Bit Error Probability of M-QAM Subject to Impulsive Noise and Rayleigh Fading

Hugerles S. Silva, Marcelo S. de Alencar, Wamberto J. L. de Queiroz, Danilo B. T. de Almeida and F. Madeiro

***Abstract*— In this paper exact expressions are presented for the bit error probability (BEP) of M-ary Quadrature Amplitude Modulation (M-QAM) signals subject to Double Gated Additive White Gaussian Noise (G^2AWGN) combined with Rayleigh fading. In this study, the Rayleigh fading channel can be seen as a channel subject to an additive noise, R, obtained by the ratio between the received signal and the fading amplitude. Besides, for each model of impulsive noise considered, the probability density function (PDF) of R is presented, which is defined as the ratio between the random variable of the impulsive noise and the random variable of the fading.**

***Keywords*— Rayleigh fading, bit error probability, impulsive noise.**

I. Introduction

The impulsive noise may significantly affect the performance of communication systems. The sources generating this type of noise are numerous [1]. Impulsive noise may be natural, due to atmospheric phenomena, such as lightning strikes, or artificial. Noises caused by electrical equipment in factory environments, fluorescent and incandescent lamps, light switches, for example, are impulsive [2]. Studies on impulsive noise began with Middleton in 1951 [3]. Since them, several studies have arisen on Double Gated Additive White Gaussian Noise (G^2AWGN) binary [4], [5]. Impulsive noise is of interest to several researchers [6], [7], [8], [9], [10], [11].

Nowadays, with the advances of the researches on Fifth Generation (5G) systems, studies of impulsive noise on communication links in millimeter waves have been presented. In [12], for instance, a discussion is presented on the effects of noise of an impulsive nature, characterized statistically by Gaussian mixtures, in the performance of systems that operate in the spectral range corresponding to millimeter waves. The author also addresses blanking and clipping filter to minimize these effects.

A variety of automation techniques in industrial environments can benefit from the high bandwidth of the spectrum in the millimeter range for 5G systems [13]. In these environments, however, besides the permanent noise with greater power, due to the larger bandwidth of the channel, there are also sources such as motors, inverters, ignition systems and other elements of heavy machinery that contribute to the emergence of impulsive nature noise.

Hugerles S. Silva, Marcelo S. de Alencar, Wamberto J. L. de Queiroz e Danilo B. T. de Almeida¸ Center of Electrical Engineering and Informatics, Federal University of Campina Grande (UFCG), Campina Grande-PB, Brazil, E-mails: {hugerles.silva,danilo.almeida}@ee.ufcg.br, {malencar,wamberto}@dee.ufcg.br. Francisco Madeiro, Polytechnic School of Pernambuco, University of Pernambuco, Recife, Brazil, E-mail: madeiro@poli.br.

In the spectral range below 1 GHz, in which the operation of wireless sensor networks and internet links between devices (in the scenario of Internet of Things, IoT) is taken into account, one of the main sources of impulsive noise is the fluorescence lamp [14]. It is therefore plausible to consider models of impulsive noise in communication channels so that the transmission and reception systems can be designed to minimize the noise effects.

In this work, new closed expressions for calculating the bit error probability (BEP) of M-QAM signals subject to G^2AWGN noise combined with the Rayleigh fading are presented. The computation of the BEP is performed from an alternative method, which consists of dividing the received signal by the estimated fading envelope and considering the resulting signal subject to a new noise, R, which is modeled as the ratio between the random variable that represents the impulsive noise and the random variable the represents the fading.

This alternative approach to calculating the BEP was introduced in [15], in which the ratio of two random variables was used to determine closed expressions of the BEP for Quadrature Phase Shift Keying (QPSK) modulation scheme subject to AWGN and Rayleigh fading. A study of the influence of AWGN noise and Rayleigh fading on the performance of an optimal maximum a posteriori probability receiver for M-ary Pulse Amplitude Modulation (M-PAM) signals and Rectangular Quadrature Amplitude Modulation (R-QAM) is presented in [16]. This performance is evaluated by means of BEP curves obtained by exact expressions, which were also determined using the ratio of random variables.

