Pre-FFT Adaptive Spatial Filter Applied to OFDM Systems

Marcelo A. C. Fernandes and Dalton S. Arantes

***Abstract*— This paper proposes an adaptive spatial filter scheme in time-domain (pre-FFT), aimed to improve the performance and the capacity of OFDM systems. This strategy uses a supervised adaptive algorithm, with frequency domain multiplexed pilots of the OFDM system as reference. The simplicity of the proposed structure, as well as the method used to obtain reference signals for the adaptive spatial filter are important aspects that distinguish this paper from other publications. Another difference is the discrete characterization of the OFDM system, which resulted in a more appropriate model for the development of this proposal. Details on the operation of the proposed scheme, as well as the BER performance curves, are presented here. The proposal investigated here allows significant reduction in the guard interval of the OFDM system, thereby increasing its robustness or transmission capacity.**

***Keywords*— OFDM, Spatial Adaptive Filter, LMS, Guard Time.**

I. Introduction

In digital communication systems, signals are corrupted by various factors, the most common including white noise and the multipath effect [1]. White noise, which can be treated as Gaussian distribution random variables, can be efficiently overcome with the use of channel encoders [1]. However, the multipath phenomenon, which is caused by different reflections of transmitted signals, is not efficiently handled by these encoders. The referred phenomenon causes the effect of Intersymbol Interference (ISI) [1], which is characterized by overlapping symbols from the same information source, that is, they interfere with each other in the time-domain. The ISI limits channel capacity and is one of the major problems of the current digital communication systems [1]. To minimize the problem of ISI, devices such as spatial filters can be used in the reception process [2] [3].

OFDM systems have reduced transmission capacity due to the use of the Guard Interval (GI), which in some cases is nearly 25% of the band [4]. On the other hand, the use of a cyclic prefix such as the GI indirectly eliminates ISI, reducing the problem to changes in phase and amplitude that can be solved with channel estimators in the frequency domain [4]. These estimators, which are part of the conventional OFDM receiver, require adaptive interpolation algorithms associated with frequency domain multiplexed pilot signals. The referred pilot signals can be scattered, such as in the ISDB-T, DVB and SBTVD [5] [6] or fixed, as in IEEE 802.11 standard [7].

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Adaptive spatial filters in OFDM receivers has been the focus of many previous papers that addressed different methods. The methods differ as to the positioning of the spatial filter which may be located at pre-FFT (Fast Fourier Transformation) or at post-FFT. In [8] the MMSE beamforming algorithm for pre-FFT OFDM system is applied to a channel assumed to be frequency selective fading. However, the scheme needs the channel estimation to calculate the parameters of the spatial filter. The pre-FFT beamforming based on eigenanalysis and post-FFT subcarrier diversity is proposed in [9] [10]. In this case, the eigenvalues and the corresponding eigenvectors are used to determine the parameters of the spatial filter. In [11] is proposed a Post-FFT beamformer combines based on estimated one-tap channel coefficient associated a Pre-FFT switched-beam device. Other approaches using the pre-FFT and post-FFT beamforming can be found in [12] [13] [14].

Unlike the proposals observed in the literature, this work presents a new implementation of the pre-FFT beamforming, which uses an adaptive spatial filter trained with the Least Mean Square (LMS) algorithm acting directly on the samples of the OFDM symbol in the time domain. The scheme proposed in this paper is not dependent on the channel estimation and use two training modes, supervised and unsupervised, where the error calculation is in the frequency domain (open eye condition). In the supervised mode, the error signal uses the OFDM pilot (spread or grouped) and in the unsupervised mode, the decision directed (DD) algorithm is used.

II. Channel Communication

Figure 1 shows the structure of a discrete baseband system with a source of information transmitting symbols a(m), from an alphabet $A_M = a_0, a_1, \ldots, a_{M-1}$ of M symbols. The symbols are transmitted over a period of T_s seconds and represented by words of k bits. T_s is the sampling period of the symbols, or symbol interval.

