# A Factor Graph Approach to Nonuniform Power Distribution

Igor M. Guerreiro, Dennis Hui and Charles C. Cavalcante

*Abstract*— **This work addresses distributed processing methods to precoder selection in wireless systems. To explore the spatial correlation of the medium, an additional step of nonuniform power distribution over data streams is proposed. The min-sum algorithm in factor graphs is then applied to reach a near-optimal solution. The system capacity is optimized in a distributed manner assuming that it is simply the sum of individual sum rates. Also, spatially-correlated MIMO channel matrices drawn from measured data are considered. Evaluations on the potential of weighted precoder are provided and its performance is compared to the case with uniform power distribution. Simulation results are presented and discussed. As expected, the min-sum algorithm obtains gain in system capacity over a baseline greedy approach.**

*Keywords*— **Factor graphs, distributed processing, precoder selection, MIMO systems.**

## I. INTRODUCTION

Factor graph and the associated sum-product algorithm have been widely used in probabilistic modeling of the relationship among inter-dependent (random) variables or parameters. There are numerous successful applications [1] including, most notably, various fast-converging algorithms for decoding low-density parity check (LDPC) codes and turbo codes, generalized Kalman filtering, fast Fourier transform (FFT), etc. Similar (but different) applications of factor graphs have also been recently proposed for the problem of fast beam coordination among base-stations in [2]–[5]. The basic idea in those works is to model the relationship between the local parameters to be coordinated among different communication nodes of a network and their respective performance metrics or costs using a factor graph [1]. In [2], [3], the belief propagation algorithm is adopted to solve the downlink transmit beamforming problem in a multi-cell multiple-input-multiple-output (MIMO) system for single data stream transmission, considering a one-dimensional cellular model in [2] and a hexagonal cellular model in [3]. Moreover, in [4], [5] some message-passing algorithms (including the sum-product algorithm) are deployed to coordinate parameters of downlink beamforming in a distributed manner in a multi-cell single-input-single-output (SISO) system.

In this work we address the problem of distributed precoder selection along with nonuniform transmit power distribution over data streams. Different from the work in [3], we assume multi-stream precoders to be selected from a pre-defined codebook and also spatially correlated MIMO channels. The distributed approach is founded on the min-sum algorithm in factor graphs [1], [6]. Besides, different from our work in [6], we propose the use of a weighting matrix to allocate different power levels to data streams. Such a matrix is combined with precoding matrices in the 3GPP long-term evolution (LTE) codebook [7]. The nonuniform transmit power distribution explores the spatial correlation of the MIMO channel where multiple eigenmodes usually have different gains.

The rest of this paper is organized as follows: Section II presents the system model and generally introduces the distributed parameter coordination problem. Then, the case study on nonuniform power distribution is addressed. In Section III, we address the distributed approach based on the min-sum algorithm as a solution to the problem in hand. Some simulation results are presented in Section IV. Finally, we conclude this work in Section V.

## II. SYSTEM MODEL

Consider a multi-cell wireless communication network with several communication nodes. Here, a communication node represents a pair of base-station (BS) and its associated user equipment (UE) in downlink. Let $N$ be the number of cells. Then, assume there are $N$ communication nodes, each one in a cell, sharing a certain set of resources. This condition can be typically obtained with the use of some sort of multiple access technique, such as orthogonal frequency-division multiple access (OFDMA).

The $i$th BS, or simply BS $i$, transmits precoded and spatially multiplexed vector $\mathbf{x}_i$ to the $i$th UE, or simply its associated UE $i$. The vector $\mathbf{x}_i$ is defined as

$$\mathbf{x}_i = \sqrt{\frac{P_T}{N_s}}\mathbf{W}_i\mathbf{s}_i \,, \tag{1}$$

where $\mathbf{s}_i$ is the $N_s \times 1$ spatially multiplexed (SM) symbol vector, $N_s$ is the number of data streams, $P_T$ stands for the transmit power, $\mathbf{W}_i \in \mathcal{W}$ is the $N_t \times N_s$ precoding matrix satisfying

$$\mathrm{tr}(\mathbf{W}_i{\mathbf{W}_i}^H) = N_s \,,$$

and $N_t$ is the number of available transmit antennas. Here, $\mathcal{W}$ is a finite set of all precoding matrices (precoder codebook) available for every communication node in the network.

