Parameter Optimization in ACO-MuD DS/CDMA

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***Abstract*— A simple and efficient methodology for tuning the input parameters applied to the ant colony optimization multiuser detection (ACO-MUD) in direct sequence code division multiple access (DS-CDMA) is proposed. The motivation in using a heuristic approach is due to the nature of the NP complexity posed by the wireless multiuser detection optimization problem. The challenge is to obtain suitable data detection performance in solving the associated hard complexity problem in a polynomial time. Previous results indicated that the application of heuristic search algorithms in several wireless optimization problems have been achieved excellent performance-complexity tradeoffs. Regarding different system operation and channels scenarios, a complete input parameters optimization procedure for the ACO-MUD is provided herein, which represents the major contribution of this work. The performance of the ACO-MUD is analyzed via Monte-Carlo simulations. Simulation results show that, after convergence, the performance reached by the ACO-MUD is much better than the conventional detector (CD), and somewhat close to the optimum likelihood detector (OMuD). Flat Rayleigh channels is initially considered, but the input parameter optimization methodology is straightforward applicable to selective fading channels scenarios, as well as to joint time-spatial wireless channels diversities.**

***Keywords*— Ant colony intelligence; multiuser detection; input parameters optimization; computational complexity; DS-CDMA.**

I. Introduction

In the direct sequence/code division multiple access (DS/CDMA) technology, all the users share the entire frequency band available at the same time. This is possible due the spreading sequence with short chip period, which is used in order to spread the user information along all the available bandwidth spectrum, as well as serves as an identification code for each user, providing some level of multiple access interference (MAI) immunity. The application of sequences with low cross correlation allows to support a considerably number of users simultaneously, as well as the possibility of operation in the asynchronous configuration mode, meting the requirements of wireless mobile communication uplink. However, as the system loading[1] increases, the utilization of sophisticated detectors become necessary, such as multi user detection (MuD) [1], in such a way to obtain a reasonable separation among the several user' signals, each one under an intense multiple access interference level generated by $K-1$ interfering users. The best performance is achieved by the optimum multiuser Detector (OMuD) or maximum likelihood (ML) detector, which complexity grows exponentially with the number of users, $\mathcal{O}(2^K)$ [1].

In the last decade, proposals based on heuristic methods have been reported to solve the MuD problem, getting performance close to the ML performance with polynomial computational complexity [2], [3]. The use of heuristic search algorithms is motivated by the fact that optimization problems related to wireless communication systems results in non-polynomial (NP-hard) problems, e.g, MuD optimization problem [4]. So, from a practical point-of-view, the challenge is to obtain satisfactory results for these high computational complexity problems in a polynomial time. In the multiuser detection context, the heuristic based algorithms (Heur-MuD) most commonly used includes the evolutionary programming (EP) based algorithms, specially the genetic algorithms (GA) [2], [5], particle swarm optimization (PSO) [6], [7], ant colony optimization (ACO) [8] and the local search method (LS) [9], [10]. Furthermore, the input parameters optimization of the heuristic-based algorithms is of paramount importance in order to obtain reliable results. Specifically on MuD optimization problem, in [11] a detailed study about the input parameters of the particle swarm heuristic algorithm applied to DS/CDMA multiuser detection problem has been conducted. Hence, present work aims to develop an input parameters analysis for the ant colony optimization (ACO) heuristic-based algorithm applied to DS/CDMA multiuser detection problem.

The first algorithm using the ACO heuristic approximation was proposed in 1991 by Colorni [12], and since that many variant algorithms were described in the literature. Recently, this ant behavior-based technique has been widely applied to multiple access multiuser detection [8], [13]–[15].

The computational complexity of DS/CDMA ACO multiuser detection was analyzed in [13], noting that with a few iterations the ACO-MuD algorithm was able to reach the near-optimal performance spending only a small fraction ($\approx 5\%$) of computational effort necessary to perform an exhaustive search. Furthermore, [8] analyzes the ACO-MuD applied to multi carrier DS/CDMA systems (MC-DS/CDMA). ACO-MuD in this context is able to reach the optimal performance, regardless the adopted number of carriers. An heuristic ACO-based multiuser detector for space-time block coding (STBC) systems with receiver diversity was proposed in [16]. Numerical results have indicated a very close performance to the optimal one; also, the STBC ACO-MuD does not present the bit error rate saturation (BER-floor), a degradation performance effect that occurring at high SNR region.

