Blind Equalization Based on Complexity Measures: Is It Feasible?

Patrick F. Coutinho, Diogo C. Soriano, Filipe I. Fazanaro, Ricardo Suyama, Romis Attux

Abstract— **This work investigates an alternative approach to the problem of blind equalization. The approach is based on complexity measures and is inspired by preceding successful application of the same framework to the problem of blind source separation. We draw the relationship between algorithmic complexity, a measure for randomness within the area of algorithmic information theory, and recurrence quantification analysis, a tool for recurrent structure analysis of dynamical data. The evaluation of the hypotheses is carried out in the context of chaotic signals. The results show that such approach is effective under some circumstances (minimum-phase or stable and invertible channel).**

Keywords— **Algorithmic complexity, Blind channel equalization, Chaotic signal processing, Recurrence plot.**

I. Introduction

Intersymbol interference (ISI) is a problem that may affect information transmission in communication systems. It can be seen as the effect of a system (channel) operating over an information signal $s(n)$. The received signal $x(n)$ is, therefore, a distorted version, which can compromise the process of message reconstruction. The main goal of equalization is to counterbalance the channel effects by means of the use of an appropriate filter, the equalizer [1].

Unsupervised (or blind) equalization is interesting in that it does not require the availability of a desired sequence during parameter adaptation. There are many unsupervised approaches, including the Bussgang and Shalvi-Weinstein families [2]. Interestingly, the very rich notion of algorithmic complexity allows an alternative path to be followed. The basic idea, founded on the works of Pajunen [3] and Soriano et al. [4] about blind source separation, is to consider the superposition of different versions of a signal as being more complex than the signal itself. This creates a novel framework for channel inversion, which was initially and partially analyzed in [5]. This work can be seen as an extension of these efforts towards a more general view of the novel framework and of its concrete possibilities. It is our belief that this extension is of relevance, as this alternative to classical theory has the potential of leading to theoretical and practical insights and to a natural treatment of the problem of deterministic signal processing.

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The work is structured as follows. In section II, it is discussed the notion of algorithmic complexity and it is shown how to estimate it by means of recurrence plots and data compression. In section III, we deal with the problem of blind equalization based on complexity measures in the context of chaotic signals. Section IV exhibits some results of numerical experiments. Finally, section V brings the conclusions.

II. Algorithmic Complexity and its Estimation

A. Algorithmic Complexity

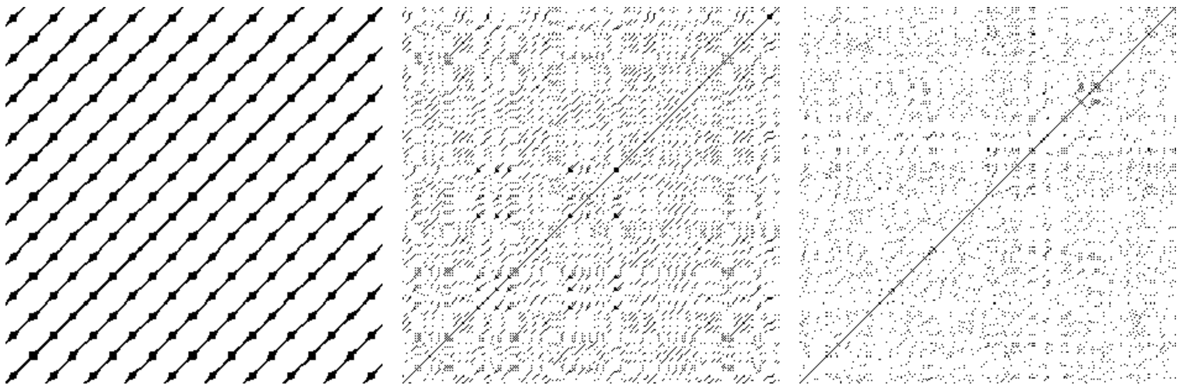
Compared to Shannon's notion of information, algorithmic complexity (AC) is a paradigmatic shift in the realm of information theory. In his seminal paper, Shannon [6] was concerned about the average information content of a source, which is built upon a set of messages and is associated with the concept of entropy. In simple terms, entropy measures the "uncertainty" of a random variable based on its probability distribution.

