LSMI Beamformer with Adaptation based on Estimation Error

Filipe de C. B. da Silveira, Ricardo Zelenovsky and Mylène C. Q. de Farias

***Abstract*— In adaptive beamforming, computational cost is a critical factor for the beamformer performance. The newest methods employed in robust adaptive beamforming use complex calculations to achieve the highest SINR values, but the computational cost usually increases. So, methods to reduce this cost maintaining relatively high SINR are desired. The idea is to use simple beamforming methods when the SINR performance is reasonable and a more sophisticated method when it degrades below a certain value. Nonetheless, an algorithm that switches between the delay-and-sum and loaded sample-matrix inversion method is presented.**

***Keywords*— Adaptive beamforming, uniform linear array, sample-matrix inversion, loaded SMI, minimum variance distortionless response beamformer and loading factor γ.**

I. Introduction

In the last decades, antenna arrays have been a relevant research topic in various applications [1]: radar; sonar; wireless communications; etc. They are used in beamforming solutions and several approaches have been developed to increase their performance in various scenarios. Methods for direction-of-arrival (DOA) estimation - delay-and-sum (DS), Capon and MUSIC [2] - and for robust adaptive beamforming (RAB) - minimum variance distortionless response (MVDR) and least-mean-square (LMS) [3], [4] - have been proposed. When the DOA of the signal-of-interest (SOI) is known, the simplest method to aim the array is the DS one.

When the environment variables - noise, DOA, array imperfections, etc. - are unknown, adaptive beamforming is used [5], [6]. It consists of a versatile approach to detect and estimate the SOI [7]. The main goal in adaptive beamforming is to maximize the beamformer output signal-to-noise-ratio (SINR). However, typical applications include the SOI in their training snapshots, which can severely degrade the SINR performance as the SOI component can be mistakenly interpreted as an interferer by the algorithm.

In RAB, the minimum variance distortionless response (MVDR) is one of the commonly used methods [4]. Two typical variants of the MVDR use samples to maximize the SINR: the sample matrix inversion (SMI); and the loaded SMI (LSMI) [8]. The former uses the inverted interference-plus-noise covariance matrix; the latter just loads this matrix diagonal with a constant loading factor. This factor improves the algorithm robustness and its value has been empirically suggested in previous works [3].

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These RAB methods - SMI or LSMI - are computationally expensive when compared to the simpler methods [9], such as the DS one, as they use complex operations, such as matrix inversion. These algorithms are very robust and they can achieve very high SINR values, but sometimes the DS method is able to achieve similar SINR performance with a much lower computational cost. Therefore, an algorithm capable of switching between the two methods in order to take advantage of the DS simplicity and the LSMI robustness is suggested.

This paper is divided as follows: Section II gives a synthesis of concepts related to adaptive beamforming; Section III proposes an adaptive beamformer based on the estimation error of the SMI and LSMI methods; Section IV proposes a shifting DS-LSMI beamformer; Section V presents the conclusions.

II. Background

In this paper, a uniform linear antenna array (ULA) with M omni-directional antenna elements is used. The narrowband signal received by the ULA at the time instant k is represented by equation (1), where $\mathbf{x}(k)$ is the snapshot captured by the ULA. This snapshot is composed of $\mathbf{s}(k)$, $\mathbf{i}(k)$ and $\mathbf{n}(k)$, which are the signal, interference and noise $M \times 1$ vectors respectively and k is an index that represent the time instant that they were captured.

$$\mathbf{x}(k) = \mathbf{s}(k) + \mathbf{i}(k) + \mathbf{n}(k) \tag{1}$$

The signal is assumed to be uncorrelated with the interferences and the noise; a point source - far-field consideration - is considered so the signal and the interferers arrive at the ULA as a plane wave. The contribution of the signal $s(k)$ is expressed by equation (2).

