HMM Models and Estimation Algorithms for Real-Time Predictive Spectrum Sensing and Cognitive Usage

Luiz Renault L. Rodrigues and Ernesto Leite Pinto

***Abstract*— This work investigates the use of Non-Stationary Hidden Markov and Hidden Bivariate Markov Models through simulations and real-time application to predict RF channel occupancy in cognitive radio systems. Real data collected in public safety frequency band during the Rio 2016 Olympic and Paralympic Games were used to test a simple cognitive spectrum sharing scheme here proposed with the main goal of maximizing secondary usage of available spectrum. Several algorithms for parameter estimation are compared in this context. Remarkably good performance is obtained with a windowed version of the Baum algorithm and Non-Stationary Hidden Markov Model.**

***Keywords*— Cognitive Radio, HMM, HBMM, NS-HMM.**

I. Introduction

Cognitive radio (CR) system recently emerged as an attempt to address the spectrum scarcity problem through the smart reuse of frequency bands without interfering the licensed or primary user (PU). The CR as a secondary user (SU) performs, among several tasks, the observation of the surrounding environment, as part of the learn, plan, decide and act cycle proposed by Mitola [1].

The environment observation process was extensively treated in the literature as spectrum sensing [2] and consists on acquiring knowledge about surrounding users behavior to be used for spectrum sharing scheduling.

In public safety and military communications, it's not unusual to consider that the sensing process concurs with data transmission since a large variety of radio equipment used in that application operate in half duplex mode. This means that the CR is usually assumed to be either listening to the spectrum or transmitting data. In this context, [3] exposes the existence of two dynamic spectrum access (DSA) techniques: spectrum sensing (SS) and sensing plus prediction (SPP).

SS-based DSA employs various signal detection methods to instantaneously identify the RF channel state and decide on the beginning of the CR transmissions. Since SS is imperfect, the CR will interfere the PU communication when the channel is erroneously detected as idle but is being used or when the PU attempts to use the channel while the CR is transmitting. Another result of SS imperfection, although not harmful to the PU, is the detection of the busy state while the channel is idle, which results in under-usage of the channel availability.

With several advantages, SPP-based DSA uses the history of PU spectrum occupancy to model and predict its behavior. The SU transmission length is selected on the basis of the predicted channel idle state duration, giving rise to new possibilities of dealing with sensing imperfections. Thus SPP can significantly increase SU data throughput [3] and reduce interference to the licensed system. The key factor in applying this technique is the capacity to model and predict the state duration.

For this intent, Hidden Markov Models (HMM) are often used [4]–[6]. HMM allows to estimate the channel occupation state sequence, given some observable information statistically linked to it such as the received channel power and to predict future state transitions based on the system dynamic behavior.

The main limitation of HMM is the fact that state duration is modelled by geometric distribution, which is recognized not to be satisfactory in several applications, including spectrum sensing [6]. To overcome this issue, HMM based models were recently proposed, standing out the Hidden Bivariate Markov Models (HBMM) [6] and Non-Stationary Hidden Markov Models (NS-HMM) [7], [8].

This work addresses an the application of such models to predict spectrum occupancy to maximize CR data throughput and minimize interference to the PU. It is organized as follows.

Next section shows an overview of the evaluated models. Section III details a simple spectrum sharing scheme here proposed for performance evaluation purposes. Simulation results are presented in section IV. Signal capture and real-time modelling results are discussed in section V. At last, conclusions are presented with suggestions for future work.

II. Evaluated Models

Consider a CR that senses the spectrum through the received RF channel power level expressed in dBm, captured in discrete time steps $k \in \{1, 2, ...\}$. These observations will be here denoted by o_k. Also, let $s_k \in \{1, ..., c\}$ be the hidden process of channel occupation state, at time k, with 1 denoting the idle state and 2 to c standing for channel use by the primary or other secondary users. Like previous works [6], [7], we assume that $c = 2$ and also that the observation at time k, o_k, is conditionally independent and normally distributed given the occupation state s_k, with conditional mean μ_{s_k} and variance $\sigma^2_{s_k}$.

In this scenario, the investigated models can be trained and employed to detect and predict the channel state, not directly observed, using only the channel power observations. The HMM, HBMM and NS-HMM models represent distinct levels of time dependency and are briefly described below.

