Power Spectrum Detection Using Clustering

Luiz Paulo de A. Barbosa, Edmar C. Gurjão, Francisco M. de Assis

***Abstract*— Spectrum detection is the basic tool to permit cognitive radio to utilize an empty channel and opportunistically transmit. Considering the sparse utilization of the frequency spectrum, in this paper we propose the use of k-means clustering algorithm to create an sparse representation of the Power Spectrum Density (PSD) of a received signal, and a method to extract the spectral information from it. Preliminary results show the possibility of to identify the occupied channels using this sparse representation followed by some simple processing. The proposed method have low complexity, and under proper conditions it can achieve approximately 99% of correct channel detection on average.**

***Keywords*— Spectrum, Detection, Clustering, Sparse, Representation.**

I. INTRODUCTION

New systems and applications like 5G, Internet of Things (IoT) and others based in wireless transmission, pushed back the discussion about frequency spectrum utilization. In Cognitive Radio (CR) [1], [2] such discussion has evolved considering that a channel not assigned to or not in use by a Primary User (PU) can be opportunistically explored by Secondary Users (SU). Such utilization needs techniques for spectrum detection, i.e., methods to analyze and to detect if a given channel is free at a given moment.

In CR, spectrum detection [3] is the basic tool to permit a secondary user to utilize an empty channel and opportunistically transmit. Several spectrum sensing techniques have been proposed [4], [5], such as energy detection method [6], which compares the energy measured in the channel with a chosen threshold in order to decide about channel occupancy. However, this method has a poor performance on low Signal to Noise Ratio (SNR) regime. Searching for a signal characteristic or statistical characteristics in the received signal Signal Feature and Cyclostationary Methods [7] respectively give better performance that Energy Detection. Signal feature implies to know exactly the characteristics of the transmitted signal and Cyclostationary has a high complexity. Variations of these methods have been proposed, and it can be observed that great part of these methods extract characteristics of the raw received signal.

Observing the frequency representation of the received signal as the Power Spectrum Density (PSD) produced by measurement equipment, like Spectrum Analyzers [8], it can be noticed that the PSD of a received signal can be modeled as a sparse signal. For example, consider the illustration of a set of active channels, Figure 1, represented with rectangles to depict their bandwidths and relative power. In this graphic representation it is clear that only a fraction of the spectrum is utilized and that several channels with different bandwidths could be transmitting in the spectrum holes. This conclusion could be reached also by processing the PSD of such system, illustrated in Figure 2, by choosing a threshold to represent the noise floor in the PSD, and by keeping every point above this threshold, or better, keeping only their positions in the frequency axis. This would lead to an sparse representation of the PSD, and one that preserves the information about spectrum occupation which is necessary to perform spectrum sensing in CR. In this paper we propose a method to extract and to process an sparse representation of a PSD in order to detect the channels in use. The proposed method presents a good performance, as shown by the analysis performed on simulation results.

The rest of this paper is organized as follows, in Section II the system model is presented and in Section III the proposed channel detection method is described. In Section IV the experimental procedure to test the proposed method is presented, and in Section V the obtained results are presented and discussed. Conclusion are drawn in Section VI.

II. SYSTEM MODEL

The system model is composed by a set of transmitters $x_i(t)$, $i = 1, 2, ..., N$, transmitting in channels with central frequencies f_{ci} and bandwidths B_i. Transmitters have different bandwidths and in certain moment they can be off, so the associated channel is free. Signal at receiver is given by

$$s(t) = \sum_{i=1}^{N} \alpha_i x_i(t) + n(t) \tag{1}$$

where α_i is the channel attenuation and $n(t)$ the Additive White Gaussian Noise with $\mathcal{N}(0, \frac{\sigma_y}{10^{(SNR/20)}})$ for SNR in dB, and σ_y denoting the standard deviation of $y(t) = \sum_{i=1}^{N} \alpha_i x_i(t)$.