II. System Model

In a DS/CDMA system deploying BPSK modulation under non-line-of-sight (NLOS) fading channels the time continuous baseband signal at the receiver side can be described as:

$$r(t) = \sum_{k=1}^{K} A_k b_k s_k(t-\tau_k) h_k(t) + \chi(t) \qquad (1)$$

where K is the number of active users in the system; $t \in [0, T_b]$ and T_b is the bit period; A_k is the transmitted signal amplitude

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[1]The number of users by the processing gain ratio, $L = K/N$.

of the kth user, given by $A_k = \sqrt{\frac{E_k}{T_b}}$, where E_k is the bit energy and P_k the power of the signal received by the kth user; s_k is the spreading sequence assigned to the $k-$th user; $b_k \in \{\pm 1\}$ is the $k-$th user's transmitted bit information, assumed independent and equiprobable distributed; $h_k(t)$ is the complex channel coefficients for the kth user, and $\chi(t)$ is the time continuous additive white Gaussian noise (AWGN), representing the thermal noise and other uncorrelated noise sources, with bilateral power density $N_0/2$.

Multiplying the received signal by the spreading sequence of the interest user (matched filter to this sequence), the conventional detector (CD) provides the information de-spreading. In this way and using matrix notation, the output of the matched filter bank (MFB) is $\mathbf{y} = \mathbf{RCAb} + \boldsymbol{\chi}$, where $\mathbf{y}$ is the $K \times 1$ output vector, $\mathbf{R}$ is the $K \times K$ correlation matrix, $\mathbf{C} = \mathrm{diag}(c_1, c_2, ..., c_k)$ the $K \times K$ channel coefficients diagonal matrix, $\mathbf{A}$ is the diagonal matrix of received amplitudes, and $\mathbf{b}$ is the $K \times 1$ vector containing one information bit for each user. $\boldsymbol{\chi}$ is the sampled AWGN $K \times 1$ vector with bilateral power spectral density $N_0/2$. At the MFB output follows the hard decisor which takes decision according to the signal polarity: $\mathbf{b}_{\rm cd} = \mathrm{sgn}(\mathbf{y})$, where the modified signum function $\mathrm{sgn}(.)$, which returns the polarity of its input.

The conventional detector for DS/CDMA uplink receiver (MFB) considers the MAI as a additional background noise, being not able to separate multiple access interference (MAI) from the interest signal. On the other hand, the multiuser detectors (MuD) takes advantage of MAI as a way to takes its performance closer to the optimal. In [1], it was shown that the optimal multiuser detector (OMuD), or maximum likelihood (ML) detector, calculates the cost function of all the possible candidate-solutions, and return as the optimal solution the argument of the higher value found. The cost function can be expressed as:

$$f(\boldsymbol{\vartheta}) = \Re\{2\mathbf{y}^T\mathbf{C}^H\mathbf{A}\boldsymbol{\vartheta} - \boldsymbol{\vartheta}^T\mathbf{CARAC}^H\boldsymbol{\vartheta}\} \qquad (2)$$

where $\Re(.)$ is the real operator and $\boldsymbol{\vartheta}$ the $K \times 1$ information bits candidate vector. Consequently, the estimated transmitted bit vector for the K users is defined as:

$$\widehat{\mathbf{b}}_{\rm opt} \;\; = \;\; \arg \max_{\boldsymbol{\vartheta} \in \{\pm 1\}^K} \; f(\boldsymbol{\vartheta}) \qquad (3)$$

Since the optimal detector (ML) calculates the cost function for all the possible solutions, it is immediate that its performance grows exponentially with the users number K, because the number of possible combinations is given by 2^K.

III. Heuristic ACO-MuD

The ant colony optimization is based on the foraging behavior of the ant colony in nature. In search of food, the ants of a colony are scattered randomly in their neighborhood. When an ant is successful in it search for food, it come back nest and leaves pheromones in the way. This pheromone will induce the other ants to take this same way in the search for food, further strengthening the pheromone trail. If the food at the end of a certain way runs out and the ants stops taking it, this pheromone will be evaporated.