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A. Hidden Markov Models (HMM)

In an HMM model, The state transition probabilities are constant over time and depends only on the previous state, i.e. $Pr(s_{k+1} = m \mid s_k, s_{k-1}, ..., s_1) = Pr(s_{k+1} = m \mid s_k)$, $k \in 1, ...$

Let $g_{nm} = Pr(s_{k+1} = m \mid s_k = n)$ denote the probability of being in state m at $k+1$ given the model was in state n at k, and $\mathbf{G} = [g_{nm}]$ be the $c \times c$ transition probability matrix, with $n, m \in \{1, ..., c\}$ and c the number of possible states.

It can be shown [6] that the probability of being in state n for exactly $d \in \{1, ...\}$ consecutive time slots is given by $P_n(d) = g_{nn}^{d-1}(1 - g_{nn})$, which corresponds to a geometrical distribution with mean $\frac{1}{1-g_{nn}}$.

The complete set of HMM model parameters is $\boldsymbol{\phi} = [\boldsymbol{\pi}, \mathbf{G}, \boldsymbol{\mu}, \boldsymbol{\sigma}^2]$, with $\boldsymbol{\pi} = [\pi_n = Pr(s_1 = n)]$ the initial state distribution vector, $\boldsymbol{\mu} = [\mu_n]$ and $\boldsymbol{\sigma}^2 = [\sigma_n^2]$, $n \in \{1, ..., c\}$, respectively the observation conditional mean and variance vectors. The total amount of parameters is $c^2 + 3 \cdot c$.

B. Hidden Bivariate Markov Model

In an HBMM model, as described in [6], auxiliary subjacent states are appended to every channel occupation state, thus enabling distinct state duration distributions.

Let $z_k = (s_k, x_k)$ be the double stochastic process so obtained, with s_k denoting the channel occupation process, as described above, and $x_k \in \{1, ..., r\}$ the subjacent process, being r the number of subjacent states. This process is Markovian and the observation process o_k is kept to be conditionally independent given s_k.

State transition probabilities are constant. Let $g_{nm}^{ij} = Pr(z_{k+1} = (m, j) \mid z_k = (n, i))$ be the transition probability from occupation state n and subjacent state i at time k to occupation state m and subjacent state j at time $k+1$. The $c \cdot r \times c \cdot r$ transition probability matrix is $\mathbf{G} = [\mathbf{G}_{nm}]$, with $\mathbf{G}_{nm} = [g_{nm}^{ij}]$ a $c \times c$ matrix, $n, m \in \{1, ..., c\}$ and $i, j \in \{1, ..., r\}$. The number of subjacent states r is said to be the HBMM model order. The HBMM model is equivalent to a HMM when $r = 1$.

Despite the duration of state $z = (n, i)$ being geometrically distributed, the duration of state $s = n$ follows a phase type distribution with r phases which can, in principle, approximate a given distribution to within any desired level of accuracy [6].

The occupation state duration probability for the HBMM model can be calculated as $P_n(d) = \boldsymbol{\pi}_n \mathbf{G}_{nn}^{t-1}(\mathbf{I} - \mathbf{G}_{nn})\mathbf{1}$, where $\mathbf{1}$ is a column vector with ones and $\boldsymbol{\pi}_n$ the stationary distribution of being in state $z_k = (n, i)$, for $i \in \{1, ..., c\}$ and a given n.

The complete set of HBMM model parameters is $\boldsymbol{\phi} = [\boldsymbol{\pi}, \mathbf{G}, \boldsymbol{\mu}, \boldsymbol{\sigma}^2]$, with $\boldsymbol{\pi} = [\pi_n^i = Pr(z_1 = (n, i))]$ the initial state distribution vector, $\boldsymbol{\mu} = [\mu_n]$ and $\boldsymbol{\sigma}^2 = [\sigma_n^2]$ respectively the observation conditional mean and variance vectors, $n \in \{1, ..., c\}$ and $i \in \{1, ..., r\}$. The total amount of parameters is $c^2 \cdot r^2 + c \cdot r + 2 \cdot c$, largely increasing the computational complexity implied by the use of this model.

C. Non-Stationary Markov Model

Non-Stationary Markov Model [7], [8] can also be used to overcome the geometrically distributed state duration limitation of HMM. In this model, the state transition probabilities from a given state, say n, also depend on the time d spent since the model entered the state n.