The i-th signal transmitter is modeled by a chirp signal, given by

$$x_i(t) = \cos(2\pi f_i(t)t), \tag{2}$$

where

$$f_i(t) = f_{0i} + \frac{(f_{1i} - f_{0i})}{T} t \text{ and } i = 1, 2, \ldots, N. \tag{3}$$

Particularly, defining $f_{ci} = (f_{0i} + f_{1i})/2$ and making N = 9, one possible choice of frequencies can be 50, 100, 175, 225, 300, 375, 400, 425, and 475 kHz with associated bandwidths B_i of 25, 25, 50, 25, 100, 10, 10, 10, and 50 kHz. In Figure 1, an illustration of this configuration with rectangles representing the possible active channels, their bandwidths and

Luiz Paulo de A. Barbosa, Department of Computer and Technology at Federal University of Rio Grande do Norte - UFRN, Caicó - RN, Brazil and PhD Candidate at Postgraduate Program in Electrical Engineering, PPgEE/UFCG, Edmar C. Gurjão, Francisco M. de Assis, Department of Electrical Engineering at Federal University of Campina Grande - UFCG, Campina Grande - PB, Brazil, E-mails: luizpaulo@dct.ufrn.br, {ecandeia, fmarcos}@dee.ufcg.edu.br.

relative power is presented. An example of estimated PSD at the receiver, computed using the Welch method [9], is shown in Figure 2.

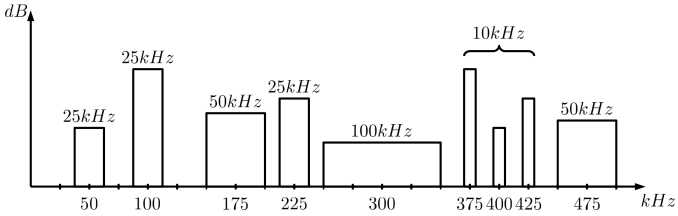

Fig. 1. Illustration of a possible choice of active channels for N = 9.

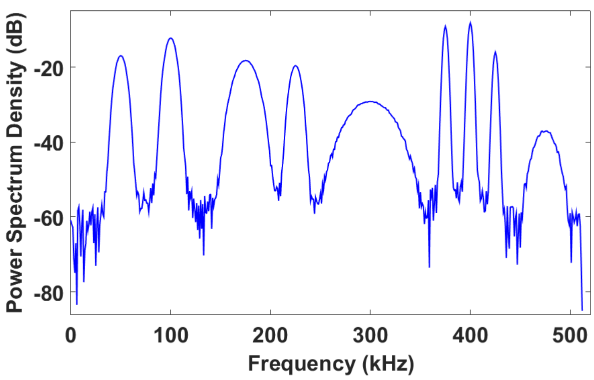

Fig. 2. PSD graph of a possible choice of active channels with N = 9 and SNR = 40 dB. Indication of the magnitude of the vertical axis in dB mean $10\log(W/Hz)$.

Channels with different bandwidths consider a receiver with a frequency range over various fixed channel bandwidths systems. This assumption is based on receivers for Software Defined Radio systems like Universal Software Peripheral Radio (USRP) [10], which are designed for RF applications from DC to 6 GHz, including multiple antenna systems. Also, modern Spectrum Analyzers using parallel signal acquisition can simultaneously cover a great range of frequencies.

One of the main motivation to spectrum reutilization in CR was low spectrum utilization by assigned users, or primary users, in a certain area. The PSD of such spectrum can be modeled as a sparse signal, and in next section we propose a method that uses a sparse representation and processing of such PSD to detect occupied channels.

III. Channel Detection Method

The proposed method is based on processing the PSD at the receiver. To detect occupied channels, initially k-means algorithm [11] is applied to cluster the PSD points, using only the magnitude values of the vertical axis, and therefor producing a set $C = \{c_j \mid c_j$ is the center point of cluster $j,\ j = 1, 2, \ldots, k\}$. In sequence, the elements of C are used to define an adjustable threshold, $\eta(C)$, to serve as reference for the noise floor

$$\eta(C) = c_{min} + \beta \left| \frac{(c_{min} + c_{max})}{2} \right|, \qquad (4)$$

where c_{min} and c_{max} are the minimum and the maximum elements of set C, and β and adjust parameter discussed below.

The PSD points with magnitude above $\eta(C)$ are automatically marked and an sparse representation of the PSD is obtained. Spectrum analysis is performed by processing the sparse representation.