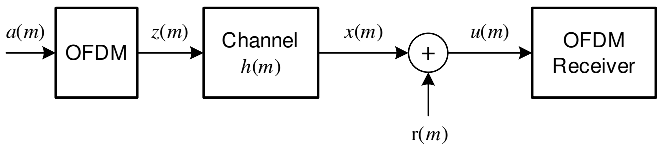

Fig. 1. Discrete baseband communication system with ISI and white noise

The symbols, processed by the OFDM transmission method, are transmitted through a channel, $h(m)$, subject to the phenomenon of the ISI and to Additive White Gaussian Noise (AWGN), $r(m)$. Channel impulse response, $h(m)$, is given by

$$h(m) = \sum_{i=0}^{L-1} \alpha_i(m)\delta\left(m - \tau_i(m)\right), \tag{1}$$

where L is the number of paths in the channel, $\alpha_i(m)$ is the complex gain of the ith path and $\tau_i(m)$ is an integer value representing the delay of the ith path at time m.

The receiver, shown in Figure 1, processes the signal $u(m)$, resulting from the channel and expressed by

$$u\left(m\right) = \sum_{i=0}^{L-1} \alpha_i(m)z(m - \tau_i(m)) + r(m), \tag{2}$$

where $z(m)$ is the symbol at the output of OFDM modulator. Equation 2 can also be written in vector form, as follows

$$u\left(m\right) = \mathbf{h}_g^T\left(m\right)\mathbf{z}_d\left(m\right) + r(m), \tag{3}$$

where $\mathbf{h}_g\left(m\right)$ is the vector of complex channel gains, with ISI, length L, given by

$$\mathbf{h}_g\left(m\right) = \begin{bmatrix} \alpha_0(m) \\ \vdots \\ \alpha_{L-1}(m) \end{bmatrix}, \tag{4}$$

where

$$\alpha_i(m) = \rho_i(m)e^{-j\theta_i(m)} \tag{5}$$

and $\mathbf{z}_d\left(m\right)$ is the vector of channel delays applied to the transmitted signal $z(m)$, given by

$$\mathbf{z}_d\left(m\right) = \begin{bmatrix} z\left(m - \tau_0(m)\right) \\ \vdots \\ z\left(m - \tau_{L-1}(m)\right) \end{bmatrix}. \tag{6}$$

III. Spatial Adaptive Filter

There are two types of adaptive antenna, namely switched beam or antenna arrays, both of which are able to increase the gain in the desired direction and apply nulls to the other directions [2]. The antennas are usually arrayed in the form of a vector of linearly and equally spaced (LES) elements that direct the gain of the antenna in only a certain direction while nulling others. The maximum number of nulls is given by $K-1$, where K is the number of elements in the array. The analysis is simplified here by assuming that the space between the antenna elements is $\lambda/2$, and that there is no mutual coupling between them [2]. Figure 2 shows an LES array where the antenna elements are arranged along the x-axis, with spacing of Δx. It is assumed that all the multipaths arrive at the array in the horizontal plane, with angle of arrival (AOA) of θ radians with respect to the to the y-axis orthogonal to the x-axis. Each vth element of the antenna array is weighted by a complex gain f_i, and the spacing Δx should generally be greater than or equal to $\lambda/2$. The signal $u(m)$ received by the mth antenna element is given by

$$u_v(m) = u(m)e^{-j\beta v\Delta x\cos(\theta)} = y(m)g(\theta), \tag{7}$$

where $\beta = (2\pi)/\lambda$ and $g(\theta)$ is known as the signature of the signal on the vth antenna element.

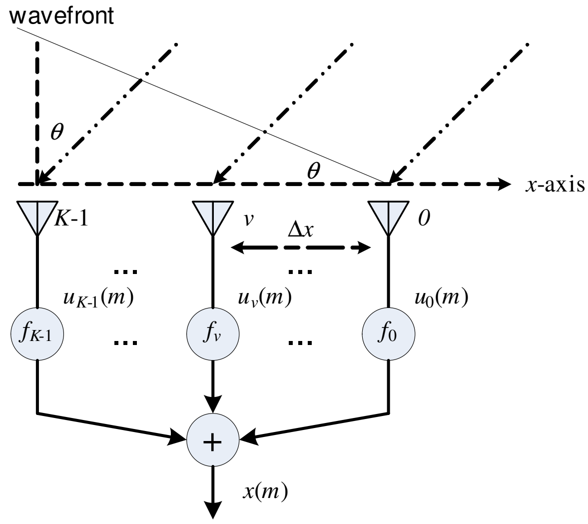

Fig. 2. Structure of a linear vector of M equally spaced antennas.