Let $g_{nm}^{(d)} = Pr(s_{k+1} = m \mid s_{k-d+1}^{k} = n, s_{k-d} \neq n)$ be the transition probability to state m at time $k+1$ given that the model has been in state n for d consecutive time slots. The $c \times c \times d$ transition probability matrix is denoted by $\mathbf{G} = [\mathbf{G}^{(d)}]$, with $\mathbf{G}^{(d)} = [g_{nm}^{(d)}]$ a $c \times c$ matrix, $n, m \in \{1, ..., c\}$ and $d \in \{1, ..., D\}$. D is said to be the NS-HMM model order. The probability of the NS-HMM being in occupation state n for d time slots is given as $P_n(d) = (1 - g_{nn}^{(d)}) \prod_{t=1}^{d-1} g_{nn}^{(t)}$.

To remove the maximum state duration limitation seen in other works [7], the NS-HMM proposed in [8] has state transition probabilities beyond D constant, or $g_{nm}^{(d)} = g_{nm}^{(D)}$ for $d \geq D$. This makes the state duration distribution arbitrary for $1 < d < D$, with exponential decay for $d \geq D$. With this modification the modelled state duration is no longer limited and the NS-HMM model is equivalent to a HMM when $D = 1$.

The NS-HMM model parameters set is $\boldsymbol{\phi} = [\boldsymbol{\pi}, \mathbf{G}, \boldsymbol{\mu}, \boldsymbol{\sigma}^2]$, with $\boldsymbol{\pi} = [\pi_n = Pr(s_1 = n)]$ the initial state distribution vector, $\boldsymbol{\mu} = [\mu_n]$ and $\boldsymbol{\sigma}^2 = [\sigma_n^2]$ respectively the observation conditional mean and variance vectors, $n \in \{1, ..., c\}$. The total amount of parameters is $c^2 \cdot D + 3 \cdot c$, thus inducing less computational efforts than the HBMM model.

D. Parameter Estimation

The HBMM and NS-HMM parameters can be estimated using adaptations of the Expectation Maximization (E-M) Baum algorithm for HMM, as seen in [6], [8], which updates the parameter set estimate $\hat{\boldsymbol{\phi}}$ while increasing the likelihood upon a training observation data. The Baum algorithm iterates on the entire training sequence, being a non-causal parameter estimator normally suitable to offline applications [9].

For online processing and real-time applications, recursive estimators for the HBMM model can be found in the literature, which can iterate over a block of observations [9] or over a single observation [10]. They will be referred to in this work, respectively as Rydén and Stiller-Radons algorithms.

As an attempt to use Baum algorithm in real-time, a windowed application was proposed in [8], in which a certain number of past samples over an observation window is used to estimate the model parameters sequentially. As time progresses, new observations are added and the oldest discarded to keep the window length constant. Good results were produced with windows lengths longer than 20 times the average state duration. An additional advantage of this technique is that it can track slow variations over time.

E. State Estimation

Since in spectrum sensing applications the main interest resides in identifying the channel availability, the probability of being in idle state at time $k+t$, $t \geq 0$, given the observations up to time k and an estimate of the model parameters, $Pr(s_{k+t} = 1 \mid o_1^k, \hat{\boldsymbol{\phi}})$, can be compared to a threshold (γ) in order to decide on the channel state. In this work, we used $\gamma = 0.5$ to implement a maximum a-posteriori decision rule [6].

The required statistics $\{Pr(s_k = n \mid o_1^k, \phi)\}$, can be obtained using the forward recursion from the Baum algorithm. Even if online parameter estimation algorithms are used, this forward recursion will be needed for state estimation.

Detailed information on parameter and state estimation can be found in [6], [8] for the evaluated models.

III. Spectrum Sharing Scheme

In practical situations, where many radio systems operate in half duplex mode, spectrum sensing concurs with SU transmissions. Thus, when a CR starts to transmit data, the PU activity cannot be detected until the transmission stops. If the PU attempts to use the channel during this period, a potentially harmful interference (collision) can occur. This highlights the importance of predictive spectrum sensing as a mean to establish the length of the CR transmission period to preventively avoid collisions.

We propose here a simple cognitive spectrum sharing strategy with maximum secondary usage of an RF channel licensed to a primary system. Every time the CR detects the channel as idle, the transmission is started and lasts for the predicted idleness duration. After that the CR restarts the detection process.