In addition to this introductory section, this article is divided into six more sections. In Section II the mathematical model for the communication system is presented. Section III presents the stochastic process and the probability density function (PDF) for the G^2AWGN noise. In Section IV expressions for the PDF of the additive noise, obtained by the ratio between the random variable that characterizes the impulsive noise and the random variable that characterizes Rayleigh fading, are presented. Section V describes the BEP of the M-QAM signals subject to impulsive noise G^2AWGN and Rayleigh fading. In Section VI, BEP curves as a function of the signal-to-noise ratio (SNR) under the effect of the impulsive noise and Rayleigh fading are presented, and conclusions are presented in Section VII.

II. Mathematical Model for the Received Signal

Consider a communication system in which the received signal is composed by the transmitted signal affected by the fading and the impulsive noise, that is,

$$Y(t) = \gamma X(t) + \eta(t), \tag{1}$$

in which $X(t)$ is the transmitted signal, $\eta(t)$ is the impulsive noise, $Y(t)$ is the received signal and γ is the fading [17]. The fading is caused by multipath propagation and imposes random variations of intensity to the transmitted signal. For the received signal model described by Equation (1), non frequency selective slow fading is assumed. This implies that the multiplicative parameter γ can be considered constant at least during a signaling interval.

Lopes and Alencar [15] have shown that the Rayleigh fading channel can be viewed as a channel subject to an additive noise, R, by performing the division of the received signal $Y(t)$ by γ. This additive noise is modeled by a random variable defined as the ratio of the random noise variable $\eta(t)$ by the random variable of fading γ [15], [16].

III. Mathematical Model for the Impulsive Noise

The general model of the Double Gated Binary Gaussian Impulsive Noise with random occurrences of pulses and bursts and with the presence of the permanent noise is given by [4]

$$\eta(t) = \eta_g(t) + C_1(t)C_2(t)\eta_i(t), \tag{2}$$

in which $\eta_i(t)$ represents a zero-mean complex white Gaussian random process with variance σ_i^2, $\eta_g(t)$ is the background Gaussian noise with zero-mean and variance σ_g^2 and $C_1(t)$ and $C_2(t)$ are auxiliary random signals which characterize the bursts and pulses, respectively, and assume values from the discrete set {0,1}. The signal $C_1(t)$ is written as

$$C_1(t) = \sum_{k=-\infty}^{\infty} m_k P_{R_1}(t - kT_1), \tag{3}$$

in which m_k is the k-th bit of the alphabet {0,1} with probability distribution $p(m_k = 1) = p_1$ and $p(m_k = 0) = 1 - p_1$. The pulse $P_{R_1}(t)$ assumes unit amplitude at $0 \leq t \leq \beta T_1$, with β assuming values between zero and one. The random signal $C_2(t)$ assumes the values zero and one randomly and is represented by

$$C_2(t) = \sum_{l=-\infty}^{\infty} m_l P_{R_2}(t - lT_2), \tag{4}$$

in which m_l is the l-th bit of the alphabet {0,1} with probability distribution $p(m_l = 1) = p_2$ and $p(m_l = 0) = 1-p_2$. The pulse $P_{R_2}(t)$ assumes unit amplitude at $0 \leq t \leq \alpha T_2$, with α assuming values between zero and one.

The probability density function of the impulsive noise for this model, $f_{\eta(t)}(\eta)$, is given by [18]

$$\begin{aligned} f_{\eta(t)}(\eta) \quad = \quad & \frac{\alpha\beta p_1 p_2}{\sqrt{2\pi(\sigma_g^2 + \sigma_i^2)}} \exp\left[-\frac{\eta^2}{2(\sigma_g^2 + \sigma_i^2)}\right] \\ + \quad & \frac{(1 - \alpha\beta p_1 p_2)}{\sqrt{2\pi\sigma_g^2}} \exp\left[-\frac{\eta^2}{2\sigma_g^2}\right]. \end{aligned} \tag{5}$$

From the general model G^{2}AWGN presented in Equation (2), it is possible to obtain seven simpler noise models. Two distinct models are determined when the permanent component of noise is absent, with total noise being pure; and when the permanent component is present in the system, the total noise being composed. When $p_1 = 1$, it means that the bursts are deterministic, $0 < p_1 < 1$ means that the bursts are random, $p_2 = 1$ means that the pulses are deterministic and $0 < p_2 < 1$ means that the pulses are random.