The combined output of the signals of the K elements, $x\left(m\right)$, is represented by

$$x\left(m\right) = \sum_{v=0}^{K-1} f_v\left(m\right)u_v\left(m\right), \tag{8}$$

Substituting Equation 7 in Equation 8 gives

$$x(m) = \sum_{v=0}^{K-1} f_v(m)u(m)e^{-j\beta v\Delta x\cos(\theta)} = u(m)o(\theta), \tag{9}$$

where

$$o\left(\theta\right) = \sum_{v=0}^{k-1} f_v\left(m\right)e^{-j\left(2\pi\frac{v}{\lambda}\right)\Delta x\cos(\theta)} \tag{10}$$

is the array factor that determines the pattern and direction of the gain. By adjusting the weightings $f_v\left(m\right)$ of the array, it is possible to select any direction for maximum gain [2].

Rewriting Equation 7 and substituting $u(m)$ by Equation 2 gives

$$u_v(m) = \sum_{i=0}^{L-1} \alpha_i(m)z(m - \tau_i(m))e^{-j(v\beta)\Delta x\cos(\theta_i)} + r_v(m), \tag{11}$$

where $r_v(m)$ is the noise associated with each antenna element. Rewriting Equation 8 gives

$$\begin{aligned} x(m) \quad = \quad & \sum_{v=0}^{K-1}\sum_{i=0}^{L-1} f_v(m)\alpha_v(m)z(m - \tau_i(m)) \\ & e^{-j\beta v\Delta x\cos(\theta_i)} + r_v(m). \end{aligned} \tag{12}$$

It can be seen from Equation 9 that each ith path is weighted by its angle of arrival (AOA), θ_i, and by its position in the spatial vector. This enables the spatial processor to distinguish and eliminate undesirable paths. Various algorithms exist that can be used to automatically adjust the weightings, $f_v(m)$, and here the technique governed by the LMS algorithm [3] was employed, in which an estimated signal error, $e(n)$, is obtained using a previously known training signal, $a_{tr}(n)$. The error signal is given by

$$e(n) = a_{tr}(n) - \tilde{a}(n). \quad (13)$$

IV. Discrete Model of the OFDM System

The OFDM transmission technique basically consists in transforming a Single Carrier Signal (SC), of bandwidth B Hz, into a signal formed by C carriers of bandwith B/C Hz. Unlike the FDM technique, in which information from different sources are frequency domain multiplexed, OFDM parallelizes one data source in several other data sources transmitted in orthogonal sub-carriers [4].

One advantage of this type of transmission is that each sub-carrier may have a bandwith smaller than the coherence bandwith of the channel. In other words, each symbol may have a period longer than the delay spread of the channel, improving the robustness of the system against ISI [4]. The OFDM symbol period, T, is given by

$$T = T_s \cdot C, \quad (14)$$

where T_s is the symbol period of data source, which also coincides with the OFDM symbol sampling period.

The OFDM transmission technique could, theoretically, be implemented through a bank of oscillators for the generation of orthogonal sub-carriers, which would be impractical for a large number of sub-carriers. However, the OFDM transmitter can be implemented using Discrete Fourier Transform (DFT).

The signal of the pth sample of the mth OFDM symbol, in the IDFT output, is given by

$$b_p(m) = \frac{1}{C}\sum_{k=0}^{C-1} a_k(m)\, e^{-j\frac{2\pi k}{C}p}, \quad (15)$$

where C is the number of sub-carriers and $a_k(m)$ is the kth complex symbol to be transmitted (also called sub-carrier) in the mth OFDM symbol. In a vector representation of the signals of the OFDM transmitter, it follows that at each time m the transmitter stores C symbols $a_k(m)$ in the vector $\mathbf{a}(n)$, which is given by

$$\mathbf{a}(n) = \begin{bmatrix} a_0(s) \\ \vdots \\ a_{C-1}(s) \end{bmatrix} = \begin{bmatrix} a(s) \\ \vdots \\ a(s-C+1) \end{bmatrix}, \quad (16)$$

where $s = n - m$ and n represents the index of the samples in the period of the OFDM symbol, shown in Equation 14. This vector is then processed by the IDFT generating a new OFDM symbol, $\mathbf{b}(n)$, characterized as

$$\mathbf{b}(n) = \mathbf{W}_{IDFT}\mathbf{a}(n), \quad (17)$$

where $\mathbf{W}_{IDFT}$ is the IDFT matrix of gains, which is represented as

$$\mathbf{W}_{IDFT} = \begin{bmatrix} \mathbf{w}_0 \\ \vdots \\ \mathbf{w}_{C-1} \end{bmatrix}, \quad (18)$$

where

$$\mathbf{w}_p = \begin{bmatrix} e^{-j\left(\frac{2\pi 0}{C}\right)p} & \cdots & e^{-j\left(\frac{2\pi(C-1)}{C}\right)p} \end{bmatrix}. \quad (19)$$