For BPSK signaling, uplink receiver side and just one antenna at the base-station (BS) receiver and each of K users' transmitter, i.e., from the interest user receiver viewpoint at BS, we have a single-input-single output (SISO) communication system, with $K-1$ interfering users. So, the multiuser detection problem at the BS receiver side is constituted by 2^K possible candidate solutions in (3). This solutions are seen by the algorithm as all the possibles vector-candidates (or trails) that the ants can travel. The quality of each trail is evaluated by the cost function, defined in eq. (2). The algorithm steps aiming to find a solution that maximizes (2), or analogously, find the fastest trail for the ants until the food.

The MFB outputs serve as initial information to the ants. So, the log-likelihood function (LLF) for the kth user is $\boldsymbol{\mathcal{L}}_{\rm k}(\pm 1) = 2\Re\{\pm \boldsymbol{\mathcal{A}}(k)\mathbf{y}(k)\} - \boldsymbol{\mathcal{A}}(k)^2\mathbf{R}(k,k)$, where $\boldsymbol{\mathcal{A}}(k) = \mathbf{A}(k,k)\mathbf{C}(k,k)$ is the kth signal received amplitude, including the channel effects (fading, path loss and shadowing). The desirability function is defined using the LLF function: $\boldsymbol{\mathcal{D}}_{\rm k}(\pm 1) = 1 + e^{-\boldsymbol{\mathcal{L}}_k(\pm 1)}$. From the desirability function, the intrinsic affinity function is defined, which influences the trail decision of each ant along the algorithm iterations:

$$\boldsymbol{\eta}_{\rm k}(\pm 1) = \left[\boldsymbol{\mathcal{D}}_k(+1) + \boldsymbol{\mathcal{D}}_k(-1)\right] / \boldsymbol{\mathcal{D}}_k(\pm 1) \qquad (4)$$

The signals at the matched filter bank output are assumed as initial information. It is then necessary to take into consideration that the decision taken by the ants be influenced by the paths taken previously, which resulted in better results. This way, the solution found by the algorithm will evolve along the iterations. So, in order to quantify this evolution, the $2 \times K$ pheromone table $\boldsymbol{\mathcal{P}}$ is created, in which the first row refers to the probability of positive bits, and the second row refers to the probability of the negative bits. Its elements are initialized with probability λ. Along the iterations, this table is being filled according to the quality of the paths taken by each ant and a tendency, measured in terms of increasing probability of that specific bit be 1 (positive bit) or 0 (negative bit).

The first step of the table updating takes into account the paths chosen by each ant in that iteration, and how successful these chooses were (measured by the cost function evaluation). A pheromone amount which is equivalent to the cost function value regarding the path taken by the ant is multiplied by the γ coefficient, and incrementally accumulated at the respective positions in the $\boldsymbol{\mathcal{P}}$ matrix:

$$\boldsymbol{\mathcal{P}}_{i+1} = \boldsymbol{\mathcal{P}}_i + \gamma \cdot f(trail(m)) \cdot \boldsymbol{\mathcal{T}}(trail(m)) \qquad (5)$$

where $trail(m)$ is the path taken by the mth ant in a given iteration and $\boldsymbol{\mathcal{T}}(trail(m))$ is a $2 \times K$ filled with 1 in the positions related to the path taken by the ant and 0 in the others.

The second step of the table updating takes into account the best path found by the ACO-MuD algorithm until that moment, named herein $\boldsymbol{\vartheta}_{\rm best}$. Similar to the adopted procedure in the first update stage, now, a pheromone amount which is equivalent to the cost function of $\boldsymbol{\vartheta}_{\rm best}$ is multiplied by a coefficient σ and deposited at the respective positions of $\boldsymbol{\mathcal{P}}$:

$$\boldsymbol{\mathcal{P}}_{i+1} = \boldsymbol{\mathcal{P}}_i + \sigma \cdot f(\boldsymbol{\vartheta}_{\rm best}) \cdot \boldsymbol{\mathcal{T}}(\boldsymbol{\vartheta}_{\rm best}) \qquad (6)$$

Aiming to scape from possibles local optima (maxima), at each new iteration i, the pheromone table is multiplied by a coefficient $(1-\varepsilon)$, being ε the pheromone evaporation rate:

$$\boldsymbol{\mathcal{P}}_{i+1} = (1-\varepsilon)\cdot\boldsymbol{\mathcal{P}}_i \tag{7}$$

Hence, an excessive amount of pheromone is avoided to be accumulated over any possible trail.