IV. PDF of the Noise R

This section presents the PDF of the random variable of the noise R, which is modeled as the ratio between two random variables, one characterized by a Gaussian mixture and the other characterized by the Rayleigh distribution. This division is performed to make the fading channel as a channel subject to an additive noise R. Mathematically, that ratio can be expressed as

$$R = \frac{\eta}{\gamma}, \tag{6}$$

in which η denotes the G^{2}AWGN impulsive noise, with PDF given by Expression (5), and γ denotes the fading amplitude, with PDF [19]

$$f_\gamma(\gamma) = 2\gamma e^{-\gamma^2} u(\gamma), \tag{7}$$

in which $\mathrm{u}(\cdot)$ denotes the unit step function.

Under those conditions, the PDF of R is given by [20]

$$f_R(r) = \int_{-\infty}^{\infty} |\gamma| f_{\eta,\gamma}(r\gamma, \gamma) d\gamma, \tag{8}$$

in which $f_{\eta,\gamma}(r\gamma, \gamma)$ is the joint PDF of η and γ. Since η and γ are independent random variables, it follows that

$$\begin{aligned} f_{\eta,\gamma}(r\gamma, \gamma) &= f_\eta(r\gamma) f_\gamma(\gamma) \\ &= \frac{2\gamma\alpha\beta p_1 p_2}{\sqrt{2\pi(\sigma_g^2 + \sigma_i^2)}} e^{-\left(\gamma^2 + \frac{r^2\gamma^2}{2(\sigma_g^2+\sigma_i^2)}\right)} u(\gamma) \\ &+ \frac{2\gamma(1 - \alpha\beta p_1 p_2)}{\sqrt{2\pi\sigma_g^2}} e^{-\left(\gamma^2 + \frac{r^2\gamma^2}{2\sigma_g}\right)} u(\gamma). \end{aligned} \tag{9}$$

Hence, the PDF of R can be written as

$$\begin{aligned} f_R(r) &= \int_{-\infty}^{\infty} |\gamma| f_\eta(r\gamma) f_\gamma(\gamma) d\gamma \\ &= \frac{\alpha\beta p_1 p_2 (\sigma_g^2 + \sigma_i^2)}{(r^2 + 2(\sigma_g^2 + \sigma_i^2))^{\frac{3}{2}}} + \frac{(1 - \alpha\beta p_1 p_2)\sigma_g^2}{(r^2 + 2\sigma_g^2)^{\frac{3}{2}}}. \end{aligned} \tag{10}$$

V. BEP of M-QAM Signals in Channel with Impulsive Noise and Rayleigh Fading

In this section, the PDF given by Expression (10) is considered to obtain closed form expressions for the BEP of M-QAM symbols subject to impulsive noise G^{2}AWGN and Rayleigh fading.

In the $I \times J$-QAM scheme, the waveforms consist of two quadrature carriers modulated in amplitude, independently, expressed by

$$s(t) = A_I \cos(2\pi f_c t) - A_J \mathrm{sen}(2\pi f_c t), \;\; 0 \leq t \leq T, \tag{11}$$

in which A_I and A_J respectively represent the amplitudes of the in-phase and quadrature components of the symbol to be transmitted, f_c is the carrier frequency and T is the duration of the symbol [19]. In this scheme, $\log_2 M$ bits of information are mapped into a two-dimensional constellation using a Gray code. In Equation (11), A_I and A_J are independently selected from the set $\{\pm d, \pm 3d, \cdots, \pm(\sqrt{M}-1)d\}$, in which $2d$ is the minimum Euclidean distance between the components of two distinct symbols of the constellation, with d given by

$$d = \sqrt{\frac{3\log_2 M \cdot E_b}{2(M-1)N_0}}, \tag{12}$$

E_b is the bit energy and N_0 is the power spectral density of the noise [17].

Cho and Yoon [21] proposed a exact expression for calculating the BEP of the QAM scheme with arbitrary dimension of the constellation, considering AWGN [21] noise. In this section, the expressions obtained by Cho and Yoon are considered to obtain new expressions for the BEP of the square M-QAM scheme in a channel with impulsive noise G^2AWGN and Rayleigh fading.