A. PSD Sparse Representation

The sparse representation is an index vector I with ones at the marked positions. i.e. points above the threshold $\eta(C)$ in the PSD, and zero elsewhere. Such points are shown in the top of Figure 3, in green, where 0 means zero value in the index set. By processing I it is possible to identify the occupied channels and to estimate their central frequencies.

In Figure 3 the orange dashed horizontal line is the adjustable threshold $\eta(C)$ and the other horizontal dashed lines, in black and magenta, indicates respectively whrere c_{min} and $\left|\frac{(c_{min}+c_{max})}{2}\right|$ are in the vertical axis. The green circles are the automatically selected points used to create the sparse representation and the blue vertical dashed lines are the estimated central frequency $\hat{f}_{ci}$ for each detected channel.

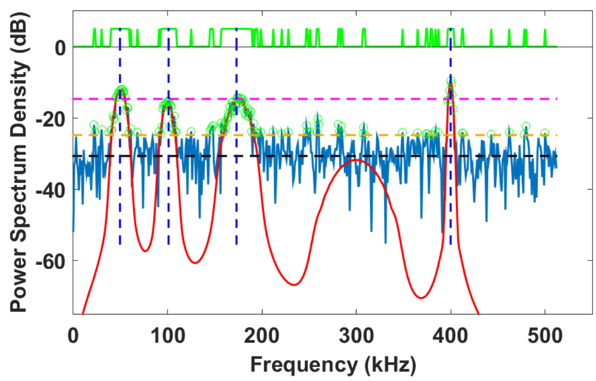

Fig. 3. Example of low SNR case illustrating band rejections parameter γ and threshold adjustment parameter β. The dashed horizontal line, in orange, is the adjustable threshold $\eta(C)$. The black and magenta horizontal dashed lines indicates respectively where c_{min} and $\left|\frac{(c_{min}+c_{max})}{2}\right|$ are in the vertical axis. The green circles are the selected points. Above the 0dB line a sparse representation I of the PSD is shown, also in green. The vertical dashed lines, in blue, are the estimated central frequency $\hat{f}_{ci}$ for each detected channel. In solid red, is the original signal $y(t)$ and in solid blue $s(t) = y(t) + n(t)$ is represented.

To extract information about spectral occupancy from the sparse representation, the sequences of nonzero values (in this case ones values) in it were analyzed. For the i-th sequence, its length or distance d_i with $i = 1, 2, \ldots, D$, between the first and last nonzero values are computed, and *a priori* it is the i-th detected channel. In order to eliminate the influence of spurious sequences and to optimize performance an adjustable rejection parameter γ is introduced. It represents a minimum size of d_i to detect a channel and declare it as occupied. Therefore, if a distance d_i is less than γ, these sequences are ignored by the method. For the remaining ones the central frequency of each channel $\hat{f}_{ci}$ is estimated using the location of the central point of the sequence. This procedure is illustrated in Figure 4, where for the two sequences in the sparse representation, only the first will satisfy the criteria $d_i \geq \gamma$ and the correspondent channel will be declared occupied.

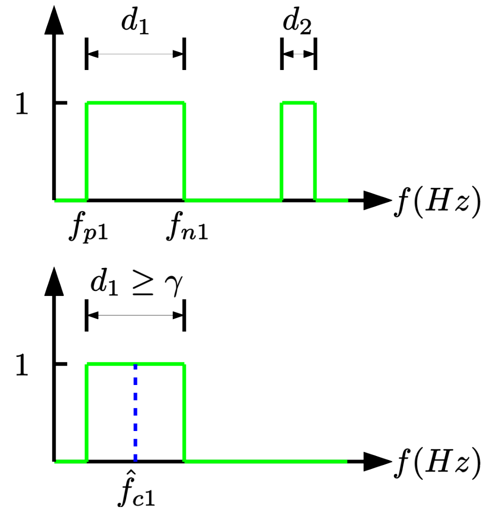

Fig. 4. Example of how the distances d_i and parameter γ are used to implement the rejection of spurious sequences. The vertical dashed line, in blue, is the estimated central frequency $\hat{f}_{c1}$ for the detected channel.

Rejection of sequences with less than γ elements is important in low SNR regime and a given choice of β, since noise components can exceed the threshold $\eta(C)$ and consequently appear as spurious sequences in the sparse representation.