The OFDM symbol can be characterized as

$$\mathbf{b}(n) = \begin{bmatrix} b_0(s) \\ \vdots \\ b_{C-1}(s) \end{bmatrix} = \begin{bmatrix} b(s) \\ \vdots \\ b(s-C+1) \end{bmatrix}. \quad (20)$$

The GI insertion in a vectorial representation can be characterized by

$$\mathbf{GI}(n) = \begin{bmatrix} GI(n-C) \\ \vdots \\ GI(n-C-N_{GI}+1) \end{bmatrix}, \quad (21)$$

where N_{GI} is the length of the GI. Combination of matrices 20 and 21 gives

$$\mathbf{z}(n) = \begin{bmatrix} \mathbf{b}(n) \\ ---- \\ \mathbf{GI}(n) \end{bmatrix}, \quad (22)$$

where $\mathbf{z}(n)$ is the OFDM symbol composed of $C + N_{GI}$ samples. Figure 3 illustrates in a block diagram the OFDM transmitter in baseband.

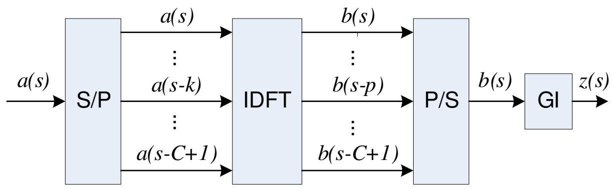

Fig. 3. Block diagram of a baseband OFDM transmitter.

The OFDM symbol is then transmitted through a multipath channel with AWGN noise, as shown in Figure 1 and characterized by Equation 3. Assuming that the size of the GI, N_{GI}, is a percentage of C and greater than or equal to channel length L, it can be shown that the $C + N_{GI}$ samples of the received OFDM symbol are given by

$$\mathbf{x}(n) = \mathbf{h}_g(n)^T\mathbf{Z}_d(n) + \mathbf{r}(n), \quad (23)$$

where $\mathbf{Z}_d$ is the delay matrix formed by the vectors shown in Equation 6, which can be represented by

$$\mathbf{Z}_d(n) = \begin{bmatrix} \mathbf{B}'_d(n) & | & \mathbf{B}''_d(n) & | & \mathbf{GI}_d(n) \end{bmatrix}, \quad (24)$$

where $\mathbf{B}'_d(n)$ is the é ISI portion within the OFDM symbol characterized as

$$\mathbf{B}'_d(n) = \begin{bmatrix} b(s-d_{0,0}) & \cdots & b(s-d_{0,C-L-1}) \\ \vdots & \ddots & \vdots \\ b(s-d_{L-1,0}) & \cdots & b(s-d_{L-1,C-L-1}) \end{bmatrix}, \quad (25)$$

whereas $\mathbf{B}''_d(n)$ is the ISI portion between the OFDM and GI symbols, being described as

$$\mathbf{B}''_d(n) = \begin{bmatrix} b(s-d_{0,C-L}) & \cdots & b(s-d_{0,C-1}) \\ \vdots & \ddots & \vdots \\ GI(s-d_{0,C-L}) & \cdots & GI(s-d_{L-1,C-1}) \end{bmatrix}, \quad (26)$$

and the $\mathbf{GI}_d(n)$ matrix describes the interference within the GI. In the $\mathbf{B}'_d(n)$ and $\mathbf{B}''_d(n)$ matrices the $d_{i,j}$ variable is characterized as

$$d_{i,j} = \tau_i(m) - j. \tag{27}$$

In the receiver shown in Figure 4, the $\mathbf{GI}_d(n)$ matrix is discarded, with the received signal characterized as

$$\mathbf{x}(n) = \mathbf{h}_g(n)^T \left[\begin{array}{c|c} \mathbf{B}'_d(n) & \mathbf{B}''_d(n) \end{array}\right] + \mathbf{r}(n). \tag{28}$$