$$\mathbf{x}_s(k) = s(k)\,\mathbf{a}(\theta_s) \tag{2}$$

In equation (2), the $\mathbf{a}(\theta_s)$ represents the steering vector for the signal arriving from the direction θ_s [2] and it can be modeled by equation (3), where $\phi_s = 2\pi\frac{d}{\lambda}\sin(\theta_s)$.

$$\mathbf{a}(\theta_s) = \begin{bmatrix}1 & \exp^{-j\phi_s} & \cdots & \exp^{-j(M-1)\phi_s}\end{bmatrix}^T \tag{3}$$

The beamformer output at the time instant k is given by equation (4), where $\mathbf{w}$ is a $M \times 1$ complex weight vector of the ULA and $(\cdot)^H$ denotes the Hermitian transpose operation.

$$y(k) = \mathbf{w}^H\mathbf{x}(k) \tag{4}$$

If the weight vector $\mathbf{w}$ delays the various sensors outputs in a way that the signal components are aligned to the direction of the target, then the beamformer uses the DS method to estimate the output signal [7]. Therefore, the weight vector for the DS method can be expressed by equation (5).

$$\mathbf{w}_{DS} = \mathbf{a}\left(\theta_s\right) \tag{5}$$

Another way to calculate the weight vector $\mathbf{w}$ is to maximize the beamformer output SINR. This implies solving the optimization problem described by equation (6).

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w} \quad s.t. \quad \mathbf{w}^H \mathbf{a}\left(\theta_s\right) = 1 \tag{6}$$

The solution is known as minimum variance distortionless response (MVDR) beamformer and its weight vector will be given by equation (7), in which α represents a scaling factor immaterial to the SINR.

$$\mathbf{w}_{MVDR} = \alpha \mathbf{R}_{i+n}^{-1} \mathbf{a}\left(\theta_s\right) \tag{7}$$

In practice, it is not possible to know the actual $\mathbf{R}_{i+n}$ as the antenna snapshots also include the signal component. So, this matrix is commonly replaced by a data sample covariance matrix given by equation (8).

$$\hat{\mathbf{R}} \triangleq \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}\left(k\right) \mathbf{x}^H\left(k\right) \tag{8}$$

The SMI beamformer is obtained by replacing the interference-plus-noise covariance matrix $\mathbf{R}_{i+n}$ in the MVDR beamformer from equation (7) by the sample estimate of the data covariance matrix from equation (8) [3].

The LSMI beamformer is a robust approach to the SMI beamformer and it is based on the diagonal loading of the sample covariance matrix [10]. This means that the matrix $\hat{\mathbf{R}}$ is replaced by a diagonally loaded matrix $\hat{\mathbf{R}}_{DL}$ in the weight vector expression given by equation (7), as in equation (9), where γ is the diagonal loading factor and $\mathbf{I}$ is the identity matrix with the dimensions corresponding to the size of $\hat{\mathbf{R}}$.

$$\hat{\mathbf{R}}_{DL} = \hat{\mathbf{R}} + \gamma \mathbf{I} \tag{9}$$

Using equation (9), it is possible to define the weight vector expression for the LSMI beamformer in equation (10).

$$\mathbf{w}_{LSMI} = \hat{\mathbf{R}}_{DL}^{-1} \mathbf{a}\left(\theta_s\right) = \left(\hat{\mathbf{R}} + \gamma \mathbf{I}\right)^{-1} \mathbf{a}\left(\theta_s\right) \tag{10}$$

It is clear that the DS method provides the simplest solution and the LSMI method has a higher computational cost as it needs to calculate the inverse of the autocorrelation matrix $\mathbf{R}_{DL}$.

III. SMI and LSMI Beamformers Based on Reception Error

When digital modulation - such as binary phase-shift keying (BPSK) or quadratic phase-shift keying (QPSK) - is used, a reception error (REr) can be easily estimated. This error is defined as the displacement between the output of the bit detector (-1 or $+1$) and the received signal, which may be corrupted by noise and interferences. For simplicity, the BPSK modulation is chosen for the simulations. In this situation, the received signal sample has a complex value which will be decided by the two possible outcomes based on the euclidean distance. This scenario is shown on Fig. 1.

