Spectrum Sensing over κ-μ Shadowed Fading Channel with Noise Uncertainty

Wanessa de Alvarenga Silva, Kim Moraes Mota, and Ugo Silva Dias

***Abstract*— In this paper, the performance analysis of energy detection of cognitive radio systems operating over κ-μ shadowed fading channel distribution is presented and investigated. The composite multipath/shadowing model provide remarkably accurate statistical characterization due considering both small-scale and large-scale fading in their physical model. In this way, admitting the noise power estimation error and noise uncertainty, the worst case of the miss detection and false-alarm probabilities are derived for cooperative and non-cooperative spectrum sensing systems. Field measurements are used to investigate, in practice, the usefulness of the κ-μ shadowed composite fading, in cooperative spectrum sensing scenarios and an *excellent* fitting is found. Comparisons are performed against other fading model, in which it is possible to notice advantage in using the κ-μ shadowed distribution.**

***Keywords*— Cognitive radio, κ-μ shadowed fading channel, Spectrum sensing, Noise uncertainty.**

I. INTRODUCTION

Wireless services are emerging rapidly and further increasing the demand for spectrum, including wider bands [1]. Although the spectrum is almost all booked for high speed wireless communication, the occupancy rate of its channels are pretty low in most of the time [2]. Thus, there are a lot of bands with low or zero usage that could allow an improvement of the quality and capacity of services in which such frequencies allocated are insufficient.

The opportunistic access in the idle frequencies can be performed by cognitive radio systems. These systems must be reliable and efficient in the spectrum sensing process. At the same time, the device must be capable to adapt its transmission and/or reception parameters to operate under optimum conditions in a manner that will not cause interference to other devices.

The spectrum sensing in fading channels has been studied in [3]–[5], using an energy detection scheme. Probabilities of false alarm, detection and missing were derived for some specific small-scale fading models, such as Rice, Nakagami-m and Rayleigh. However, studies regarding composite fading channels over generalized distributions, which provides remarkably accurate statistical characterization due considering both small-scale fading and large-scale fading (shadowing) in its physical model, are still a shortage in the literature. In addition, important issues like the impact of noise uncertainty in the detection capability and the system performance in regions of low signal-to-noise ratio (SNR) are rarely found.

In [6], a general physical fading model, namely the κ-μ model, is derived. This fading model describes the small-scale variations of the fading signal under a line-of-sight (LOS) condition and is appropriately used to evaluate the channel statistics.

Shadowing effect can be introduced in an LOS multipath fading model in two basic ways. The first way, proposed in [7]–[10], is based on the assumption that the total power, which is associated to both the dominant components and the scattered waves, is subject to random fluctuations. The second way, proposed in [11] with the Rice shadowed model, relies on the assumption that only the dominant components are subject to random fluctuations. The first class of composite multipath/shadowing models is named multiplicative shadow fading models, whereas the second class of models are named LOS shadowed fading models [12]. Due to its generalized nature for fading phenomenons, composite models are largely used to evaluate the channel statistics, as in [13].

In this paper, the κ-μ shadowed fading channel, proposed in [12], is considered in order to investigate the performance analysis of the spectrum sensing activity under realistic and accurate fading environments. The κ-μ shadowed fading provides a general multipath model, which describes both small-scale and large-scale variations of the fading signal for a line-of-sight propagation scenario controlled by two shape parameters, κ and μ. Its features allow great flexibility in the most common types of practical environments [14]. Some classical fading distributions are included in the κ-μ distribution as particular cases, e.g., Rayleigh, Nakagami-m, and Rice. In fact, the fit of the κ-μ distribution, to experimental data, is better than that achieved by the aforementioned classical distributions. Admitting the noise power estimation error and noise uncertainty, the worst case of the miss detection and false-alarm probabilities are derived for cooperative and non-cooperative spectrum sensing systems. Field measurements are used to investigate, in practice, the usefulness of the κ-μ shadowed composite fading in cooperative spectrum sensing scenarios.

The remainder of this paper is organized as follows. In Section II, the generalized κ-μ shadowed fading model is revisited. In Section III, the spectrum sensing over fading channels is presented, specially the detection and false alarm probabilities. Cooperative spectrum sensing is modeled in Section IV. Section V describes the results for spectrum sensing over κ-μ shadowed fading, considering the noise uncertainty in the system, in addition, comparisons against field trials is shown. Finally, Section VI summarises the main conclusions of previous sections.