After DFT, the C carriers are recovered, where the mth carrier estimated is given by

$$\widetilde{a}(s) = \mathbf{w}^*_m(n)^T \mathbf{x}(n) \tag{29}$$

and the set of C carriers can be represented by the vector

$$\widetilde{\mathbf{a}}(n) = \begin{bmatrix} \mathbf{w}^*_0(n)^T \mathbf{x}(n) \\ \vdots \\ \mathbf{w}^*_{C-1}(n)^T \mathbf{x}(n) \end{bmatrix}. \tag{30}$$

When Equation 29 is rewritten, we have

$$\widetilde{a}(s) = \mathbf{h}_g(n)^T \left[\mathbf{B}'_d(n) \,|\mathbf{B}''_d(n)\right] \mathbf{w}^*_m(n) + (\mathbf{w}^*_m)^T \mathbf{r}(n). \tag{31}$$

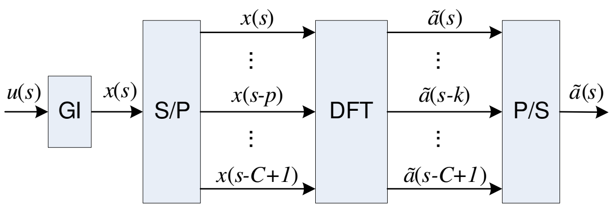

Fig. 4. Block diagram of an OFDM baseband receiver.

V. Spatial Filter for the OFDM System

According to the channel matrix $\mathbf{Z}_d(n)$, ISI correlates the carriers of the OFDM symbol, destroying the orthogonality among them. Thus, another possibility of reducing ISI is solving the problem directly through time domain spatial filters. These adaptive filters act as orthogonalizers that eliminate the correlation between the carriers. The great advantage of this technique is the increased capacity of transmission, which can be obtained by the reduction or even elimination of the GI.

Figure 5 illustrates the spatial filter scheme for the OFDM system, where the adaptation strategy is given by

$$\widehat{a}(s) = \begin{cases} \text{decisor}\{\widetilde{a}(s)\}, \textit{ if } \widetilde{a}(s) \neq a_p(s) \\ a_p(s), \textit{ if } \widetilde{a}(s) = a_p(s) \end{cases}, \tag{32}$$

where $a_p(s)$ a pilot symbol that is spread or grouped within a OFDM symbol. This strategy is essential because it allows the use of this new proposal of reception without any change in the system transmitter. An example of vector $\widehat{\mathbf{a}}(n)$ for a set of pilot sub-carriers could be described by

$$\widehat{\mathbf{a}}(n) = \begin{bmatrix} \text{decisor}\{\widetilde{a}(s)\} \\ \vdots \\ a_p(s-j) \\ \vdots \\ \text{decisor}\{\widetilde{a}(s-v)\} \\ \vdots \\ a_p(s-M+1) \end{bmatrix}. \tag{33}$$

VI. Simulations and Results

A simulator especially developed for the SBTVD was used in the simulation tests of the reception system presented in this paper [6], which enabled the comparison of the results of the proposed receiver to the conventional receiver. The simulation parameters used are shown in Table I, and it can be seen that the OFDM system was simulated with a GI $1/4$ aimed at the performance analysis of the spatial filter, given the better parametrization of OFDM in relation to the delay spread of channels. The results were simulated for the stationary Brazil A and E channels, detailed in Table II. The performance curves of the bit error rate (BER) as a function of E_b/N_0 were obtained for all channels.

TABLE I

Parameters used in the simulation.

System	SBTVD
Number of antennas	4
Mode	2k
Modulation	16-QAM
Guard interval (GI)	$1/4$
Pilots	Spread
Channel estimation	ideal
Channel coding	none

TABLE II

Power profile and AOA of the Brazil A and E simulated channels.

