Then, we define the Markov chain initial state k based on the marginal probabilities in $\boldsymbol{v}$, and determine the succession of states using the transition probabilities $P_{l,k}$ for obtaining all the values of h_i. After that, we build the matrices necessary to specify the ILP described in section IV, and

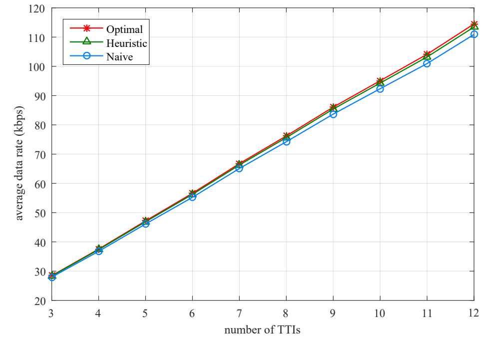

Fig. 1. Average data rate versus number of TTIs, for $M = 16$, $N = 15$, $J = 10$, $T = 3$ to 12, $h_{\max} = 112$ mW, $b_{\max} = 208$ mW, and $S = 7$.

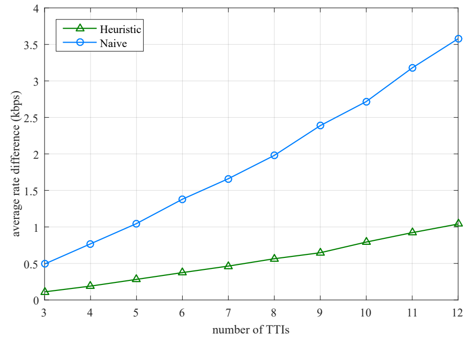

Fig. 2. Average rate difference versus number of TTIs, for $M = 16$, $N = 15$, $J = 10$, $T = 3$ to 12, $h_{\max} = 112$ mW, $b_{\max} = 208$ mW, and $S = 7$.

solve the optimization problem through the CPLEX solver, that returns the values of $x_{i,n,j,m}$. The total throughput of the system is calculated according to equation (2). The sub-optimal solution is obtained next by running Algorithm 1, and by applying HH algorithm we obtain a result denoted as naive solution. Figure 1 shows the results for ten different values of T, where the performance metric is the average data rate over all the 3,000 instances evaluated by each solution. And as T increases the difference between optimal and heuristic grows, since the error of estimating q_i in procedure *AdjustReq* is greater as T becomes larger. However, our results remain very close to optimal as the plot indicates. In fact, the average difference between optimal and heuristic solutions exceeds 1 kbps only for $T = 12$, as indicated by Figure 2. The best results are obtained for $T = 3$, with average difference of just 110 bps. The naive solution (HH) has poor performance when compared to optimal solution, as Figure 2 shows that the difference for $T = 12$ is greater than 3.5 kbps. Furthermore, the complexity/performance gain is high, since for $T = 8$ the average execution time of a single scenario is 1338 ms for optimal solution, and only 16.7 ms for heuristic, that is 80 times faster in this case. All these results were obtained from a Windows 10 computer with a 2.4 GHz core i7 intel processor using the software CPLEX.

VII. Conclusions

In this paper we studied the problem of resource allocation for rate maximization in OFDMA systems with an EH Base Station transmitting to several users. Firstly, we formulated the problem in an offline scenario in the form of an ILP. Secondly, we proposed a heuristic solution that, according to simulation results, achieves near-optimal performance at the cost of low computational complexity in the simulated scenario. We also implemented the discrete relation between SNR and data rate through MCSs, that is a more realistic assumption than the continuous mapping proposed in several studies referenced by this work. Finally, our EH model follows a Markov chain that, differently from other models found in literature, has states represented by continuous intervals, giving the advantage of simulating the actual nature of the energy arrivals. To the best of our knowledge, the problem formulation proposed in section IV has not been presented in literature yet, and the algorithm in section V introduces a novel solution for problems that deal with EH technology in wireless communications.

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