Once factors, which influence the path choice of the ants along the iterations, have been defined, it is possible to define the bit choice probability:

$$\mathtt{P}_k(\pm 1) = \frac{[\boldsymbol{\mathcal{P}}_k(\pm 1)]^\alpha\,[\boldsymbol{\eta}_k(\pm 1)]^\beta}{[\boldsymbol{\mathcal{P}}_k(+1)]^\alpha\,[\boldsymbol{\eta}_k(+1)]^\beta + [\boldsymbol{\mathcal{P}}_k(-1)]^\alpha\,[\boldsymbol{\eta}_k(-1)]^\beta} \tag{8}$$

where α and β parameters provide more or less importance (weighting factors) to the pheromone amount and the initial information, respectively. Note that α is related to the algorithm convergence speed, while β is related to the reliability that can be assigned to the MFB output, which must be set a low value in hostile conditions of channel and/or system loading ($L > 0.5$).

At each iteration, the choice of a certain bit related to each ant trail will be taken from the probability defined in (8). If some trail is more successfully than $\boldsymbol{\vartheta}_{\rm best}$, the best-candidate solution is updated. After the algorithm performs a specified number of iterations $N_{\rm iter}$, the solution found by the algorithm is returned by the vector $\boldsymbol{\vartheta}_{\rm best}$.

IV. ACO-MuD Input Parameters Optimization

Essentially, there are four input parameters in the ACO-MuD algorithm, α, β, γ e σ; the values assigned to these parameters can dramatically affect algorithm's performance.

The parameter α is related to the weight given to the information registered in the pheromone table during the probability calculation. As α grows, more and more ants choose to take the better path identified in the table (higher probability value). Thus, the algorithm's convergence speed is improved, because the ants tend to choose the same way quickly. This affects the convergence time and, as a consequence, the algorithm's complexity.

The parameter β is related to the *a priori* information during the probability calculation. β increasing implies in more ants following the initial solution trend, i.e., choosing the solution given by the MFB outputs $\mathbf{y}$, in the MuD context. However, if the initial information is not reliable, i.e., in multiuser scenarios which SNR is low and/or system loading is high ($L \geq 0.7$), high values of β could induce the ants to choose a mistaken path, increasing the system's BER.

On other hand, the parameters γ and σ are related to the pheromone accumulation according to the quality of paths taken by each ant and the best path found so far $\boldsymbol{\vartheta}_{\rm best}$, respectively. According the study about the ACO algorithm parameters applied to the traveling salesman problem done in [17], the ACO algorithm performance is not affected by the values assigned to γ and σ parameters. Indeed, our simulation results described in the next subsections, related to these two input parameters optimization applied to the MuD problem, offer support to the conclusions found in [17].

Next, it is carried out a complete analysis optimization on the four input parameters of ACO algorithm, specifically applied to the MuD problem considering the reverse link of DS/CDMA systems under flat frequency fading channels and different mobility conditions for the mobile terminals. Monte-Carlo simulation method is deployed in order to determine the optimum values of the ACO-MuD input parameters. A $20dB$ SNR, Gold spreading sequences with length (processing gain) 31 and system loading $L_{\%} = 100\cdot\frac{K}{N} = 100\%$ have been adopted. For the others ACO-MuD parameters, the following values have been assumed: initial pheromone probability, $\lambda = 0.01$; population = 30 ants, ε = 0.5, and $N_{\rm iter}$ = 20 iterations.

The optimization is made starting from presetting initial values for the four main parameters, for instance, $\alpha = 1$, $\beta = 1$, $\sigma = 8$ e $\gamma = 1$. Keeping three parameters fixed and ranging the fourth, a first set of curves for the ACO-MuD input parameter optimization could be obtained. Then, the four optimized parameters at this first step of optimization are updated. Hence, a second set of curves for the optimized input parameters could be obtained, now in a narrower values range, being the optimized values of the first step the middle of the values range. The values obtained at this second optimization step are then assumed as optima for the ACO-MuD algorithm at that channel condition and system operation point. The ACO-MuD performance is compared with the single-user bound (SuB), i.e., when only one user is active in the system.