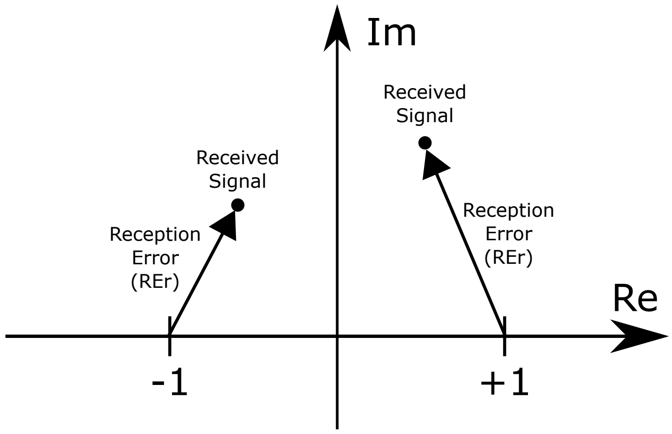

Fig. 1. Two examples of reception error.

Conventional adaptive algorithms recalculate the weight vector $\mathbf{w}$ each time a new sample is perceived. This is rather computationally expensive as the receiver has to recalculate the inverse of the autocorrelation matrices for the LSMI method, even when the difference between the old and the new matrices and the SINR gain are minimal. Therefore, the purpose of the new algorithm is to reduce the amount of adaptation iterations based on the received signal quality using the following idea: if the REr is within a certain threshold, no adaptation is needed; otherwise, an adaptation is done.

The algorithm used can be specified as follows: (i) calculate the output $y = \mathbf{w}^H \mathbf{x}$; (ii) estimate the REr; (iii) adapt the weight vector $\mathbf{w}$ if $|REr| < a$, else keep it.

The simulations have the following characteristics: an ULA of $M = 16$ antennas with 0.5λ inter-space is used to perceive the signals; $n = 2$ users, SOI and interferer, both with the same power and coming from $30°$ and $70°$ respectively and using BPSK modulation; and $N = 2,000$ samples are captured. The noise is modeled as additive white Gaussian noise (AWGN) and its power will vary in order to verify how efficient the methods are.

A. Complex Reception Error

The BPSK detector uses the euclidean norm to decide which value was transmitted: -1 or $+1$. The REr is determined by the distance between the received signal sample and the BPSK detector output, as shown in Fig. 1. The idea is to adapt whenever the euclidean norm of the REr is greater than the value R as it is represented in Fig. 2; if this received value is inside one of the circles whose radii are R, no adaptation is needed. The value for the radius R is not critical and it is chosen as $R = 5$. Therefore, it should be chosen as a function of the computational cost and the SINR performance: the smaller is the value for the radius R, the bigger is the computational cost.

Table I presents the bit error rate (BER) for different noise variances and the number of adaptations done by each method,

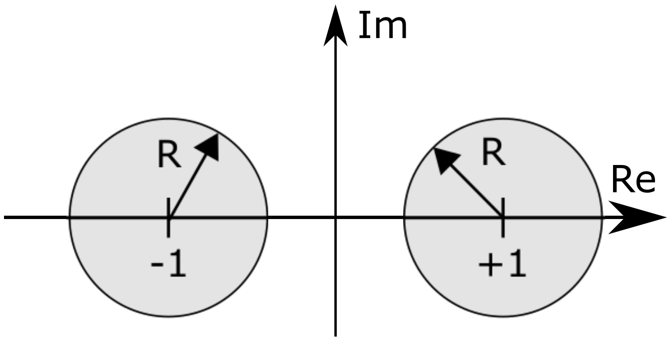

Fig. 2. REr estimation criterium.

TABLE I
NUMBER OF ADAPTATIONS AND ERROR PERCENTAGE WHEN VARYING THE NOISE VARIANCE USING COMPLEX RECEIVED ERROR.