Brazil A	μs	0	$0,15$	$2,22$	$3,05$	$5,86$	$5,93$
	dB	0	-13,8	-16,2	-14,9	-13,6	-16,4
	AOA	$0°$	$30°$	$60°$	$90°$	$300°$	$330°$
Brazil E	μs	0	1	2			
	dB	0	0	0			
	AOA	$0°$	$60°$	$90°$			

The results of BER performance according to E_b/N_0, shown in Figures 6 and 7, indicate that the performance of the proposed scheme led to considerable gains compared to the other scenarios, especially in the much more complicated case of the Brazil E channel. The results for the scenario with the conventional receiver with GI and ideal channel estimation characterize a lower bound for the OFDM system. On the other hand, the proposed receiver has a higher transmission capacity because it does not use the GI. The pre-FFT adaptive spatial filter does not require the elimination of the GI and the channel estimator, but it may increase the robustness of the ODFM system against noise and interference.

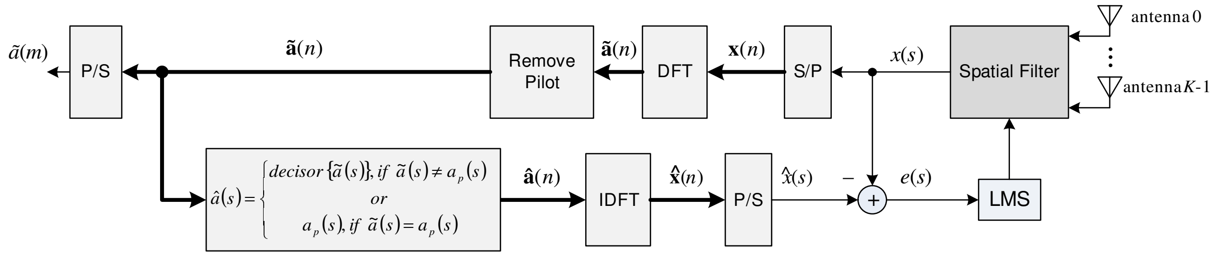

Fig. 5. Block diagram of the adaptive spatial filter scheme proposed for the OFDM system.

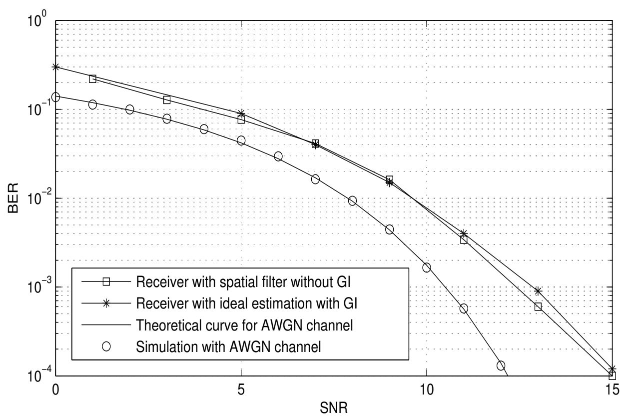

Fig. 6. BER performance curve for Brazil A channel. Spatial filter with $K = 4$ antennas.

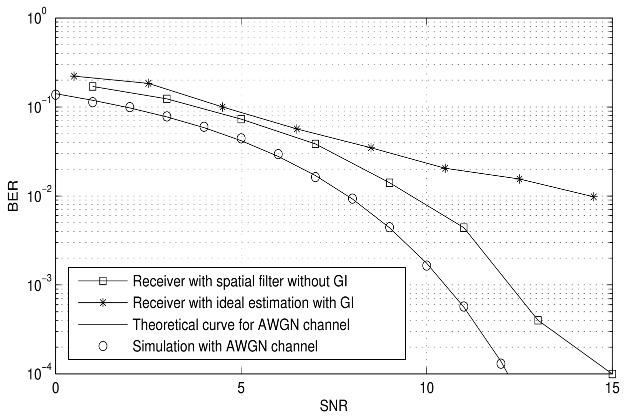

Fig. 7. BER performance curve for Brazil E channel. Spatial filter with $K = 4$ antennas.

VII. Conclusions

This paper proposed an adaptive spatial filter scheme for OFDM systems, with pre-FFT processing. This spatial filter scheme allows a very low or practically null Guard Interval (GI) of the OFDM system, and still ensuring a performance similar to that of the ideal receiver with GI. This strategy allows for a simple and efficient solution for the OFDM receiver compared to pre-FFT spatial filters structures found in the literature. The results suggest the feasibility of implementing the proposed scheme in different OFDM receivers, such as for Digital TV and wireless LANs standards. Another important aspect concerns the simplicity of the adaptive structure that does not change the conventional algorithms, reinforcing the feasibility of the proposed scheme.

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