Consider a noise AWGN with zero mean and power spectral density $N_0/2$. Let P_e be the BEP, E_b be the bit energy, M the number of symbols on the QAM constellation and $P_e(k)$ the BEP for the k-th bit, with $k \in \{1, 2, \cdots, \log_2 M\}$. The expression for the BEP of the M-QAM scheme for a AWGN channel obtained by Cho and Yoon is given by [21]

$$P_e = \frac{1}{\log_2 \sqrt{M}} \sum_{k=1}^{\log_2 \sqrt{M}} P_e(k), \tag{13}$$

with

$$\begin{aligned} P_e(k) &= \frac{1}{\sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} \{w(i,k,M) \\ &\times \operatorname{erfc}\left((2i+1)\sqrt{\frac{3\log_2 M \cdot E_b}{2(M-1)N_0}}\right)\right\}, \end{aligned} \tag{14}$$

in which

$$w(i,k,M) = (-1)^{\left\lfloor \frac{i \cdot 2^{k-1}}{\sqrt{M}} \right\rfloor} \cdot \left(2^{k-1} - \left\lfloor \frac{i \cdot 2^{k-1}}{\sqrt{M}} + \frac{1}{2} \right\rfloor\right), \tag{15}$$

M is the order of the constellation, E_b/N_0 denotes the SNR per bit, $\lfloor x \rfloor$ denotes the largest integer less than or equal to x, and $\operatorname{erfc}(\cdot)$ represents the complementary error function.

The contribution of Cho and Yoon was to write the BEP of the M-QAM scheme transmitted in an AWGN channel in terms of the weighted sum of complementary error functions. The weights $w(i,k,M)$ incorporate the effect, in the BEP, of the k-th bit position in symbol with $\log_2 M$ *bits*. Notice that the term $\operatorname{erfc}((2i+1)\sqrt{3\log_2 M \cdot E_b/2(M-1)N_0})$, in Equation (14), corresponds to twice the probability that the additive noise exceeds [22]

$$(2i+1)\sqrt{3\log_2 M \cdot E_b/(M-1)N_0}. \tag{16}$$

Assuming the channel with G^2AWGN impulsive noise and Rayleigh fading, the probability that the additive noise $r(t)$ exceeds $(2i+1)\sqrt{3\log_2 M \cdot E_b/2(M-1)N_0}$ is given by Equation (18), with

$$a(i,M) = \frac{3(2i+1)^2 \log_2 M}{M-1}. \tag{17}$$

Using Equation (18) and the weights of Equation (15), the expression for the BEP of M-QAM scheme subject to impulsive noise G^2AWGN and Rayleigh fading is given by Equation (13), with $P_e(k)$ given by Equation (19). In [16] expressions are presented for the BEP of M-QAM scheme considering channel with AWGN and Rayleigh fading. In Equation (18), the term SNR represents the ratio of the signal power to the power of the background Gaussian noise, that is always present in the system, and SNI is the impulsive signal to noise ratio, defined as the ratio between the power of the signal and the power of the impulsive noise that acts in the system.

VI. Results

In this article, the values adopted for the parameters of $\eta(t)$ and the modulation schemes are based on the results of theoretical and experimental studies with impulsive interference in the European Digital Video Broadcasting (Terrestrial) (DVB-T) by the United Kingdon Digital Television Group (DTG) [10].

The BEP curves of the 64-QAM modulation scheme under the effect of the G^2AWGN impulsive noise and Rayleigh fading for four distinct values of the SNI are shown in Figure 1. The values adopted for p_1, p_2, β and α were 0.5. It is noted that the BEP decreases with increasing SNR for fixed SNI values. For the blue curve, in which SNI = 10 dB, it is observed that for SNR values above 20 dB there is a small reduction in the BEP with increasing SNR. For SNR values above 10 dB, when the SNR is larger than the SNI, for a fixed signal power it follows that the power of the impulsive noise is greater than the power of the permanent noise. Therefore, the action of the impulsive noise $\eta_i(t)$ further contributes to increase P_e.

Curves of the BEP of the 64-QAM modulation scheme under the effect of the G^2AWGN impulsive noise and Rayleigh fading, considering $\alpha = 0.5$, $\beta = 0.5$, SNI = 20 dB and different probabilities of the signals $C_1(t)$ and $C_2(t)$ assume value one are shown respectively in Figures 2 and 3. As the value of p_1 or p_2 increases, the BEP obtained for fixed values of SNR is greater. It is noted that for values of SNI = 20 dB, P_e is not less than 10^{-3} for SNR < 30 dB, for values of p above 0.1.

Considering the noise G^2AWGN and Rayleigh fading, the evaluation of the influence of the order M of the modulation scheme on the BEP is shown in Figure 4 for four values of M, considering SNI = 20 dB, p_1, p_2, β and α equal to 0.5. The higher the value of M, the greater the BEP since the symbols affected by the noise are closer, so the received signal is more susceptible to errors. It is also observed that the BEP equal to 10^{-2} is obtained with SNR $\approx$ 14 dB for M = 16 whereas it is obtained with SNR $\approx$ 27 dB for M = 1024.