If detection and prediction were perfect, the RF channel would be used at its maximum availability, without interference to the primary user. Since it's subject to error, the following events can occur:

- Transmission successful: the channel is correctly detected or predicted as idle;
- Collision: the channel is detected or predicted as idle but was occupied and the CR starts the transmission;
- Transmission loss: the channel is detected as occupied but was idle;
- Channel busy: channel detected correctly as occupied;

Figure 1 illustrates the channel reuse scheme and the possible outcomes.

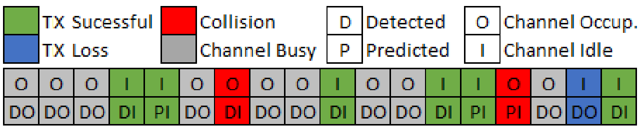

Fig. 1. Proposed channel reuse scheme and outcomes.

The underestimate of the idle state duration would also reduce transmission throughput, since the CR needs to restart the detection after stopping transmission and immediately resume the transmission if the channel is then detected as free. However, as the detection time will be shown to be much shorter than the time slot in the application here addressed, this reduction will be neglected.

This is in our view an effective tool to investigate the potential of predictive spectrum sensing and sharing in cognitive radio systems. It is worthy to notice that this scheme could also be applied in parallel in multi-carrier systems.

IV. Numerical Results

Simulations were performed to compare the different models and parameter estimation algorithms and provide a guideline for initial parameterization in real-time processing. The results were averaged over 1000 runs of the simulation experiments using the model seen in [6] to generate observation samples of RF power level of length 2000.

Initially, only Baum algorithms were used to evaluate the performance of HBMM and NS-HMM. The stop criteria for parameter estimation was based on the likelihood change between iterations, limited to a maximum number of 300. Model orders of 1, 2, 3, 5, 6, 10 and 15 were employed.

Table I shows the number of training iterations performed to satisfy the stop criteria. NS-HMM exhibits much faster convergence than HBMM model, which is very desirable for real-time applications. This can be attributed to the lower amount of parameters of NS-HMM.

TABLE I

Average training iterations required to match stop criteria.

Order	1	2	3	5	6	10	15
Models							
HBMM	1	50	55	72	79	71	74
NS-HMM	1	1	1	1	1	1	1

It is remarkable that regardless of the model order, the NS-HMM needed only 1 iteration to converge. In fact, it was only observed to need more training iterations to converge when the observation samples were severely degraded [8].

Despite HBMM model have required more training iterations, at the end both models produced similar likelihood and state estimation errors. Lower state prediction errors were obtained when estimated state duration distributions fitted well. This has been observed in situations where the model order exceeds the majority of the real state durations, which in the model of [6] was 6.

To evaluate state estimation accuracy, we compared the detected/predicted states $\hat{s}_t$ with the real ones s_t, $t = k, ..., k+9$ and $k = 1, ..., K-9$, with K the number of samples in the sequence. The average state estimation error probabilities were estimated as the number of incorrect state decisions divided by the total amount of decisions. Results are shown in Table II. The maximum standard deviation found was in the order of 10^{-2} times the estimated error probabilities, which is a good indicator of the estimate confidence.

TABLE II

Average state estimation error probabilities.

Order	1	2	3	5	6	10	15
Models							
HBMM	0,205	0,201	0,201	0,172	0,160	0,157	0,157
NS-HMM	0,205	0,201	0,201	0,184	0,161	0,158	0,158

Figure 2 shows real and estimated duration distributions for state 2. It is worthy to notice the exponential decay for duration probabilities above the NS-HMM model order, allowing to better model larger state durations with lower model orders. This is a major advantage of the model proposed by [8] over the model seen in [7].

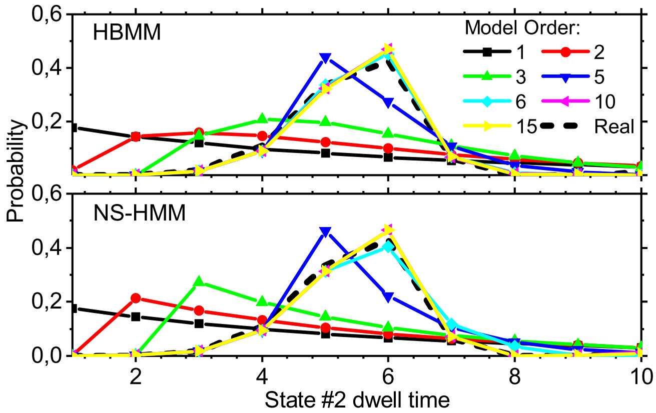

Fig. 2. State 2 duration distribution for different models and orders.