σ_n^2	i_{SMI}	BER_{SMI} (%)	i_{LSMI}	BER_{LSMI} (%)
1	432	0.35	600	0
2	438	0.40	1377	0
3	436	0.60	379	0.05
4	429	0.95	31	0.20
5	430	1.55	31	0.45
10	442	5.15	31	2.95
20	449	11.10	31	10.05
30	442	15.75	31	14.00
40	458	18.75	31	17.45
50	457	21.45	31	19.40

denoted by i_{SMI} and i_{LSMI}. Using the proposed adaptation criteria, for example, the first row on Table I shows that: the SMI had a total of $i_{SMI} = 432$ adaptations and a $BER = 0.35\%$; the LSMI had a total of $i_{LSMI} = 600$ adaptations and a $BER = 0$. The results are rather intuitive: as the noise variance becomes greater, the errors of both beamforming methods become greater as well. However, the rate at which each one increases is different as the LSMI seems to be less affected by the increase in the noise variance. Two phenomena are important to note in the simulation: for low noise variances, the LSMI presents a low error rate when compared to the SMI; on the other hand, their error rates become close to each other for high noise variances. It is worth noticing that the computational cost was reduced with a little performance degradation.

B. Real Reception Error

Here, the adaptation is based on the real value of the received signal for the detection. The real part distance is compared to both possible outcomes (-1 or $+1$) and the closest one is chosen as the desired value. It is easy to understand that, as the received signal gets closer to the vertical axis, the probability to occur an estimation error becomes greater, which increases the need for an adaptation.

The criterium is specified as follows: if the real value is greater than a certain threshold, the weight vector is kept; if it is smaller, the weight vector is recalculated.

Table II presents the simulation with $R = 0.6$ for different noise variances. The results follow the same intuitive behavior as the previous case: the error becomes greater as the noise variance is increased.

TABLE II
NUMBER OF ADAPTATIONS AND ERROR PERCENTAGE WHEN VARYING THE NOISE VARIANCE USING REAL RECEIVED ERROR.

σ_n^2	i_{SMI}	BER_{SMI} (%)	i_{LSMI}	BER_{LSMI} (%)
1	500	0.40	821	0
2	541	0.40	1597	0
3	564	0.65	461	0.05
4	583	1.00	31	0.20
5	612	1.55	31	0.45
10	731	4.95	31	2.95
20	800	11.05	31	10.05
30	832	15.35	31	14.00
40	846	18.80	31	17.45
50	845	21.20	31	19.40

Nevertheless, it is important to note that: for low noise variances, the BER pratically the same for both complex and real approaches; for high noise variances, the real approach presents lower error rate than the complex one. Once again, it is worth noticing that the computational cost was reduced with a little performance degradation.

IV. SHIFTING DS-LSMI BEAMFORMER

The DS beamformer provides good SINR performance when the DOI of the SOI does not change. Now, the LSMI beamformer, which is a robust approach to beamforming, is capable of achieving higher SINR performance at certain moments, but the downside is the high computational cost inherent to the process. Therefore, both methods have their advantages and disadvantages depending on the scenario conditions. This way, a beamformer method, named shifting DS-LSMI (S-DS-LSMI), capable of shifting between the two methods - DS and LSMI - in order to take advantage of the simplicity of the DS approach and the higher SINR performance of the LSMI approach is proposed.

In order to validate the proposed beamformer, simulations in which the SINR performance is measured when the variance of an interferer is varied in unitary steps from 1 to 10 were done. A hipothetical solution, named hipothetical DS-LSMI (H-DS-LSMI), which always chooses to use the method that gives the highest SINR, is used to evaluate the S-DS-LSMI performance.

The proposed S-DS-LSMI beamformer uses two decision regions to decide between the DS and the LSMI approaches. These regions are shown in Fig. 3: if a received signal falls within the interval $[-a; +a]$, the beamformer uses the LSMI algorithm; if it falls outside the region, within the intervals $[-\infty; -a] \cup [+a; +\infty]$, it uses the DS one.