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II. THE κ-μ SHADOWED FADING MODEL

The κ-μ shadowed distribution is a general fading model that can be used to represent both small-scale and and large-scale variations of the fading signal in a line-of-sight condition [6]. The multipath clusters are assumed to have the scattered waves with identical powers but within each cluster a dominant component is found that presents an arbitrary power. The κ-μ shadowed probability distribution function (PDF) of the signal-to-noise ratio, γ, is obtained in [12] as

$$\begin{aligned} f_\Gamma(\gamma) = \frac{\mu^\mu m_s^{m_s}(1+\kappa)^\mu}{\Gamma(\mu)\,\bar{\gamma}\,(\kappa\,\mu + m_s)^{m_s}} \left(\frac{\gamma}{\bar{\gamma}}\right)^{\mu-1} \exp\left(-\mu(1+\kappa)\frac{\gamma}{\bar{\gamma}}\right) \\ \times {}_1F_1\left(m_s;\mu;\frac{\kappa(1+\kappa)\mu^2}{\kappa\,\mu + m_s}\frac{\gamma}{\bar{\gamma}}\right), \end{aligned} \tag{1}$$

in which $\bar{\gamma}$ is the average signal-to-noise ratio, $\bar{\gamma} = E(\gamma)$, $E(\cdot)$ denotes the expectation. κ and μ represent real non-negative shaping parameters, modeling the fast fading. The parameter κ is defined as the ratio between the total power of the dominant components and the total power of the scattered waves, whereas the parameter μ denotes the number of multipath clusters. The m_s parameter is a real non-negative number that characterizes the shadowing effect. $\Gamma(\cdot)$ is the Gamma function [15], ${}_1F_1(\cdot;\cdot;\cdot)$ denotes the hypergeometric function [15] and $I_\nu(\cdot)$ is the modified Bessel function of the first kind and ν-th order [15].

The κ-μ shadowed distribution includes, as special cases, other important distributions. By setting $\kappa = 0$, in κ-μ shadowed fading model, the Nakagami-m shadowed distribution is obtained, being the Nakagami-m parameter specified as $m = \mu$. The Rice shadowed fading channel can be obtained from the κ-μ shadowed distribution by setting $\mu = 1$, with κ representing the Rice parameter k. Furthermore, as $m_s \to \infty$, Equation (1) approaches the well-know κ-μ distribution [6], once increasing the value of the parameter m_s the shadowing effect is reduced.

III. SPECTRUM SENSING OVER FADING CHANNELS

Cognitive radios perform spectrum sensing to identify the available spectrum bands and the spectrum decision process selects one from these available bands for opportunistic use. The decision process with two hypotheses is given as

$$\begin{aligned} H_0 &: y(t) = n(t) \\ H_1 &: y(t) = h\,x(t) + n(t), \end{aligned} \tag{2}$$

in which H_0 is the idle channel hypothesis, H_1 is the occupied channel hypothesis, $y(t)$ is the signal detected by the secondary user also denoted as the random variable Y and $x(t)$ is the signal transmitted by the primary user. The channel gain is represented by h and additive white gaussian noise (AWGN) by $n(t)$.

The detection probability is defined as the probability to detect a busy channel when the channel is indeed busy. In fading environments, h varies and the detection probability using energy detectors is obtained averaging the conditional probability that the signal Y is above some threshold, assuming that the channel is indeed occupied, over the PDF of SNR, $f_\Gamma(\cdot)$, as [4]

$$P_d = P\{Y > \lambda \mid H_1\} = \int_x Q_u(\sqrt{2\gamma}, \sqrt{\lambda}) f_\Gamma(x)\,\mathrm{d}x, \tag{3}$$

in which λ is a comparison threshold and $Y > \lambda$ indicates the presence of the primary user signal, u is the time-bandwidth product and $Q_u(\cdot,\cdot)$ is the generalized Marcum Q-function rewritten as [16]

$$Q_u(a,b) = \exp\left(-\frac{a^2}{2}\right)\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{a^2}{2}\right)^k \frac{\Gamma\left(k+u,\frac{b^2}{2}\right)}{\Gamma(k+u)}, \tag{4}$$

in which $\Gamma(\cdot,\cdot)$ is the incomplete Gamma function [15].