VII. Conclusion

In this paper we present a study of the influence of Double Gated Additive White Gaussian Noise (G^2AWGN)

$$2P\left(r \geq (2i+1)\sqrt{\frac{3\log_2 M \cdot E_b}{2(M-1)N_0}}\right) = 2\int_{\sqrt{\frac{3(2i+1)^2\log_2 M\cdot E_b}{2(M-1)N_0}}}^{\infty} f_R(r)dr$$
$$= \alpha\beta p_1 p_2\left[1-\frac{1}{\sqrt{1+\frac{\mathrm{SNR}\cdot\mathrm{SNI}}{a(i,M)(\mathrm{SNR}+\mathrm{SNI})}}}\right] + (1-\alpha\beta p_1 p_2)\left[1-\frac{1}{\sqrt{1+\frac{1}{a(i,M)\cdot\mathrm{SNR}}}}\right]. \quad (18)$$

$$P_e(k) = \frac{1}{\sqrt{M}}\sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} w(i,k,M)\left\{\alpha\beta p_1 p_2\left[1-\frac{1}{\sqrt{1+\frac{\mathrm{SNR}\cdot\mathrm{SNI}}{a(i,M)(\mathrm{SNR}+\mathrm{SNI})}}}\right] + (1-\alpha\beta p_1 p_2)\left[1-\frac{1}{\sqrt{1+\frac{1}{a(i,M)\cdot\mathrm{SNR}}}}\right]\right\}. \quad (19)$$

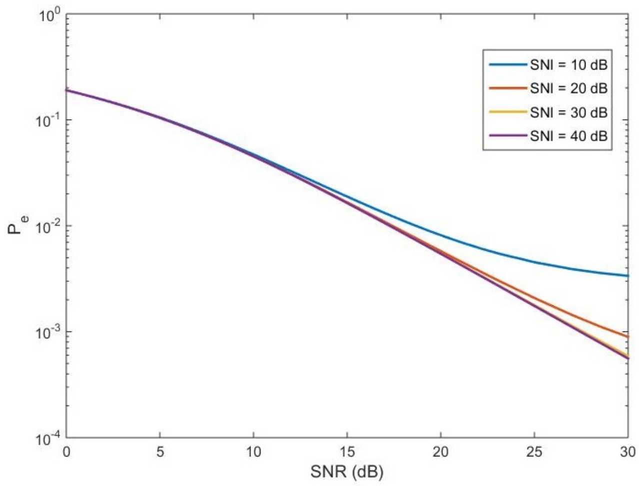

Fig. 1. Bit error probability of the modulation scheme 64-QAM under the effect ot the G^2AWGN impulsive noise and Rayleigh fading, with for distinct values signal to impulsive noise ratio.

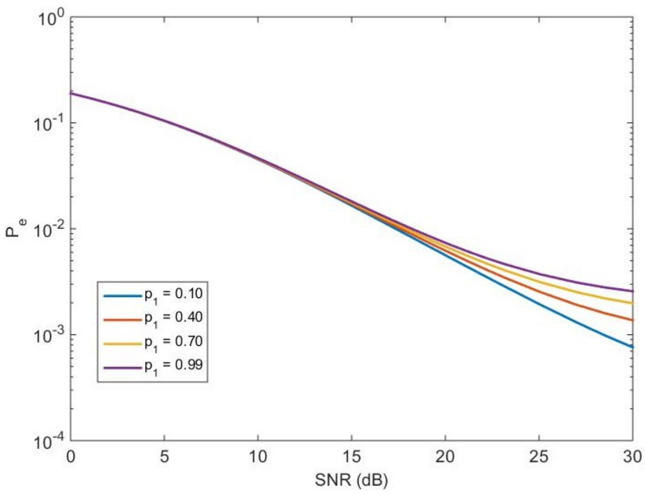

Fig. 2. Bit error probability of the modulation scheme 64-QAM under the effect of the G^2AWGN impulsive noise and Rayleigh fading, with different probabilities for p_1.