The effect of sequence selection in the sparse representation is illustrated in Figure 3, where the effect of low SNR present itself as various spurious sequences been selected and projected onto I. By properly adjusting γ this effect is minimized in the processing of the sparse representation leading to the correct detection and frequency estimation of all 4 detectable channels. A fifth channel centered at 300 kHz is originally present in $y(t)$, in red, but the choice of its attenuation factor α and low SNR combined, made this channel undetectable on $s(t)$, in solid blue, by the proposed method. This phenomenon is know as the Hidden Primary User (HPU) problem in the context of CR [12]. In Figure 5 the sparse representation of Figure 3 obtained after discarding the spurious sequences is shown.

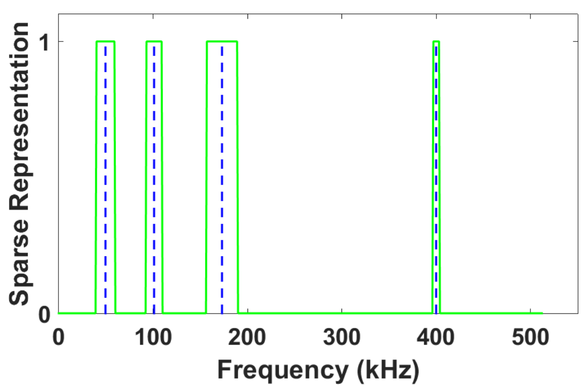

Fig. 5. Detail of sparse representation of Figure 3 after discarding spurious sequences. The vertical dashed lines, in blue, are the estimated central frequency $\hat{f}_{ci}$ for each detected channel.

IV. Experimental Procedures

For experimental purpose, the number of possible channels was set to $N = 9$ and we consider 5 primary users active at time. Considering that all possible combinations of $\binom{9}{5}$ of these channels are used, we obtained a total of 126 examples, all under the same SNR. The choice of PUs is based on the model described in Section II which is also presented in Figure 1.

The signal at the receiver is simulated by

$$s(t) = \sum_{i \in \Gamma} \alpha_i x_i(t) + n(t), \tag{5}$$

where Γ is an index vector indicating the 5 active primary users and parameters $\alpha_i \in (0, 1]$ are uniformly distributed. For $x_i(t)$, Equation (2), the frequency limits were set as $f_{0i} = f_{ci} - B_i/2$ and $f_{1i} = f_{ci} + B_i/2$ and to achieve a target SNR, specified in dB, an additive noise component $n(t)$ with distribution $N(0, \frac{\sigma_y}{10^{(SNR/20)}})$ is used. Finally, Welch method is applied to estimate the power spectral density of signal $s(t)$, Equation (5).

For each processed PSD the detection method returns a list with the estimated central frequencies $\hat{f}_{ci}$, and the corespondent channel is declared a Detected Bandwidth (DBW). Ideally these list is supposed to have five components, one for each active channel, and the estimated frequencies are expected to be contained within the limits of the bandwidths B_i of the active channels. When the detected bandwidth coincides with an active channel, such band is declared as a Matched Bandwidth (MBW). It is important to clarify that this match verification is only possible in simulation, and it is done to evaluate the performance of the method. In this scenario, we can obtain:

1) number $DBW \leq 5$, and all declared as MBW;
2) mismatch between the number of DBW and MBW.

Based on the above situations, the metrics used to access the proposed method performance are:

- μ_{DBW}, mean of the quantity of DBW and its correspondent in percentage computed relative to number of active channels, in this case 5 active channels;
- μ_{MBW}, mean of the quantitiy of MBW and its correspondent in percentage computed relative to number of active channels, in this case 5 active channels;
- the ratio ρ between μ_{MBW} and μ_{DBW} in percentage;

To compute the results in Tables I, II, and III, data was collected for SNRs of 5, 10, and 15 dB . For each SNR, empirically adjusted non optimized parameters β assuming values of 0.3, 0.4 and 0.5 and γ with possible values of 4 and 5 were tested. For all experiments, k-means algorithm was configured with parameter $k = 5$.

V. Results

The number of active channels is $Z = 5$, then, in the best case $MBW = 5$, however, depending on the SNR, spurious sequences due to noise can be marked by the proposed method, and DBW can be greater than Z. When $DBW > Z$ the proposed method will declare the excess channels as in use producing false alarms. Still, the main focus of this work is on the unused channel declaration, and the occurrence of type II errors will be assessed in this paper by the $MBW < 5$. To assess reliability of the method the measure $\rho = \mu_{MBW}/\mu_{DBW}$ is used to indicate the percentage of DBW that was indeed an active channel.