The algorithm can be summarized as follows: (i) the output $y = \mathbf{w}^H\mathbf{x}$ is calculated using $\mathbf{w} = \mathbf{w}_{DS}$; (ii) the REr is estimated; (iii) if $|Re\{REr\}| < a$, then the output is recalculated using $\mathbf{w} = \mathbf{w}_{LSMI}$, else keep $\mathbf{w} = \mathbf{w}_{DS}$; (iv) the output $y = \mathbf{w}^H x$ with the previously chosen weight vector $\mathbf{w}$ is calculated; (v) the bit detection is done using $\mathbf{w}_{DS}$ or $\mathbf{w}_{LSMI}$ as the weight vector $\mathbf{w}$ calculated using the output y.

The system configuration and the environment behavior follows the same pattern in both situations: 1 user transmitting the signal with unitary signal variance; 1 interferer with power varying in steps from 1 to 10; SOI coming from 30° and

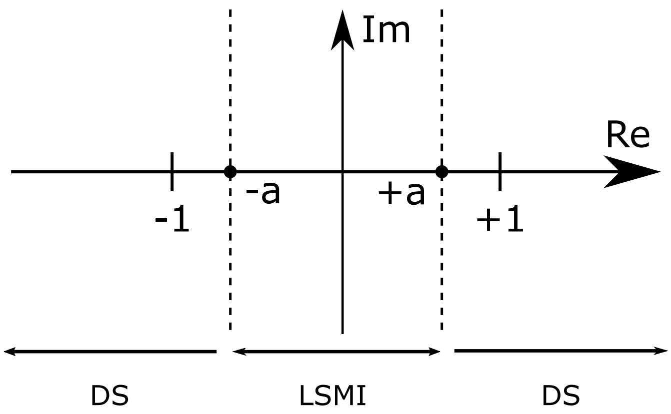

Fig. 3. Decision regions based on the complex diagram of the received bits.

the interferer, from 70°; ULA with the antennas spaced 0.5λ between them; unitary noise variance σ_n^2; 2,000 samples; the loading factor for the LSMI approach is $10\sigma_n^2$.

In the following tables, the DS and LSMI columns contains the usage percentage of the respective method and the SINR values in the columns were calculated in dB.

A. ULA with 32 antennas

The results for this case are displayed in table III.

For unitary interferer variance, it is noticeable that the S-DS-LSMI achieves almost the same SINR as the H-DS-LSMI. However, as the interferer power is augmented, the frequency that the LSMI method is used for both the S-DS-LSMI and the H-DS-LSMI increases to maintain the SINR as high as possible. In the S-DS-LSMI beamformer, the usage percentage of the LSMI method does not increase much, but this does not represent a problem as the SINR performance suffers a small degradation - approximately 0.15 dB - when compared to the H-DS-LSMI beamformer.

For extreme-cases ($\sigma_{int}^2 = 10$), the number of iterations in which the LSMI method is used increases significantly for the H-DS-LSMI, but not as much for the S-DS-LSMI. Comparing the percentages of LSMI use, it was used: 7.05% of the times for a SINR of 14.5902 dB in the S-DS-LSMI; and 30.70% of the times for a SINR of 14.7320 dB in the H-DS-LSMI. This represents a difference of 23.65% of the times of LSMI usage for a a SINR gain of just 0.1418 dB, which might not justify the increase in computational cost. In situations in which computational cost is critical, using the S-DS-LSMI may seem to be the best approach as the number of iterations that use the DS method is relatively high. Therefore, the computational cost is reduced because the DS method is used more times and it is a simpler method compared to the LSMI one.

B. ULA with 64 antennas

The results for this case are displayed in table IV.

A similar behavior to the ULA with 32 antennas is noticeable: when the interferer has unitary variance, the SINR performances are practically the same; when the interferer variance increases, the number of LSMI iterations needed to get the best SINR value increases significantly; in the S-DS-LSMI, this does not happen as the LSMI number of iterations increase in a much lower pace.