Missed detections occur when a busy channel is detected as idle and false alarms occur if an idle channel is detected as busy. The missing probability, P_m, is then defined based on equation (3) as $P_m = 1 - P_d$ and the false alarm probability, P_f, is given by

$$P_f = P\{Y > \lambda \mid H_0\} = \frac{\Gamma(u,\frac{\lambda}{2})}{\Gamma(u)}, \tag{5}$$

In presence of noise uncertainty, it is not possible to detect the primary user when sensing SNR is lower than some threshold. Assuming the presence of β dB of uncertainty in the noise power estimation, the estimated noise power is in the range $(\delta_S^2/\alpha, \alpha\delta_S^2)$, in which $\alpha = 10^{\frac{\beta}{10}}$ and δ_S^2 is the average noise power. When the noise power is overestimated as $\bar{\delta}_S^2 = \alpha\delta_S^2$, the false alarm probability can be obtained as [17]

$$\begin{aligned} P_f &= P\left\{\frac{1}{\bar{\delta}_S^2}\sum_{i=1}^{N} x^2(i) > \bar{\lambda} \mid H_0\right\} \\ &= P\left\{\frac{z}{\delta_S^2} > \alpha\bar{\lambda} \mid H_0\right\} = P_f(\alpha\bar{\lambda}), \end{aligned} \tag{6}$$

in which $z = \sum\limits_{i=1}^{N} x^2(i)$ is the received signal power considering N signal samples. Considering that the estimated noise power may take any value within $(\delta_S^2/\alpha, \alpha\delta_S^2)$, to guarantee the spectrum utilization constraints, the proper threshold should be set as $\bar{\lambda} = \alpha\lambda$. Therefore, the worst case of P_d occurs when $\bar{\delta}_S^2 = \alpha\delta_S^2$ and

$$\begin{aligned} P_{d,worst} &= P\left\{\frac{z}{\bar{\delta}_S^2} > \bar{\lambda} \mid H_1\right\} \\ &= \int_x Q_u(\sqrt{2\gamma}, \alpha\sqrt{\lambda}) f_\Gamma(x)\,\mathrm{d}x. \end{aligned} \tag{7}$$

Using (1) in (7) and after some algebraic manipulations [18], the detection probability can be obtained in exact-form as

$$\begin{aligned}P_{d,worst} = {} & \frac{\mu^{\mu} m_s^{m_s} (1+\kappa)^{\mu}}{\Gamma(\mu)\,\bar{\gamma}^{\mu}\,(\kappa\,\mu+m_s)^{m_s}} \\ & \times \sum_{l=0}^{\infty} \frac{\Gamma(l+u,(\alpha^2\,\lambda/2))}{\Gamma(l+1)\,\Gamma(l+u)} \left(\frac{\kappa(1+\kappa)\,\mu^2}{\bar{\gamma}(\kappa\,\mu+m)}\right)^{-(l+\mu)} \\ \times\, & H_{2,2}^{2,1}\left[-\frac{(\bar{\gamma}+\mu+\kappa\,\mu)(\kappa\,\mu+m)}{\kappa(1+\kappa)\,\mu^2}\middle|\begin{matrix}(1-l-\mu,1),(-l,1)\\(0,1),(m-l-\mu,1)\end{matrix}\right],\end{aligned} \tag{8}$$

in which $H_{p,q}^{m,n}$ is the Fox's H-function [19].

Using (1) in (6) and using (8) is possible to evaluate the sensing spectrum conditions under κ-μ shadowed fading channel.

IV. COOPERATIVE SPECTRUM SENSING

Cooperative spectrum sensing has been shown as an effective method to improve the detection performance by exploiting spatial diversity [20]. More specifically, cooperative sensing aims to enhance the sensing performance by implementing the spatial diversity in the observations of spatially located cognitive radios users. Thus, users can share their sensing information for making a combined decision more accurate than the individual decisions at the expense of cooperation overhead.

In this context, we consider now a secondary network with n collaborating users, sensing all the desired frequency band in a periodic regime. For simplicity we assume that all n users experience independent and identically distributed (iid) fading with same average SNR. We assume that all users employ energy-detection and use the same decision rule (i.e. same threshold λ).