To mimic real-time applications, additional tests were executed simulating consecutive observation samples availability. For the HBMM models, the recursive parameter estimators were also tested. Rydén algorithm was employed with block size of 250 samples, the same length used with Baum windowed application, which is more than 30 times the average state duration. The empirical state estimation error probabilities over time so obtained are shown in Figure 3.

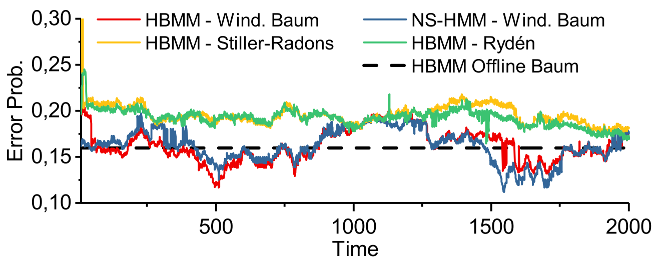

Fig. 3. State estimation error probabilities over time in online application.

In general, it was observed that the windowed Baum algorithm performed better than the recursive ones. As the forward recursion is still needed in the recursive algorithms for state estimation, the total processing times were very similar, in the order of few tenths of milliseconds. The state estimation error probabilities produced by the windowed Baum algorithm in some intervals were lower than the obtained using offline Baum algorithm. This can probably be due to the capacity of the former to better adapt to small variations over time.

The performance evaluation of the proposed channel sharing scheme is summarized in Table III, where relative occurrences of the distinct events are displayed. The high separation of the conditional distributions of the observations in the model of [6] is the reason for the flawless state detection and no transmission losses shown in that table.

Simulations showed that the processing time involved in parameter and state estimations makes the HBMM and NS-HMM models suitable for real-time applications, discussed next.

TABLE III

CHANNEL SHARING EVENTS OCCURRENCE (%).

Model-Algorithm	**Collision**	**TX Ok**	**TX Loss**	**Ch Busy**
NS-HMM-Wind. Baum	4.8	22.4	0	72.7
HBMM-Wind. Baum	3.7	22.4	0	73.8
HBMM-Rydén	4.6	22.4	0	73.0
HBMM-Stiller-Radons	4.1	22.4	0	73.5

V. REAL-TIME APPLICATION

As a demonstration of the real-time application potential, the evaluated models were used to estimate public safety RF channels occupation during 2016 Olympic and Paralympic Games in Rio, Brazil. The Brazilian's government operates in this band a P25 Trunked Radio System as a part of the National Critical Communications System [11]. A sample of the captured signal spectrum can be seen in Figure 4, which shows 17 potential licensed channels for cognitive sharing.

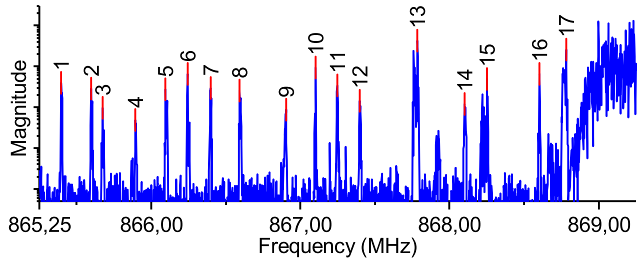

Fig. 4. Captured signal spectrum obtained in September 7, at 18:58 with approximate 2 hours of duration, using the developed framework in LabVIEW and the NI-USRP-2901 SDR equipment in the center frequency of 867.25MHz with 4MHz bandwidth.

The power observation sequence for each channel was extracted from the raw signal FFT at 0.5 samples per second, integrated within the 25KHz band. As an example of the modelled data, Figure 5, shows the observation sequence for channel 1 and its histogram.

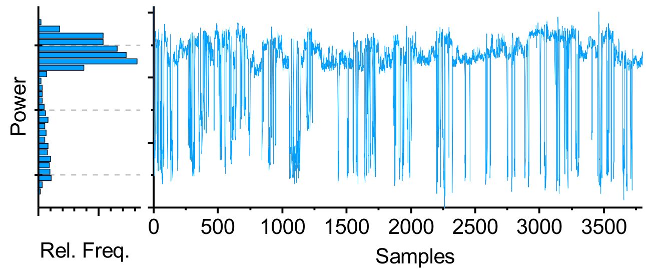

Fig. 5. Channel 1 power observations and relative frequency count.