A. ACO-MuD Performance – Low Speed Vehicular Channels

Fig. 1.a shows the first performance analysis ranging the parameters according the methodology described above. For the parameters α and β, it could be observed an optimum value trend, given by: $\alpha = 0.6$, $\beta = 6$. For the parameters σ and γ, one can see that there were not performance degradation throughout their respective range values. Hence, intermediate values have been assumed given by $\sigma = 5$ and $\gamma = 3$. Then, this set values has been deployed as the basis for the second optimization step for the parameters α and β. Results in Fig. 1.b is taken considering a narrower range centered on the respective optimum value obtained from the first optimization step.

Finally, the optimum input parameter values for the ACO-MuD operating on non-selective fading channels with mobile units moving with uniformly distributed speeds in the range $v \sim \mathcal{U}[0, 60]$ km/h and system loading of $L_{\%} = 100\%$ were obtained, as shown at the first line of Table I ($V_{\max} = 60$ km/h).

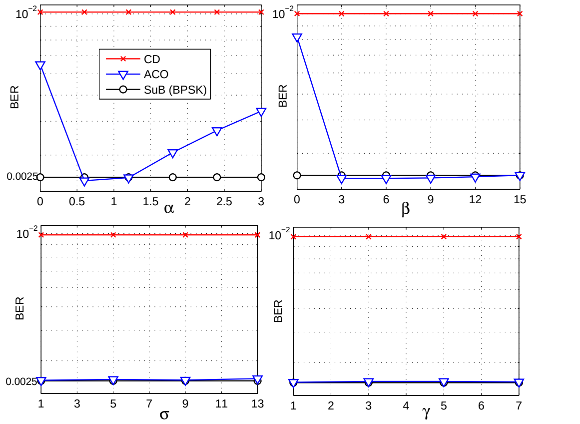
TABLE I

Optimized values for the ACO-MuD input parameters considering different system's mobility conditions.

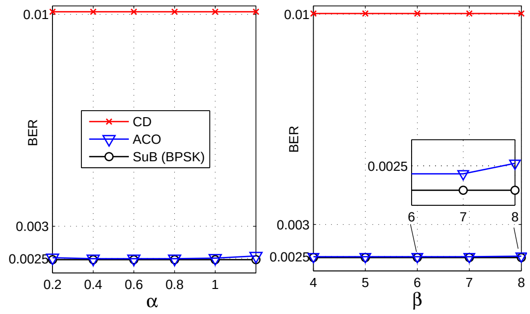
Parameters	α	β	σ	γ
$V_{\max} = 60\ km/h$	0.6	7	5	3
$V_{\max} = 120\ km/h$	0.4	7	5	3
$V_{\max} = 240\ km/h$	0.6	4	5	3

B. ACO-MuD Performance – High and Ultra-High Speed Vehicular Channels

Under higher mobility, and at same system loading $L = 1$, one can see from Fig. 2 the similar results regarding the optimal input parameters values obtained by the two-step optimization procedure for the low-speed vehicular channels, as described in the section IV-A.



a) First step input parameter optimization: $\alpha_{\rm fx} = 1$; $\beta_{\rm fx} = 1$; $\sigma_{\rm fx} = 8$; $\gamma_{\rm fx} = 1$



b) Second step optimization: $\alpha_{\rm fx} = 0,6$; $\beta_{\rm fx} = 6$; $\sigma_{\rm fx} = 5$; $\gamma_{\rm fx} = 3$

Fig. 1. Input parameters optimization: $V_{\max} = 60$km/h; SNR $= 20$ dB

According to the methodology described above, under high mobility the optimal values for the α e β parameters in the first optimization step, Fig. 2, were changed to $\alpha = 0.6$ and $\beta = 7.5$. Similarly to the low mobility case, for σ and γ parameters it was assumed $\sigma = 5$ and $\gamma = 3$. This set was used in the second optimization stage of the parameters, considering a narrower ranges centered in each optimum value found in the first step.