On the receiver side, the sampled incoming signal vector at th UE $i$ is given as being

$$\mathbf{y}_i = \sqrt{g_{ii}}\mathbf{H}_{ii}\mathbf{x}_i + \sum_{j\in\mathcal{N}_i} \sqrt{g_{ji}}\mathbf{H}_{ji}\mathbf{x}_j + \mathbf{v}_i \,, \tag{2}$$

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where $\mathbf{H}_{ji}$ denotes the $N_r \times N_t$ MIMO channel matrix from BS $j$ to the UE served by BS $i$, spatially correlated[1] and quasi-static over a data block, and $\mathbf{v}_i$ is a zero-mean circularly symmetric complex Gaussian (ZMCSCG) noise vector. The constant $g_{ji}$ is a gain that corresponds to the path loss of each signal, here modeled in a simplified way as being

$$g_{ji} = \left(\frac{1}{d_{ji}}\right)^{\alpha}, \tag{3}$$

where the constant $\alpha$ refers to the path loss exponent and $d_{ji}$ is the distance between the transmitter $j$ and the receiver $i$. The second term on the right-hand side of (2) refers to the interference caused by the neighboring communication nodes, where $\mathcal{N}_i$ stands for the neighbor list of node $i$. For each transmitter, the average transmit power $P_T$ is constant and given by

$$P_T = \mathbb{E}\left\{\mathbf{x}_i{}^H \mathbf{x}_i\right\}. \tag{4}$$

The symbols are assumed to be uncorrelated and have unit average magnitude, which means that $\mathbb{E}\left\{\mathbf{s}_i\mathbf{s}_i^H\right\} = \mathbf{I}_{N_s}$.

### A. Problem Formulation

Let $p_i$ denote a discrete parameter of the communication node $i$, whose value is drawn from a finite set $\mathcal{P}$. Assume that $\mathcal{P}$ is an index set defined as $\mathcal{P} \triangleq \{1, 2, \ldots, |\mathcal{W}|\}$, for all the communication nodes. In order to index the elements of $\mathcal{W}$, a bijective function $f : \mathcal{P} \leftrightarrow \mathcal{W}$ maps the elements of $\mathcal{P}$ onto the elements of $\mathcal{W}$ properly. Thus, each parameter $p_i$ represents a precoding matrix index (PMI) for BS $i$ indicating which precoder from codebook $\mathcal{W}$ that BS $i$ should use at a certain radio resource block to transmit signals.

Each node $i$ is associated with a list $\mathcal{N}_i$ of proper neighbor nodes (i.e. excluding node i) whose choices of parameter values can affect the local performance of node $i$. For convenience, also let $\mathcal{A}_i \equiv \mathcal{N}_i \cup \{i\}$ denote the "inclusive" neighbor list of node $i$. Let $\mathbf{p}_{\mathcal{A}_i}$ denote the vector of those parameters of nodes in $\mathcal{A}_i$, with its ordering of parameters determined by the sorted indices in $\mathcal{A}_i$. Associated with each node $i$ is a performance metric or cost, denoted by $M_i(\mathbf{p}_{\mathcal{A}_i})$, which is a function of those parameters in the list $\mathcal{A}_i$ of node $i$. Each node $i$ is assumed to be capable of communicating with all nodes in $\mathcal{A}_i$.

The local performance metric $M_i(\mathbf{p}_{\mathcal{A}_i})$ represents the negative of the data throughput [8], [9] of node $i$ and is given by

$$M_i(\mathbf{p}_{\mathcal{A}_i}) = -\log\det\left(\mathbf{I} + |g_{ii}|\mathbf{R}_i^{-1}\mathbf{H}_{ii}\mathbf{W}_i\mathbf{W}_i^H\mathbf{H}_{ii}^H\right), \tag{5}$$

where $\mathbf{R}_i$, defined herein as

$$\mathbf{R}_i \triangleq \mathbf{R}_{\mathbf{v}_i} + \sum_{j\in\mathcal{N}_i} |g_{ji}|\mathbf{H}_{ji}\mathbf{W}_j\mathbf{W}_j^H\mathbf{H}_{ji}^H \tag{6}$$

denotes the covariance matrix of the noise-plus-interference experienced by the UE served by BS $i$ in the downlink given that $\mathbf{R}_{\mathbf{v}_i}$ is the covariance matrix of the noise vector $\mathbf{v}_i$.