For threshold parameter $\beta = 0.5$ and rejection factor $\gamma = 5$, considering the performance relating Detected Bands (DBW) and Matched Bands (MBW), given by $\rho = \mu_{MBW}/\mu_{DBW}$, it was obtained 99.38, 97.79 and 94.94 % of correct DBWs for SNRs of 15, 10 and 5dB respectively. However, these configuration does not promote the best results in terms of μ_{DBW}, as ρ express the relation of the average number of DBWs and MBWs and it can occur with μ_{DBW} and μ_{MBW} well below Z.

By choosing parameters $\beta = 0.3$ and $\gamma = 4$ the method is able to detect more bands improving μ_{DBW} in exchange for a small decrease in the values of ρ. This configuration can represent a good trade off between ρ and μ_{DBW} with ρ never below 90 % of correct DBW considering all tested conditions which indicate a satisfactory reliability.

To explore the distribution of outcomes for this configuration 50 sets of 126 examples with $\beta = 0.3$ and $\gamma = 4$ where decomposed in terms of DBW, MBW and $DBW - MBW$ and averaged. The percentages are the proportions, computed relative to 126, that an outcome have a certain value of DBW, MBW and $DBW - MBW$. The SNR of 10 dB was selected because it approximates the mean of parameter ρ for this configuration.

In Figure 6 the distribution of outcomes in terms of Detected Bands (DBW) is presented. It can be observed that 88.57 % of the outcomes have DBW between 3, 4 and 5 the specific proportions are respectively 10.97, 36.38, 41.22 %. Also, outcomes with 6, 7 and 8 DBW are present in 10.49 % of cases, what implies in to consider more channels in use than it really are. Similarly in Figure 7, 99.02 % of the outcomes can be attributed to 3, 4 and 5 MBWs in the following proportions 12.03, 41.37 and 45.62 % in this order.

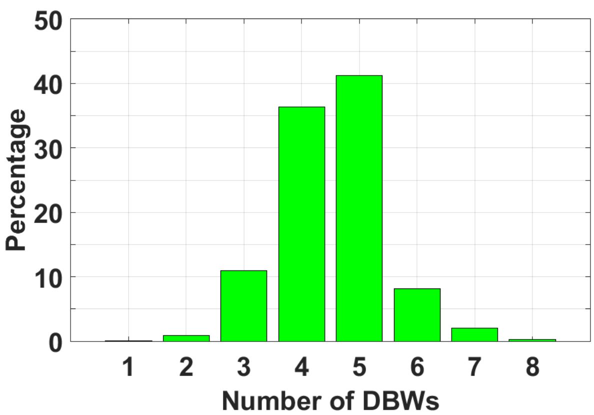

Fig. 6. Decomposition in terms of number of DBWs.

Figure 8 presents the difference between DBWs and MBWs. It can be seem that for 83.38 % of the outcomes the number of DBW is equal to the number of MBW, with the remaining 16.62 % accounting for an excess of 1, 2, or 3 DBW. This excess can be associated with a detection of

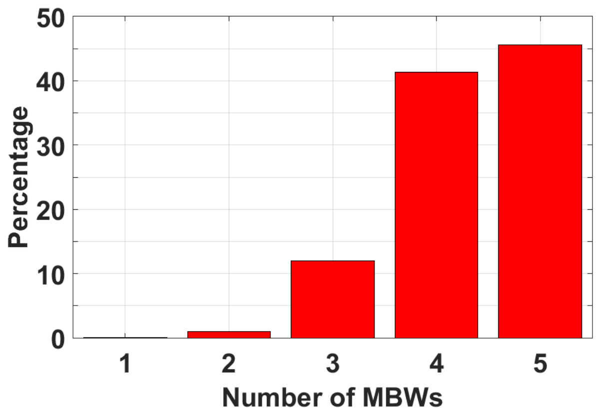

Fig. 7. Decomposition in terms of number of MBWs.

multiples bands within a single channel bandwidth or with incorrect detection as an effect of SNR and other factors.