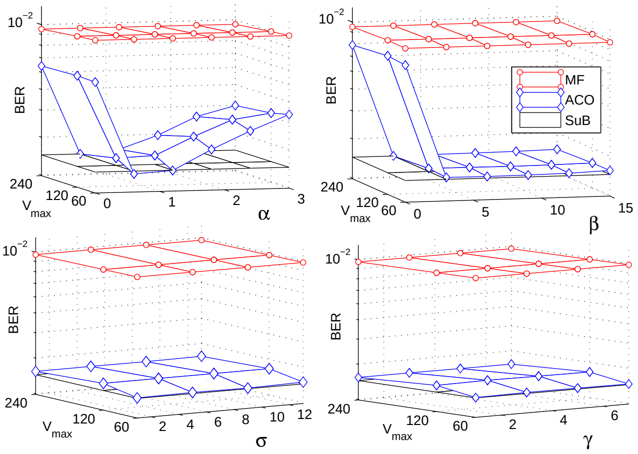


Fig. 2. ACO-MuD input parameters optimization. SNR $= 20$ dB.

Analyzing the optimized values on different mobility situations, one can conclude from Fig. 2 that there were not significant differences for distinct channel mobility conditions, from low to ultra-high speed vehicular channels. The parameters α and β, which are related to the convergence speed and the *a priori* information reliability, respectively, are more influential to the algorithm's performance (convergence), while the parameters σ and γ proved to be less sensible, corroborating the analysis carried out in [17] for a general purpose discrete ACO algorithm.

V. Numeric Results for MuD problem with Optimized ACO Input Parameters

In order to demonstrate the ACO-MuD algorithm robustness and efficiency, in this section the performance of the heuristic detector with and without optimized input parameters is compared, regarding the number of iterations (convergence speed) and signal-to-noise ratio (SNR). Fig. 3 shows the converge velocity of the ACO-MuD under SNR $= 20dB$, $K = 31$ users, $M = 30$ ants, $\varepsilon = 0.5$, $V_{\max} = 120$ km/h and with and without optimized input parameters. One can see that with optimized input parameters values found in Section IV, the optimized ACO-MuD with parameters $= 0.4$, $\beta = 4$, $\sigma = 5$ and $\gamma = 3$ achieves the BER performance bound (maximum likelihood optimum detector, i.e., performance very close to SuB) after five iterations. Thus, for the optimized ACO-MuD, $N_{\rm iter} = 5$ iterations are enough for the complete convergence under non-selective Rayleigh SISO DS/CDMA channels and full system loading ($L = 100\%$) condition.

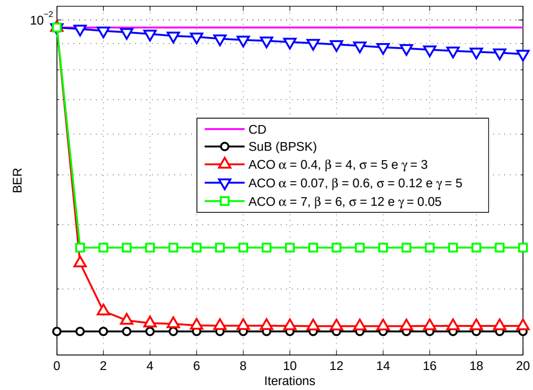


Fig. 3. ACO-MuD Convergence performance under SNR = 20 dB, flat Rayleigh channel, $L = 100\%$ and $V_{\max} = 120$ km/h.

In order to confirm the convergence performances related to ACO-MuD input parameters values, as found in Fig. 3, Fig. 4 presents the ACO-MuD BER performance for a wide range of SNR$\in$ $[0; 25]$. As one can immediately conclude, the best ACO-MuD performance (curve with marker $-\Delta-$) is achievable under optimized input parameters; besides, under with the optimized input parameters values, the ACO-MuD is able to achieve the OMuD performance for all SNR values ranging $[0; 25]$ dB.