[1]To obtain more realistic results, each MIMO channel response was drawn from a data set of measured channel matrices acquired by Ericsson Research (see more details in [6]).

Our goal is for each node $i$ to find, in a distributed fashion, its own optimal parameter $p_i^*$, which is the corresponding component of the optimal global parameter vector $\mathbf{p}^*$ that minimize the global performance metric $M(\mathbf{p})$ given by

$$M(\mathbf{p}) \equiv \sum_{i=1}^{N} M_i(\mathbf{p}_{\mathcal{A}_i}), \tag{7}$$

where

$$\mathbf{p} = \begin{bmatrix} p_1 & p_2 & \cdots p_N \end{bmatrix}^T \tag{8}$$

is a vector collecting all the parameters in the network.

### B. Weighted Precoder

The precoder codebook $\mathcal{W}$ is based on the codebook $\mathcal{W}_{\text{LTE}}$ specified by the LTE [7], which is defined as a set of complex weighting matrices for combining the $N_s$ data streams before transmission. However, the codebook $\mathcal{W}_{\text{LTE}}$ for closed-loop SM transmission contains precoding matrices that equally allocates the transmit power $P_T$ over the $N_s$ data streams. As an example, for $N_t = N_s = 2$, the codebook $\mathcal{W}_{\text{LTE}}$ is given by

$$\mathcal{W}_{\text{LTE}} = \left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\j & -j\end{bmatrix}\right\}, \tag{9}$$

where each matrix in $\mathcal{W}_{\text{LTE}}$ is a particular precoding matrix. In this case, it is straighforward to realize that the transmit power $P_T$ is equally distributed because every column of such matrices is a vector whose norm equals 1. Firstly, it is worth noting that all the precoders in (9) are design so that each one achieves the same capacity with different spatial signatures. That is, any value that vector $\mathbf{p}$ takes on will lead to the same value for the global metric $M(\mathbf{p})$, given a channel realization. Therefore, the metric in (5) along with the codebook in (9) are not suitable for precoder selection. For this reason, a subset of the codebook in (9) is defined as being

$$\mathcal{W}_{\text{LTE}}^{(1)} = \left\{\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\right\}, \tag{10}$$

which contains only one precoding matrix.

The idea here is to assign different weights to precoded data streams. The motivation for such idea is to take full advantage of possibly different quality channel responses for each antenna pair. For "better channel" links we could send more information through those since the probability of higher distortion is smaller when compared to the "worse channel" links. In our previous work [6] we have only considered the uniform power allocation case. In general, this assignment of different powers can be performed by multiplying each precoding matrix by a weighting matrix. To reach the optimality, channel knowledge at the transmitter is needed to apply the well-known water-filling solution [10]. Instead, we define a diagonal weighting matrix $\Lambda \in \mathcal{L}$ with fixed weights. The transmission mode is assumed to be fixed so that the precoding matrix rank is always two. Consequently, $N_s = 2$. Taking one in (9), the set $\mathcal{L}$ may be defined such that

$$\mathcal{L} = \left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0\\0 & \sqrt{3}\end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix}\sqrt{3} & 0\\0 & 1\end{bmatrix}\right\}. \tag{11}$$

Now let $\mathcal{W}$ be the codebook of weighted precoding matrices, defined as being

$$\mathcal{W} = \left\{ \mathbf{W}_k \mathbf{W}_l \, : \, \mathbf{W}_k \in \mathcal{W}_{\text{LTE}}^{(1)}, \mathbf{W}_l \in \mathcal{L} \right\} . \tag{12}$$

The resulting codebook $\mathcal{W}$ is, then,

$$\mathcal{W} = \left\{ \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \frac{1}{2} \begin{bmatrix} 1 & \sqrt{3} \\ 1 & -\sqrt{3} \end{bmatrix}, \frac{1}{2} \begin{bmatrix} \sqrt{3} & 1 \\ \sqrt{3} & -1 \end{bmatrix} \right\} . \tag{13}$$

As aforementioned, the precoding matrix $\mathbf{W}_i$ in (1) is drawn from set $\mathcal{W}$.