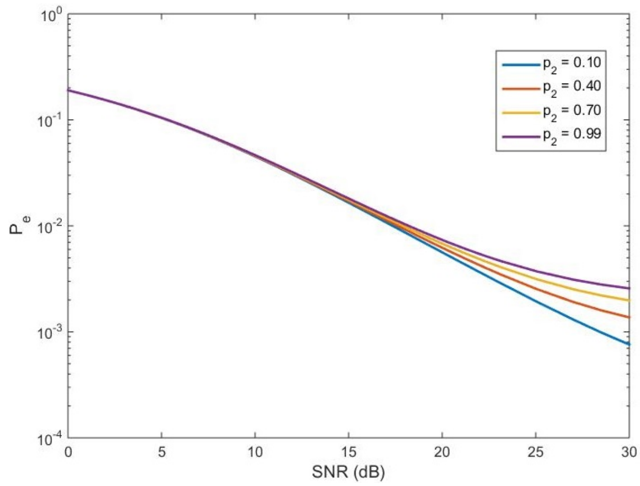

Fig. 3. Bit error probabiity of the modulation scheme 64-QAM under the effect of the G^2AWGN impulsive noise and Rayleigh fading, with different probabilities for p_2.

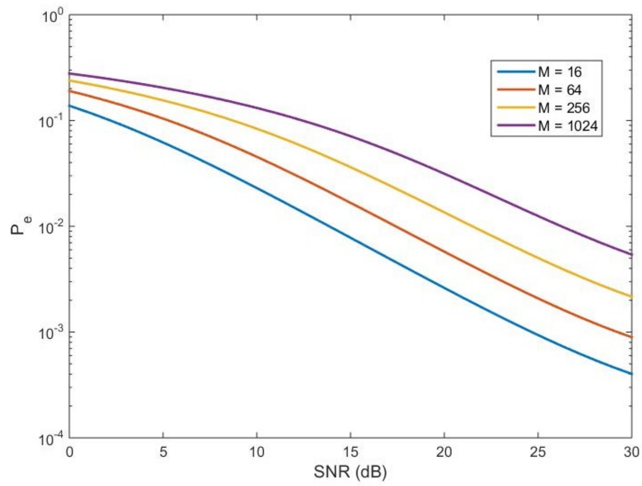

Fig. 4. Bit error probability for different values of the modulation order M under the effect of the G^2AWGN impulsive noise and Rayleigh fading.

and Rayleigh fading on the performance of an optimum maximum-likelihood receiver for the modulation scheme M-QAM. This performance was evaluated by means of the bit error probability (BEP) curves, obtained by exact expressions. To determine the BEP expressions, an alternative method was used to facilitate mathematical development. In this method the division of the received signal by the fading envelope was performed, making the channel subject to an additive noise R. In addition, new expressions have been presented for the PDF of the noise R, defined as the ratio between the random variable of the G^2AWGN, and the random variable of the envelope fading.

In the results, it was observed that some BEP curves presented an asymptotic behavior, tending to a constant value with the increase of the SNR. This is because when the power of permanent noise decreases relative to signal power, the component of impulsive noise continues to affect the performance of the receiver. With respect to different values of probabilities of occurrences of pulses and bursts in an impulsive event, it was noticed that as these values increase, the greater the BEP. This is because the increase of these parameters implies a greater probability of impulsive noise being present in the system. In relation to the effects of increasing constellation order, the results are as expected – the performance of the receiver worsens with the increase of the order M.

As future works, the authors intend to determine closed expressions for the BEP of M-QAM considering the channel subject to impulsive noise G^2AWGN and other kind of fading, such as η-μ, κ-μ and α-μ.

ACKNOWLEDGEMENTS

The authors express their thanks to Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) for the financial support to the work.

REFERENCES

[1] S. R. Al-Araji, J. E. Allos, T. I. Al-Mahdawi and A. S. M. Ali. *Impulsive Noise Reduction in Data Communication Systems Employing Smear/Desmear Technique*. IEEE International Symposium on Electromagnetic Compatibility, vol. 1, no. 1, pp. 1-8, August 1985.

[2] T. N. Zogakis, P. S. Chow, J. T. Aslanis and J. M. Cioffi. *Impulse Noise Mitigation Strategies for Multicarrier Modulation*, IEEE International Conference on Communications, vol. 2, no. 4, pp 784-788, May 1993.

[3] D. Middleton. *On the Theory of Random Noise*. Phenomenological Models I. Journal of Applied Physics, vol. 22, no. 9, pp. 1143-1152, May 1951.