From a different perspective, Kolmogorov was interested in the information content of a single message [7] and, almost at the same time, Chaitin devised a manner to quantify the randomness of a string, *i.e.*, a sequence of symbols [8]. Both ended reaching similar and intuitive idea. This idea is summarized in the concept of AC, which is defined with respect to a finite binary string s as the size of the shortest computer program that calculates it. This task must be accomplished with no additional information (there are no external inputs to the program).

AC is easily stated, however the problem with it is that it is not computable by a Turing machine [7]. Nonetheless, due to an intrinsic link with the concept of "randomness", there are efficient practices to estimate the complexity of a string. A purely random sequence needs significant information to be exactly replicated, and the computer program that generates such a sequence is, potentially, as large as the sequence itself [8]. On the other hand, non-random sequences contain patterns that can eventually be exploited to reduce the program size.

Compression algorithms try to compact binary sequences by finding an efficient code based on the underlying data patterns. Lossless methods can do it without discarding any information. Although the success of a compression task depends on the data being analyzed, lossless compression methods shorten structured sequences and give a reliable measure of their complexity. It must be said that a great variety of compression algorithms work only on integer data. This is the case *e.g.* of the famous Lempel-Ziv algorithm [7].

In the following, we turn our attention to another approach to quantify "complexity" that has its roots in the field of



(a) (b) (c)

Fig. 1: Recurrence plots shows underlying recurrences in the data. Chosen parameters values, $d_e = 2$, $\tau = 2$ and $\epsilon = 0.4$. Number of samples for each signal is 300.

dynamical systems: recurrence plots (RPs). They emphasize underlying structures of the data and can be coded by lossless compression methods.

B. Recurrence Plots

In dynamical system theory, it is usual to employ RP to analyze non-linear data. Fundamentally, an RP is a binary square matrix that expresses recurrent structures if they are present in the data. It is constructed by taking the distance between two state space vectors: whether the distance is below a threshold ϵ, the corresponding element in the matrix is set to one, and, visually, represented as a black point [9]. Otherwise, the element value is zeroed and a white point is used. Mathematically, it is represented by:

$$r_{i,j} = \begin{cases} 1, & \text{if } \|\mathbf{x}_i(k) - \mathbf{x}_j(k)\| < \epsilon \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

for $i, j = 1, ..., N$, where $r_{i,j}$ is the element in the i-th row and j-th column of the matrix, N is the number of state space vectors, and $\mathbf{x}(k)$ is the state space vector defined as:

$$\mathbf{x}(k) = [x(k) \;\; x(k-\tau) \;\; ... \;\; x(k-(d_e-1)\tau)], \quad (2)$$

in which d_e is the embedding dimension and τ the lag between samples. One of the most common norm used to calculate the distance between the state space vectors is the L_∞-norm (maximum or supremum norm) [9].

Due to the way it is constructed, the RP is a symmetric matrix with a main diagonal composed of black points. To illustrate, we compare the RPs of three distinct time series. The first is a sinusoidal signal, $s(n) = \sin(0.05n)$. The recurrent property of this signal is clearly characterized by the long and parallel diagonals in Figure 1a. The second is a chaotic signal obtained from the logistic map, a discrete nonlinear dynamic system. The time series of this signal is obtained from (3):

$$x_{n+1} = \alpha x_n (1 - x_n), \quad (3)$$

with $\alpha = 4$. The RPs of chaotic signals display typically short but parallel diagonals as can be seen in Figure 1b. The last figure (Figure 1c) corresponds to the RP of a white Gaussian noise: its RP displays a homogenous distribution of black points.