## III. Solution via Message-Passing

In our problem at hand, the global performance metric $M(\mathbf{p})$ is factorized into a sum of $N$ local performance metrics $M_i(\mathbf{p}_{\mathcal{A}_i})$, which is described in (7). Such factorization can be graphically represented as a factor graph. A message-passing algorithm [1] can be performed in such a factor graph to find the set of marginal functions of (7). In this paper, we apply the variant of sum-product algorithm that is based on the min-sum commutative semi-ring [6], [11], whose elements satisfy the distributive law. In turn, marginal functions can individually be optimized to yield the optimal parameter vector $\mathbf{p}^*$.

A factor graph is a bipartite graph consisting of a set of variable nodes and a set of factor nodes. Specifically, each variable node is associated with a parameter $p_i$ and each factor node with a local performance metric $M_i(\mathbf{p}_{\mathcal{A}_i})$. An edge connecting a factor node with a variable node exists if and only if $k \in \mathcal{A}_i$. We assume a number of communication nodes, each associated with a factor node $v(M_i)$ representing the local performance metric $M_i(\mathbf{p}_{\mathcal{A}_i})$ and a variable node $v(p_i)$ representing the parameter $p_i$. One must note that those graphs may contain loops [6] and unfortunately, the convergence of message-passing algorithms is not guaranteed for loopy factor graphs. Nevertheless, numerical results present in Section IV show the min-sum algorithm for our problem at hand converge in most of the simulation runs.

The min-sum algorithm iterates between two kinds of message computations and exchanges:

1) *Factor node to Variable node*:

$$\mu_{M_i \to p_k}(p_k) = \min_{\mathbf{p}_{\mathcal{A}_i \setminus \{k\}}} \left\{ M_i(\mathbf{p}_{\mathcal{A}_i}) + \sum_{j \in \mathcal{A}_i \setminus \{k\}} \mu_{p_j \to M_i}(p_j) \right\}, \tag{14}$$

where the notation $\setminus\{k\}$ means the underlying operator is performed over all associated variables except to variable $k$. To prevent messages from increasing endlessly, the messages are normalized to have zero mean.

2) *Variable node to Factor node*:

$$\mu_{p_k \to M_i}(p_k) = \sum_{j \in \mathcal{A}_k \setminus \{i\}} \mu_{M_j \to p_k}(p_k), \tag{15}$$

which aggregates all the incoming messages at variable node $v(p_k)$ except to the one from factor node $v(M_i)$.

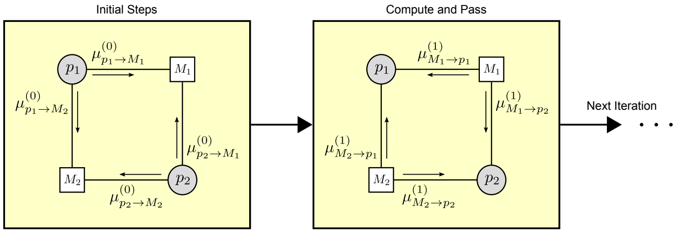


Fig. 1. Graphical model of the initial procedures of the min-sum algorithm for 2-node factor graph. Note that messages have a superscript indicating the current iteration index.

The parameter for communication node $i$ is determined at its variable node $v(p_i)$ by

$$p_i^* = \arg\min_{p_i} \left\{ \sum_{j \in \mathcal{A}_i} \mu_{M_j \to p_i}(p_i) \right\} . \tag{16}$$

The algorithm then iterates until a stopping criterion is reached, either a pre-determined maximum number of iteration $\lambda$ or when the set of parameters computed in (16) converges to a fixed state, that is, the updated messages are equal to the previous computed messages, or equivalently,

$$p_i^{(n+1)} = p_i^{(n)}, \quad \forall i = 1, 2, \ldots, N , \tag{17}$$

for sufficiently large $n$, where $n$ is an iteration index. Figure 1 illustrates part of the message-passing for a simple 2-node loopy factor graph.

Note that both messages computed in (14) and (15) depend only on the value of $p_k$. Since $p_k \in \mathcal{P}$ and $\mathcal{P}$ is assumed to be discrete and finite, each of the messages can be represented by a table of $|\mathcal{P}|$ entries. In particular, the computation in (15) is just adding up the corresponding entries of multiple tables of the same size together.