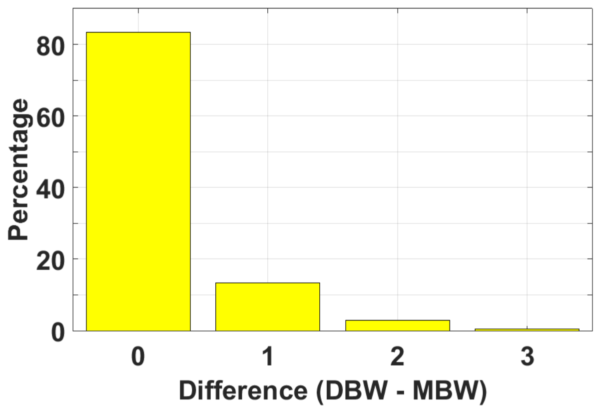

Fig. 8. Decomposition in terms of the difference between DBWs and MBWs.

As mentioned in Section IV, the parameters β and γ were chosen empirically and better results could have been achieved with a more deep investigation of their influence in the performance of the method.

Furthermore, it can be observed in Figure 3, that the sparse representation of the PSD, superior portion of the figure over 0dB line, in green, contains the information of spectrum occupancy.

TABLE I
AVERAGE VALUES FOR THE THE NUMBER OF DBW AND MBW FOR $SNR = 15dB$.

	SNR=15dB					
γ	$\gamma = 4$			$\gamma = 5$		
β	0.3	0.4	0.5	0.3	0.4	0.5
μ_{DBW}	4.55	4.38	4.21	4.44	4.28	4.07
$\mu_{DBW}(\%)$	90.94	87.61	84.11	88.71	85.68	81.44
μ_{MBW}	4.46	4.31	4.15	4.39	4.25	4.05
$\mu_{MBW}(\%)$	89.23	86.27	86.27	87.77	85.06	80.94
$\mu_{MBW}/\mu_{DBW}(\%)$	98.13	98.47	98.77	98.94	99.28	99.38

TABLE II
AVERAGE VALUES FOR THE THE NUMBER OF DBW AND MBW FOR $SNR = 10dB$.

	SNR=10dB					
γ	$\gamma = 4$			$\gamma = 5$		
β	0.3	0.4	0.5	0.3	0.4	0.5
μ_{DBW}	4.52	4.29	4.08	4.32	4.14	3.90
$\mu_{DBW}(\%)$	90.38	85.74	81.57	86.35	82.72	77.97
μ_{MBW}	4.32	4.13	3.94	4.20	4.03	3.81
$\mu_{MBW}(\%)$	86.32	82.54	78.89	83.98	80.63	76.25
$\mu_{MBW}/\mu_{DBW}(\%)$	95.50	96.26	96.71	97.26	97.47	97.79

TABLE III
AVERAGE VALUES FOR THE THE NUMBER OF DBW AND MBW FOR $SNR = 5dB$.

	SNR=5dB					
γ	$\gamma = 4$			$\gamma = 5$		
β	0.3	0.4	0.5	0.3	0.4	0.5
μ_{DBW}	4.46	4.17	3.83	4.16	3.86	3.49
$\mu_{DBW}(\%)$	89.30	83.36	76.61	83.10	77.27	69.76
μ_{MBW}	4.02	3.81	3.54	3.88	3.63	3.31
$\mu_{MBW}(\%)$	80.46	76.17	70.90	77.62	72.64	66.23
$\mu_{MBW}/\mu_{DBW}(\%)$	90.10	91.37	92.55	93.41	94.02	94.94

VI. CONCLUSIONS

In this paper we proposed a method for spectrum detection based on sparse representation of the power spectrum density in a receiver. Considering a spectral model where channel have different bandwidths, and suffer from different attenuation, obtained results shows the feasibility of the proposed method to detect used channels, the first step for spectrum detection in Cognitive Radio.

The proposed method automatically select the occupied bands using k-means algorithm and tunable thresholds. Obtained results shows that under proper conditions more than 99% of correct channel detection can be achieved.

As future works a cooperative approach with nodes in a CR network sharing the sparse representation of the spectrum will be investigated. Additionally, new techniques and algorithms will be investigated in an effort to improve spectral occupancy detection.

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