A secondary user receives decisions from $n-1$ others terminals and decides H_1 if any of the total n individual decisions is H_1. This fusion rule is known as the OR-rule or 1-out-of-n rule [21]. Thus, the detection and the false alarm probabilities for the collaborative scheme, denoted by Q_d and Q_f, respectively, may be written as

$$Q_d = 1-(1-P_d)^n, \tag{9}$$

$$Q_f = 1-(1-P_f)^n. \tag{10}$$

Despite the cooperative spectrum sensing is obviously more complicated than a single non-cooperative system in cognitive radio, the cooperative spectrum scheme has many advantages that outweigh the added complexity. Cooperative spectrum sensing promotes a considerable improvements in system performance, namely, hidden node problem reduction, increased agility system, reduction of false alarms probability and more accurate signal detection [22].

V. RESULTS AND DISCUSSIONS

Figure 1 presents the receiver operating characteristic (ROC) under the κ-μ shadowed composite fading scenarios. The u and $\bar{\gamma}$ values are assumed to be 5 and 10 dB, respectively, and the m_s parameter value is 3. It can been seen that, as already expected, increasing the predominance of the multipath clustering, by increasing the μ value, the detection probability also increases for a given false alarm probability. It is possible to see that this parameter has a very significant impact on the analysis, since in a scenario with a low parameter value has a low probability of detection and a higher missing probability. Moreover, the Figure 1 illustrates the effect of the dominant component, described by κ fading parameter, on detection characteristics. Note that, when the κ parameter increases, i.e. when the dominant components prevail, the probability of detection also increases, leading to more favorable scenarios.

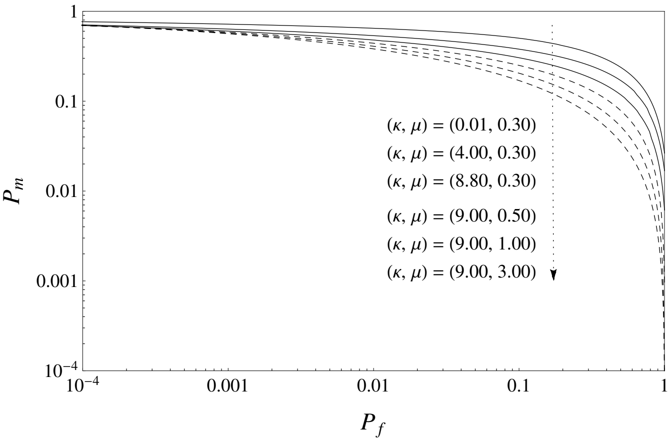

Fig. 1. ROC under κ-μ shadowed distribution ($m_s = 3$, $u = 5$ and $\bar{\gamma} = 10\,dB$).

In Figure 2, it can be clearly observed that reducing the shadowing effect — increasing the m_s parameter value — the missing probability decreases, which characterises a most favorable scenario of the wireless communication link. It is observed that κ-μ shadowed fading channel is able to report a large number of detection characteristics, allowing that the wireless receiver operates with a most suitable detection probability in different environments.

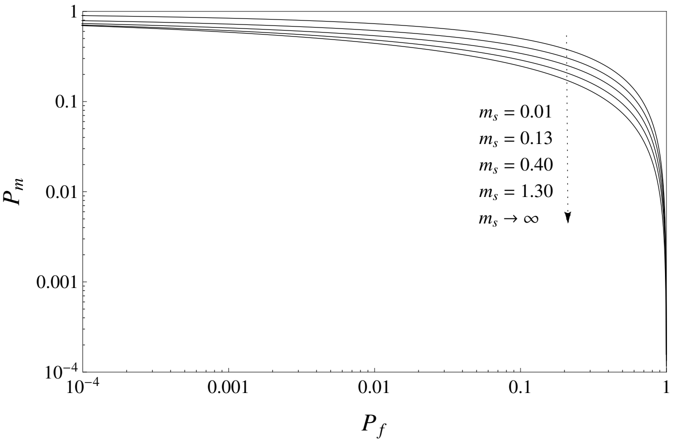

Fig. 2. ROC under κ-μ shadowed distribution ($\kappa = 1.5$, $\mu = 1$, $u = 5$ and $\bar{\gamma} = 10\,dB$).