Since the actual occupation state sequence is unknown, empirical state duration distributions were obtained using a simple Energy Detector, as described in [6], showing occurrences of lengths up to 50 time units. To calculate state estimation errors, the empirical state sequence was assumed to be real[1]. HBMM and NS-HMM models with 2 states and

[1]This was observed to be very accurate in situations with small overlap between the conditional observation distributions like in the captured samples, on the basis of additional simulations, as reported in [8].

order 50 were used to estimate the channel state. The window size used for Rydén and Baum algorithms was 400, which is more than 20 times larger than the average state duration. The resulting state dwell time distributions for channel 1 are shown in Fig. 6.

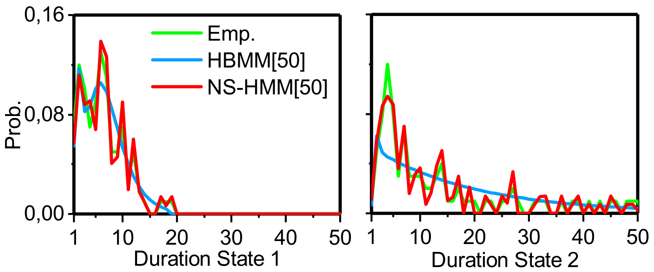

Fig. 6. Estimated state duration distributions.

Each parameter and state estimation iteration took approximately 90ms using HBMM with windowed Baum algorithm and 40ms with the NS-HMM. With 2 second sampling period, it was possible to model all 17 channels simultaneously.

Since Rydén algorithm complexity grows by the 5th power of the model order [9], it was proven to be impractical in this scenario (an iteration took hundreds of seconds). Stiller-Radons algorithm iterations took approximately 400ms. Table IV summarizes the results.

The NS-HMM model and windowed Baum estimator proposed in [8] was found to be the most time efficient and less harmful combination. In conjunction with the channel reuse scheme applied in parallel to all channels this produced successfully transmissions in 21.9% of the total time. Possible collisions were observed in 7.2% of the interval. Figure 7 shows the spectrum sharing events over time. Channels 3, 8, 13 and 15-17 were detected as being always occupied.

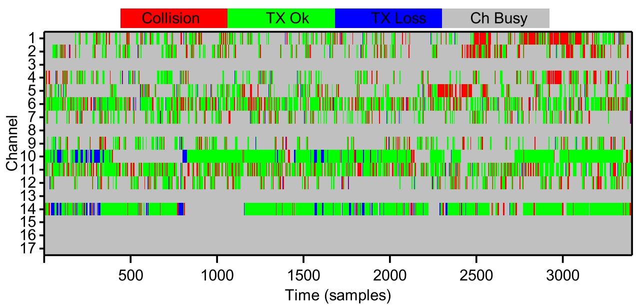

Fig. 7. Proposed spectrum reuse results for NS-HMM with order 50.

Since state detection error probabilities are usually lower than the produced in prediction, collision were most observed at the end of transmissions, indicating that the channel idle durations were overestimated. This issue can be addressed using less aggressive sharing schemes.

Channels 10 and 14 exhibit very long state durations and therefore would require unpractical model orders. The use of order values lower then necessary produced lower detection accuracy and higher transmission loss, as above observed.

TABLE IV
REAL-TIME SPECTRUM SHARING EVENTS OCCURRENCE (%).

Model-Algorithm	Collision	TX Ok	TX Loss	Ch Busy
NS-HMM-Wind. Baum	7.2	21.9	1.4	69.5
HBMM-Wind. Baum	7.9	22.2	1.1	68.8
HBMM-Stiller-Radons	8.0	22.9	0.4	68.7

VI. CONCLUSIONS

The application of HBMM and NS-HMM models in real-time spectrum sensing was tested, showing the possibility of sensing several channels in parallel. A simple spectrum reuse scheme here proposed exhibited good potential for enabling successful secondary use of a large fraction of the available spectrum.

Recursive parameter estimation algorithms, although being in principle more suitable to online applications, showed similar processing time and larger state estimation errors probabilities, while the windowed Baum algorithm was verified to be a good candidate for such application.

In future works these models and algorithms will be tested in association with Neymann-Pearson state detectors to further reduce collisions or interference to the primary user.

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