### A. Signaling Load Analysis

A reasonable way to analyze the signaling load involved in the information exchange is to count the number of real numbers which are exchanged by each node. In the message-passing algorithm, each node sends a number of $|\mathcal{P}||\mathcal{N}_i|$ real numbers and receives the same amount, per iteration. Let $\lambda_{\text{ave}}$ be the average number of iterations until convergence and assume $L = |\mathcal{N}_i|$ tends to be uniform over nodes for large $N$ in hexagon layout and equals $N_{\text{hex}}$. Then, each node exchanges

$$\mathrm{I}_{\text{GB}} \approx 2(\lambda_{\text{ave}} L |\mathcal{P}|) . \tag{18}$$

A greedy technique (e.g. noncooperative game [12]) demands much less information to be exchanged. For instance, each node may exchange only its current own choice (parameter) per iteration, that is, $\mathrm{I}_{\text{G}} \approx 2\lambda_{\text{ave}}$ real numbers. On the other hand, a centralized approach, which demands a central unit to gather the information of every node to perform a joint optimization, causes a high signaling load over the network. Roughly, each node sends its complete local performance metric and receives its optimal parameter afterwards. That is, each node exchanges

$$\mathrm{I}_{\text{C}} \approx |\mathcal{P}|^{L+1} + 1 \tag{19}$$

real numbers. Figure 2 shows the amount of real numbers exchanged per node assuming $\lambda_{\text{ave}}$ equal to five iterations and $1 \leq L \leq N_{\text{hex}} = 6$. Clearly, the function $\text{I}_\text{C}$ increases exponentially and causes more signaling load than the message-passing algorithm for $L > 3$.

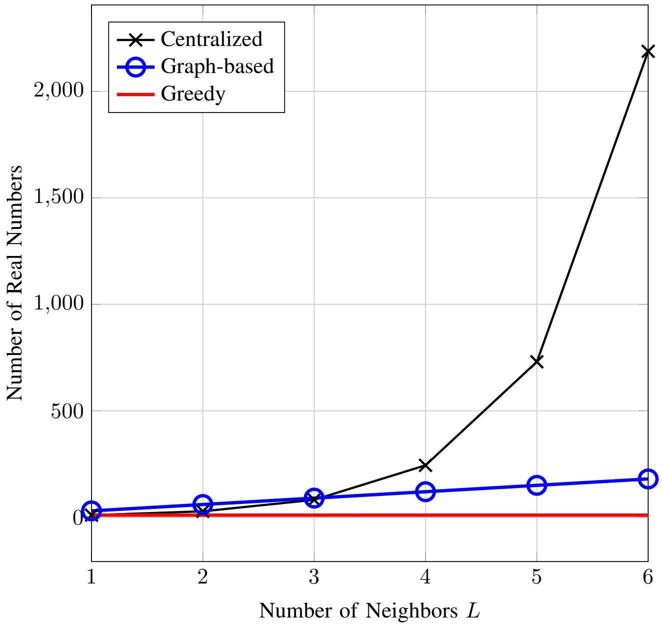


Fig. 2. Signaling load analysis showing the amount of information exchange per node against the number of neighbors. For $L > 3$, a centralized approach causes more signaling load.

## IV. Simulation Results

In this section, the global performance metric defined in (5) in the precoder selection problem is investigated in order to evaluate how it behaves statistically in terms of cumulative distribution functions (CDFs). The graph-based technique is compared with a greedy solution [6], which is expected to provide a sub-optimal result, and with the coordinate descent technique [13], which is expected to provide a near-optimal solution. For the coordinate descent technique, a total of ten iterations was considered as the stopping criterion. Moreover, the 50th CDF percentile of system capacity is evaluated to realize how much gain each distributed technique obtains over the iterations. The convergence speed, inversely proportional to the average number of iterations until convergence per simulation run, of both distributed approaches is qualitatively assessed in terms of CDF curves for only the cases in which the algorithms converge, following (17). Additionally, the convergence rate, defined as the ratio of the number of runs in which the algorithms converge to the total number of simulation runs, is shown for both distributed techniques. A total of 850 runs were conducted for statistical purposes considering a simultaneous message-passing scheduler[2] for both algorithms.

[2]Simultaneous scheduling is based on the flooding schedule [14], where all the nodes participate in the message pass at the same time at each iteration, i.e. all the nodes pass/receive messages to/from their neighboring nodes.

A hexagon layout with $N = 19$ cells and a single communication node in each cell was adopted. The position of each communication node is at random following a uniform distribution. The precoding codebook defined in (13) was used as the parameter set for weighted precoder selection. For uniform transmit power distribution (TPD), the precoding codebook in (9) was considered. The transmit power $P_T$ equals the unity. The MIMO setup is such that each transmitter has $N_t = 2$ available transmit antennas and $N_s = 2$ data streams to be transmitted, and each receiver has $N_r = 2$ receive antennas. Consequently, the parameters to be coordinated are three PMIs. Such parameters index the elements of the codebook in (13). As $|\mathcal{P}| = 3$, messages have three values.