To apply lossless compression methods to the RPs, firstly the columns of the matrix are concatenated to make a string: by doing this step, we guarantee that observed recurrent structures are explored in the compression task. The binary sequence obtained from the RP is then compressed using ZIP compression. The zipped file size is compared to the size of the original string, giving the following measure:

$$\text{ZIP}_{\text{index}} = \frac{\text{size}_{\text{ZIP}}}{\text{size}_{\text{original}}}. \quad (4)$$

Another way to estimate the complexity of RPs is by using the mathematical tool of recurrence quantification analysis (RQA) [9].Within this framework, complexity may be estimated by analysis of diagonal structures or density of recurrence points of the RP. One metric obtained from such analysis is the recurrence rate (RR), which indicates the aforementioned density and is expressed by the equation:

$$\text{RR} = \frac{1}{N^2} \sum_{i,j=1}^{N} r_{i,j}, \quad (5)$$

where N is the number of rows of the recurrence plot. Another metric is the determinism (DET), a measure of the organization of diagonals in longer structures, typical of regular observations, based on the size l of the diagonals and their histogram $(P(l))$. The equation for DET is:

$$\text{DET} = \frac{\sum_{l=l_{min}}^{N} lP(l)}{\sum_{l=1}^{N} lP(l)}, \quad (6)$$

l_{min} is the minimum diagonal length to be considered ($l_{min} \geq 2$). The values of RR and DET are higher for deterministic signals. These quantities are as viable as the compression method when it comes to estimating complexity in the context of BSS [10].

III. Blind Equalization Based on Complexity Measures

To invert the effects of the channel employing complexity as a metric to guide the equalizer design, we assume that the complexity of the distorted signal $x(n)$ is higher than that of $s(n)$. This assumption goes hand-in-hand with those presented in the case of BSS both by Pajunen [3] and Soriano et al. [4], [11]. We hypothesize that equalization is possible only in the case of minimum-phase (zeros are inside the unit circle) or stable (poles are inside the unit circle) channel and if the zero-force conditions are satisfied (channel and equalizer are inverse of each other).

In Figure 2, we analyze the behavior of sending the chaotic signal obtained from the logistic map through FIR and IIR channels. The chosen parameters for the generated RPs are $d_e = 1$, $\tau = 0$, $\epsilon = 0.4$, and the number of samples is 1000. All generated signals are normalized (zero mean and unit variance). The FIR model of the channel is (7):

$$\mathrm{H}_{\mathrm{FIR}}(z) = \cos(\theta) - z^{-1}\sin(\theta), \tag{7}$$

and the IIR model is (8):

$$\mathrm{H}_{\mathrm{IIR}}(z) = \frac{1}{\cos(\theta) - z^{-1}\sin(\theta)}, \tag{8}$$

both parameterized in θ. The figure shows the complexity measures of the signal before (horizontal dashed black line) and after being transmitted through FIR and IIR channels. The rectangles in yellow delimit maximum-phase regions for the FIR filter and unstable regions for the IIR filter. In this last case, the amplitude of the transmitted signal diverges, and the consequence is that the generated RPs are "corrupted" and no relevant information can be inferred from them. The figure corroborates what was hypothesized above. It is also interesting to note that, given the same number of coefficients, the ISI in the case of the IIR channel is more severe than that of the FIR channel, supporting what was stated in [12].

Furthermore, it is important to emphasize that the RPs are extremely sensible to the values of the parameters. There are several methods and rules of thumb that can be employed to calculate them. For example, the embedding dimensions may be chosen by means of the concepts of false nearest neighbors and mutual information [9]; the threshold, in turn, may be chosen between 20% and 40% of the signal's standard deviation [13]. Despite this, their values depend on the nature of the problem being treated [14] and, clearly, on the signal itself. There is no absolute consensus on how to choose the most appropriate values, especially to the problem of equalization.

It is worth mentioning that, for some configurations of the parameters, the hypothesis of complexity increase does not hold: the signal seems to undergo an apparent reduction in this quantity. Such an event may be observed in Figure 3 for the FIR channel. In this case, the values of d_e and τ are set to 2 and 1, respectively, while the value of the threshold is maintained equal to 0.4.