[4] E. R. Araújo, W. J. L. Queiroz, F. Madeiro, W. T. A. Lopes and M. S. Alencar. *On Gated Gaussian Impulsive Noise in M-QAM with Optimum Receivers*. Journal of Communication and Information Systems, vol. 30, no. 1, pp. 1-10, March 2015.

[5] W. J. L. Queiroz, W. T. A. Lopes, F. Madeiro and M. S. Alencar. *On the Performance of M-QAM for Nakagami Channels Subject to Gated Noise*. Telecommunication Systems, vol 68, no. 1, pp. 1-10, May 2018.

[6] M. Chan and R. Donaldson. *Amplitude, Width, and Interarrival Distributions for Noise Impulses on Intrabuilding Power Line Communication Networks*. IEEE Transactions on Electromagnetic Compability, vol. 31, no. 3, pp. 320-323, April 1989.

[7] G. A. Tsihrintzis and C. L. Nikias. *Performance of Optimum and Suboptimum Receivers in the Presence of Impulsive Noise Modeled as an Alpha-Stable Process*. IEEE Transactions on Communications, vol. 43, no. 234, pp. 904-914, August 1995.

[8] T. Y. Al-Naffouri, A. A. Quadder and G. Caire. *Impulse Noise Estimation and Removal for OFDM System*. IEEE Transactions on Communications, vol. 62, no. 3, pp. 976-989, March 2014.

[9] M. Nassar, K. Gulati, Y. Mortazavi, and B. L. Evans. *Statistical Modeling of Asynchronous Impulsive Noise in Powerline Communication Networks*. IEEE Global Telecommunications Conference (GLOBECOM), pp. 1-6, March 2011.

[10] J. Lago-Fernandez and J. Salter. *Modelling Impulsive Interference in DVB-T: Statistical Analysis, Test Waveform & Receiver Performance*. BBC R&D White Paper WHP 080, 2004.

[11] R. Schwarz. *Tolerance to Noise Tests for DTV Receivers With R&S®SFU-K41, -K42 and -K43 Part 1: Impulsive Noise*. Technical report, Rohde & Schwarz, 2007.

[12] L. M. H. Shhab, A. Rizaner, A. H. Ulusoy and H. Amca. *Impact of Impulsive Noise on Millimeter Wave Cellular Systems Performance*, in Proceedings of the IEEE 10th UK-Europe-China Workshop on Millimetre Waves and Terahertz Technologies (UCMMT), pp. 1-4, Liverpool, United Kingdom, September 2017.

[13] M. Cheffena. *Industrial Wireless Communications over the Millimeter Wave Spectrum: Opportunities and Challenges*. IEEE Communications Magazine, vol. 54, no. 9, pp. 66-72, September 2016.

[14] I. Landa, A. Blázquez, M. Vélez and A. Arrinda. *Indoor Measurements of IoT Wireless Systems Interfered by Impulsive Noise From Fluorescent Lamps*, in the Proceedings of the 11th European Conference on Antennas and Propagation (EUCAP), Paris, France, March 2017.

[15] W. T. A. Lopes and M. S. Alencar. *QPSK Detection Schemes for Rayleigh Fading Channels* in Proceedings of the IEEE International Telecommunications Symposium (ITS'02), Natal – RN, Brazil, September 2002.

[16] W. T. A. Lopes, F. Madeiro and M. S. Alencar. *Closed-Form Expression for the Bit Error Probability of Rectangular QAM Subject to Rayleigh Fading* in Proceedings of the IEEE 66th Vehicular Technology Conference, Baltimore, MD, October 2007.

[17] S. Haykin. *Communication Systems*. 4ª edition: John Willey and Sons, 2002.

[18] E. R. Araújo, W. J. L. de Queiroz and M. S. de Alencar. *Analysis of the Gated Impulsive Noise in Optimum Receivers*. V International Workshop on Telecommunications – IWT, May 6th-9th, Santa Rita do Sapucaí – MG, Brazil, 2013.

[19] A. Goldsmith. *Wireless Communications*. Cambridge University Press, 2005.

[20] A. Leon-Garcia. *Probability, Statistics, and Random Process for Electrical Engineering*. 3rd edition: Pearson Prentice Hall, 2008.

[21] K. Cho and D. Yoon. *On the General BER Expression of One- and Two-dimensional Amplitude Modulations*. IEEE Transactions on Communications, vol. 50, no. 7, pp. 1074-1080, May 2002.

[22] M. S. Alencar. *Telefonia Celular Digital*. 1ª edição: Érica, 2004.