TABLE III
PERFORMANCE OF S-DS-LSMI AND H-DS-LSMI VERSUS INTERFERER POWER.

σ_{int}^2	S-DS-LSMI %DS	S-DS-LSMI %LSMI	S-DS-LSMI SINR	H-DS-LSMI %DS	H-DS-LSMI %LSMI	H-DS-LSMI SINR
1	94.80	5.20	15.0387	92.40	7.60	15.0476
2	94.80	5.20	15.0127	91.35	8.65	15.0360
3	94.85	5.15	14.9807	90.40	9.60	15.0170
4	94.70	5.30	14.9412	88.55	11.45	14.9911
5	94.65	5.35	14.8976	85.75	14.25	14.9591
6	94.25	5.75	14.8511	83.25	16.75	14.9216
7	93.80	6.20	14.7936	80.25	19.75	14.8794
8	93.75	6.25	14.7352	76.80	23.20	14.8331
9	93.40	6.60	14.6664	73.75	26.25	14.7836
10	92.95	7.05	14.5902	69.30	30.70	14.7320

Comparing both beamformers and considering the case in which $\sigma_{int}^2 = 10$, the number of iterations in which the LSMI is used increases significantly for the H-DS-LSMI and slightly for the S-DS-LSMI. In the S-DS-LSMI, the LSMI method was used in 1.75% of the total iterations and achieved a SINR of 17.3148 dB; in the H-DS-LSMI, the LSMI method was used in 44.05% of the total iterations and achieved a SINR of 17.5264 dB. This represents a difference of 42.30% of the times the LSMI was used for just a 0.2116 dB gain in SINR, which might not be justified by the additional computational cost inherent to using the LSMI method more times. Also, it is worth noticing that the S-DS-LSMI method can achieve similar SINR performance to the H-DS-LSMI beamformer with less computational cost, because the DS method is used more frequently.

TABLE IV
PERFORMANCE OF S-DS-LSMI AND H-DS-LSMI VERSUS INTERFERER POWER.

σ_{int}^2	S-DS-LSMI %DS	S-DS-LSMI %LSMI	S-DS-LSMI SINR	H-DS-LSMI %DS	H-DS-LSMI %LSMI	H-DS-LSMI SINR
1	99.10	0.90	18.0498	99.15	0.85	18.0538
2	99.20	0.80	18.0247	95.70	4.30	18.0303
3	99.15	0.85	17.9827	90.60	9.40	17.9929
4	99.20	0.80	17.9281	85.40	14.60	17.9443
5	99.10	0.90	17.8537	81.75	18.25	17.8855
6	99.05	0.95	17.7697	74.10	25.90	17.8201
7	98.80	1.20	17.6710	69.80	30.20	17.7504
8	98.65	1.35	17.5620	65.35	34.65	17.6761
9	98.55	1.45	17.4441	58.70	41.30	17.6015
10	98.25	1.75	17.3148	55.95	44.05	17.5264

C. S-DS-LSMI versus H-DS-LSMI adaptation

In order to evaluate the performance of the S-DS-LSMI beamformer, the SINR was calculated as the snapshots were received. Fig. 4 corresponds to the H-DS-LSMI beamformer, which always chooses the method that gives the highest SINR; Fig. 5 corresponds to the case in which the beamformer chooses the method based on the S-DS-LSMI beamformer.