Note that, for this scenario, while the OMuD needs 2^{31} cost function calculations (cfc), resulting in over 2 billions of cost function calculations, on the other hand the ACO-MuD with optimized parameters evaluates a number of cfc given by the product of the ants population M and the algorithm iterations $N_{\rm iter}$. With the values assumed in the simulations, this complexity is of the order of $\mathcal{C} = M \cdot N_{\rm iter} = 150$ [cfc]. Hence, the optimized ACO-MuD is able to find solutions very close to those obtained by the OMuD, but with only a fraction of the cfc, i.e. $\approx 1.4 \cdot 10^7$ times lower than the number needed by OMuD.

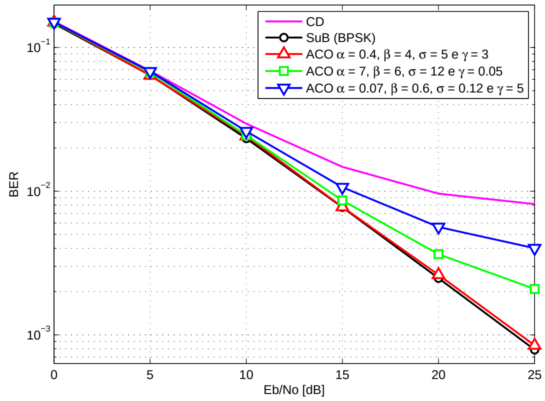


Fig. 4. BER performance for ACO-MuD under Flat Rayleigh channels, $L = 100\%$ loading, $V_{\max} = 120$ km/h, considering different values for the input parameters.

Deploying the same values for the input parameters used in Fig. 3, the excellent achievable ACO-MuD BER performance in terms of convergence speed is put into perspective in Fig. 5, considering a wide range of system SNR operation and full system loading $L_{\%} = 100\%$. The number of iterations to achieve total convergence increases with SNR values; e.g. for SNR=10 dB, $N_{\rm iter} = 2$, while for SNR=20 dB, $N_{\rm iter} = 5$ and for SNR=30 dB, $N_{\rm iter} = 9$.

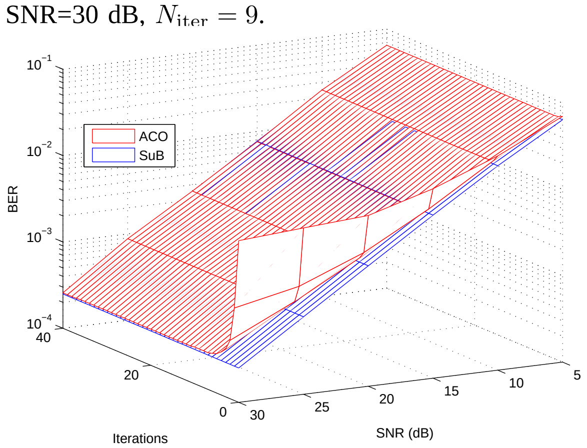


Fig. 5. ACO-MuD Convergence performance under SNR $\in [5; 30]$ dB, flat Rayleigh, $L_{\%} = 100\%$, $V_{\max} = 120$ km/h; $\alpha = 0.4$; $\beta = 7$; $\sigma = 5$; $\gamma = 3$.

VI. Conclusions

A heuristic multiuser detector based on the ant colony optimization (ACO-MuD) suitable to BPSK DS/CDMA systems under flat Rayleigh fading channels was proposed and successfully characterized. An input parameters optimization methodology for the ACO-MuD was carried out, in order to achieve the best possible performance with a fixed number of iterations. The optimized values proved to be robust enough, such a way to ensure a near-optimum performance for different system and channel operations scenarios, as well as different power control situations, without the need of significant changes in the algorithm.

Indeed, the input parameters optimization for the ACO-MuD shows that the parameters σ and γ are virtually immune to the channel mobility and loading system variation, indicating that the algorithm is able to operate robustly under different mobile channel coherence times; in practice, only the α and β parameters needs to be slightingly adjusted when drastic changes in both system operation conditions and multiple access channel occur.

The computational complexity for the proposed ACO-MuD deploying optimized input parameters, was analyzed by the number of cost function calculations. ACO-MuD complexity results very low, resulting in only a small fraction of the OMuD computational complexity ($\approx 10^{-7}$) under full system loading condition, wide range of channel mobility, but with very similar performances.

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