Figure 3 shows the detection characteristics for different number of collaborating users under a typical case of κ-μ fading channel in line-of-sight scenario, in which, $\kappa = 1.5$ and $\mu = 1$. As before, $u = 5$ and $\bar{\gamma} = 10$. The cooperative

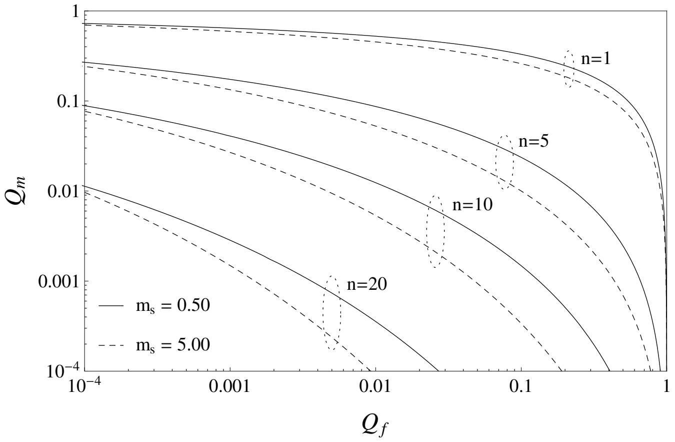

Fig. 3. Q_m vs Q_f under κ-μ shadowed distribution ($\kappa = 1.5$, $\mu = 1$, $u = 5$ and $\bar{\gamma} = 10\,dB$).

spectrum sensing can improve the detection characteristics, as expected, reducing the overall missing probability. Even if a few number of terminals are cooperating in the network, there is a relevant gain in the detection probability. A high number of collaborative terminals can be easily found in practice, for example, in a wireless sensor network.

Figures 4 and 5 show the detection probability versus average signal-to-noise ratio for different cooperative schemes under κ-μ shadowed fading model. The extremely shadowed scenario ($m_s = 0.5$), in Figure 4, and slightly shadowed scenario ($m_s = 5$), in Figure 5, are investigated. For each curve, decision threshold, λ, is chosen such that $Q_f = 10^{-1}$ and the time-bandwidth product, u, is set to 5. The uncertainty in the noise power is considered in both figures and is possible to observe that this condition promotes the decreasing of the detection probability as expected.

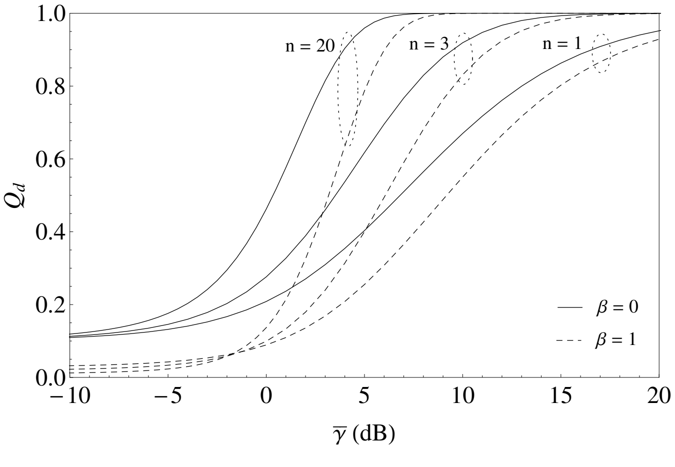

Fig. 4. Q_d vs $\bar{\gamma}$ under κ-μ shadowed fading with noise uncertainty for different number of collaborative spectrum sensors ($\kappa = 1.5$, $\mu = 1$, $m_s = 0.5$, $Q_f = 10^{-1}$, $u = 5$ and $\beta = 1$).

Comparing the Figure 4 and the Figure 5, for a same number of cooperative users, the necessary SNR required to obtain some desired detection probability is pretty lower in slightly shadowed scenario than in a extremely shadowed scenario, as expected. The shadowing effect is suppressed with the increase of collaborative terminals, confirming the great importance of cooperation among users to perform a reliable spectrum sensing. Cooperative spectrum sensing provides more accurate signal detection and a greater reliability of the overall system.