From Fig. 4, it is possible to see that, in the best scenario, there will be a lower limit for the SINR and it will be established by the DS method performance. At certain moments,

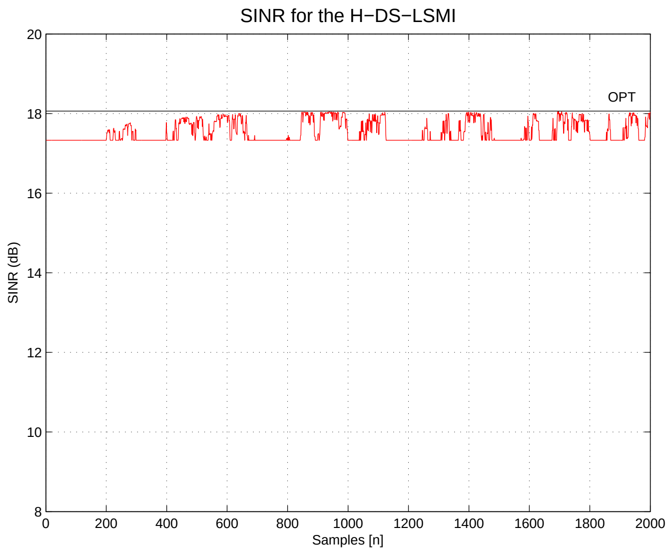

Fig. 4. Adaptation using the H-DS-LSMI.

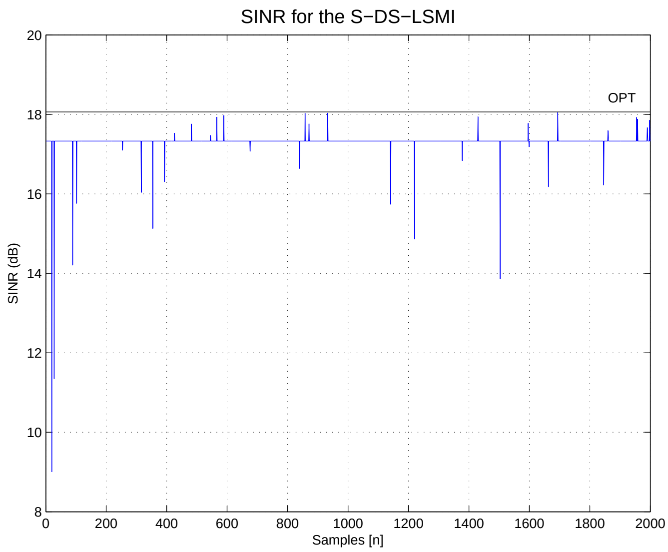

Fig. 5. SINR for the shifting DS-LSMI beamformer based on REr.

the SINR can achieve higher values using the LSMI method. Taking the average SINR value out of the $2,000$ collected samples, the values 17.5264 dB and 17.3148 dB are found for the H-DS-LSMI and the S-DS-LSMI cases respectively. Therefore, it can be seen that the SINR performance degrades slightly using the S-DS-LSMI beamformer, but this degradation is relatively small compared to the computational gain when using the DS method more frequently than the LSMI one.

V. Conclusions

In this paper, several approaches to adaptive beamforming have been proposed in order to reduce the computational cost of the algorithms.

In the first analysis, both the SMI and the LSMI beamformers have been developed and the weight vector was adapted or not based on the distance between the received sample and the expected value. For the SMI case, the number of algorithm iterations increased slightly according to the increase in noise power, as well as the BER. This result is rather intuitive, because it was expected that more adaptations would be needed if the noise power was increased. For the LSMI case, the number of algorithm iterations decreased and it remained constant for noise power values greater than $\sigma_n^2 = 4$. However, the BER kept increasing as the noise was increased. Also, this result is intuitive for the same reason as the SMI case. The main point here is the ability to reduce computational cost reducing the number of adaptations.

In the second analysis, a beamformer capable of shifting between the DS and the LSMI methods was proposed. Two cases of this beamformer were analyzed: with 32 and 64 antennas. Also, a hypothetical solution which always chooses the method that gives the best SINR was used for comparison. Comparing both the S-DS-LSMI and the H-DS-LSMI, it is easy to see that choosing the S-DS-LSMI does not provide the best SINR possible for the system. However, the number of iterations in which the LSMI method is used decreases significantly and this means that the computational cost is reduced and the loss in SINR performance is relatively minimal. This happens in both cases of antennas configuration. Therefore, this shifting beamformer might be a good solution when computational cost is a critical factor.

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