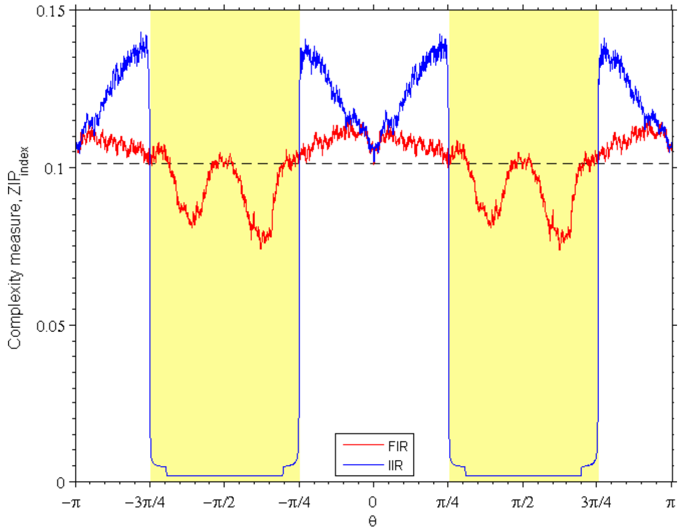


Fig. 2: ZIP$_{\mathrm{index}}$ for a chaotic signal sent through a FIR and an IIR channels. The dashed line corresponds to the complexity measure of the source signal (the signal is normalized). The region in yellow delimits the values for which the zeros or the poles of the filters are outside the unit circle. It is important to note the increase in the complexity measure for both of the channels: outside the yellow rectangles, the complexity of the transmitted signal is higher than that of the original one.

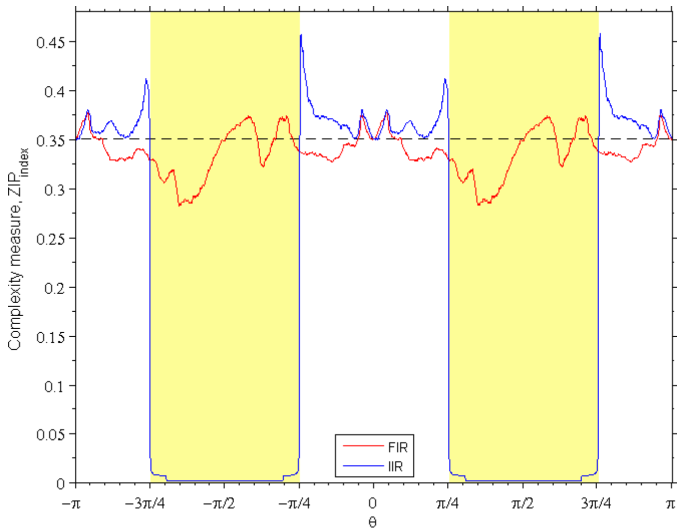


Fig. 3: The same ZIP$_{\mathrm{index}}$ as before (Fig. 2). The complexity is not correctly estimated due to the values of the parameters of the RPs. The signal undergoes a reduction in its complexity along the transmission process through the FIR channel even in minimum-phase regions (outside the yellow rectangles).

IV. Results of Mathematical Simulations

In order to exemplify the equalization process based on complexity measures, we show next (Figure 4) the results of recovering a chaotic source (logistic map). Two cases are considered: 1) the FIR equalization of an IIR channel, and 2) the reverse *i.e.* the IIR equalization of a FIR channel. The channels are modeled as (7) and (8) with $\theta = \pi/6$. The values of the parameters d_e and τ are fix and equal 1 and 0, respectively. Thus, no embedding is considered and each state space vector is composed of a single sample. The threshold ϵ varies from 0.1 to 0.5 with a step of 0.1, and the number of samples of the signal is 1000.