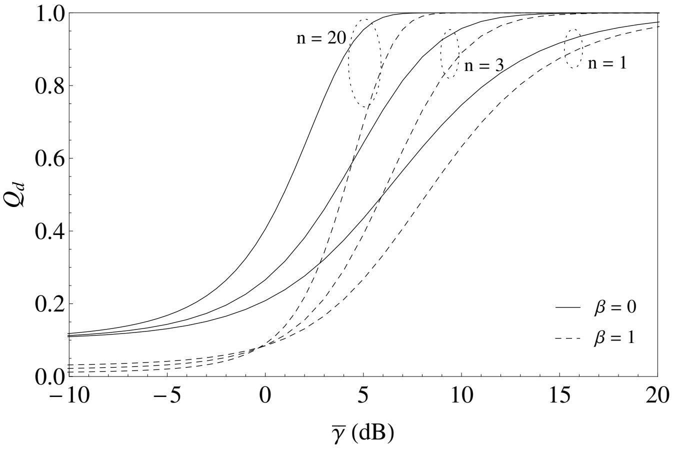

Fig. 5. Q_d vs $\bar{\gamma}$ under κ-μ shadowed fading with noise uncertainty for different number of collaborative spectrum sensors ($\kappa = 1.5$, $\mu = 1$, $m_s = 5$, $Q_f = 10^{-1}$, $u = 5$ and $\beta = 1$).

A. Comparisons Against Field Measurements

Figure 6 depicts a performance evaluation of cooperative spectrum sensing, based on field trials of primary signals of Digital Celular System (DCS 1800) [21]. The measurement platform was placed on a building roof in urban Barcelona with several transmitters located a few tens or hundreds of meters away from the antenna and with buildings reflecting and diffracting the radio signals. In order to investigate the κ-μ shadowed usefulness in spectrum sensing techniques, the fading parameters κ, μ and m_s were set to provide the best fit to the practical curve of primary signals in DCS 1800 and to prove the κ-μ shadowed suitability in fading scenarios modeling.

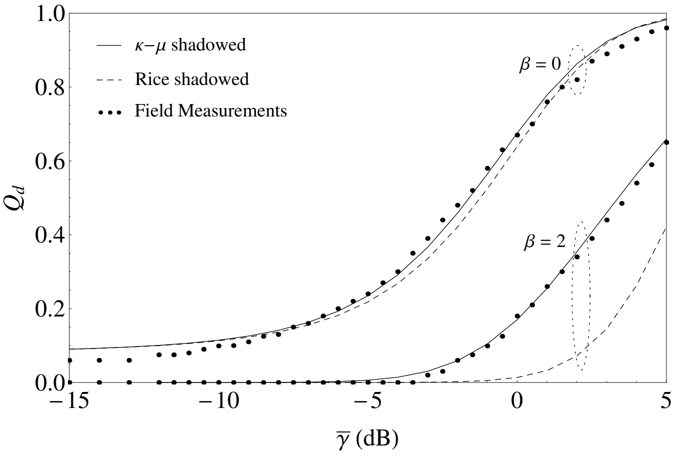

Fig. 6. Q_d vs $\bar{\gamma}$ under κ-μ shadowed and Rice Shadowed fading with noise uncertainty and field measurements in Barcelona ($m_s = 0.5$, $n = 18$ and $u = 2$).

It can be observed that the κ-μ shadowed fading model fits better than Rice shadowed fading channel, once the κ-μ

shadowed distribution provides a general multipath/shadowed model for a line-of-sight propagation scenario, controlled by three shape parameters, κ, μ and m_s. Generalized distribution tends to model better the analyzed telecommunication channels.

VI. CONCLUSIONS

This paper presented the performance analysis of energy detection for an unknown transmitted signal over generalized fading channels, modeled by the κ-μ shadowed distribution. Important statistics of the κ-μ shadowed fading model have been exploited in this paper. The receiver operation characteristics considering the influence of noise uncertainty and the influence of both combining multipath clustering and shadowed scenarios were obtained. The results were compared against field measurements to investigate the κ-μ shadowed fading usefulness. Finally, the statistical results, here derived, can be applied to the modeling and analysis of several wireless communication systems, in particular, the spectrum sensing metrics for cognitive radio systems. All results presented and analyzed are timely for emerging applications involving currently and future telecommunication systems.

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