Equalization is performed by an exhaustive search in the parameter θ - now the parameter of the equalizer. Its values are confined to the minimum-phase or stable region ranging from $-\pi/4$ to $\pi/4$ and a step of $\pi/400$ is considered. A careful attention must be paid to values around the vicinity of the unit circle, mainly for the IIR implementation of the equalizer. The figure shows that minimum complexity is attained at $\theta = \pi/6$, where the order of magnitude of the mean squared error is 10^{-6}.

V. Conclusions

In this work, the viability of employing complexity measures to the problem of blind equalization was investigated. A tacit assumption that must be acknowledged as *a priori* information permeates the work: the signal being sent is considered to have some type of recurrent structure that is lost in the transmission process. From this point of view, the *modus operandi* of the presented approach is very different from that of the conventional ones, which resort to statistical concepts.

As it was seen in mathematical simulations, when the channel is minimum-phase (FIR model) or stable (IIR model) and invertible, the source signal can be recovered perfectly. Although the tools provided by means of using RPs are powerful and simple, we stress that they are very sensible to the values of the parameters and even to the number of samples. This may mislead the adjustment of the parameters of the equalizer. It would be of interest to carry further investigation in different methodologies that are less dependent on the parameters to estimate complexity and to evaluate their applications to the blind equalization problem.

Finally, the problem of FIR equalization of a FIR channel has proven to be, in our preliminary studies, quite challenging. It remains as an open analytical task, which we intend to tackle in the near future.

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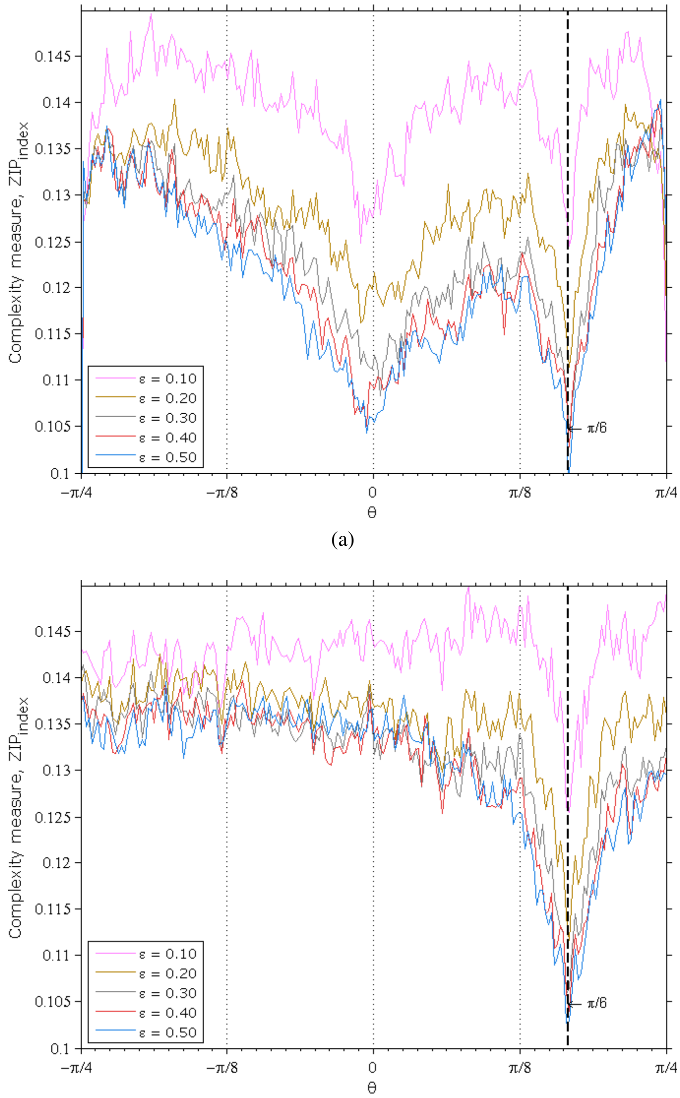

Fig. 4: Complexity measures relative to (a) an IIR equalization of a FIR channel, and to (b) a FIR equalization of a IIR channel. Global minimum is attained at the expected value of $\theta = \pi/6$.

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