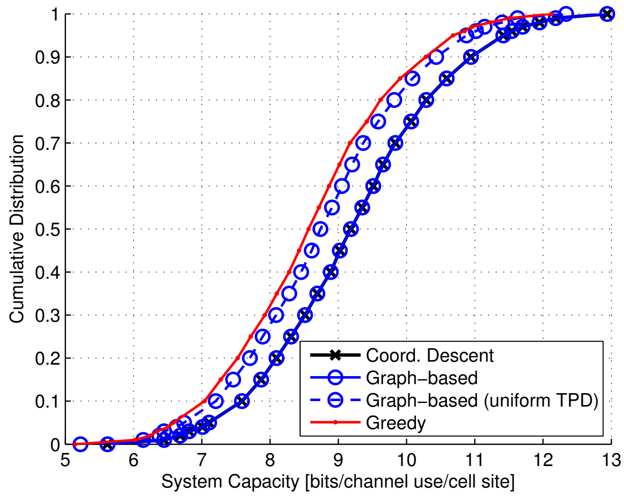


Fig. 3. Performance analysis of graph-based technique to weighted precoder selection in terms of system capacity in 19-node network.

The graph-based technique provides approximately the same performance compared to the coordinate descent technique for the nonuniform TPD case, as can be seen in Figure 3. Also, it clearly outperforms the greedy solution. Figure 3 also shows the significant gain obtained by applying nonuniform TPD against uniform TPD. This is due the effect of transmitting data streams with higher power through the channel links with better quality (smaller power dispersion) instead of selecting all streams with the same power and loosing, on average, about the same performance across all data streams.

At last, Figure 4 shows the CDF of convergence speed for all the methods with nonuniform TPD discussed above. The uniform TPD case is omitted for the sake of simplicity as it does not perform any selection of precoders. Clearly, all the methods demand less than three iterations to converge in 90% of the cases. The greedy approach converges slightly faster than the graph-based method, whereas the coordinate descent technique converges at the second iteration almost in 100% of the simulation runs. All the methods demand less than 8 iterations to converge. At last, both the graph-based and the greedy techniques do converge in 99% of the simulation runs, while the coordinate descent technique converge in 100% of the cases.

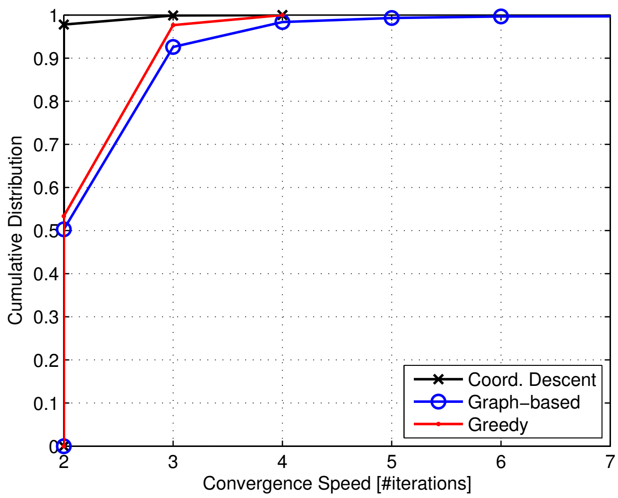


Fig. 4. Performance analysis of graph-based technique weighted precoder selection in terms of convergence speed in 19-node network.

## V. CONCLUSION

In this paper we investigated the min-sum algorithm in factor graphs applied to the problem of precoder selection. Each precoding matrix in the discrete precoder codebook allocates the transmit power over data streams in a nonuniform manner. By doing this, a percentage gain in system capacity is observed as it explores the spatial correlation of the MIMO channel. The graph-based approach outperforms the selfish/greedy technique with the cost of an increased signaling load. On the other hand, it provides a near-optimal solution with decreased complexity and signaling load compared with a centralized, jointly optimized solution. In terms of convergence speed, the graph-based method often converges by the third iteration.

As for perspectives, the application of the min-sum algorithm over continuous variable spaces emerge as an interesting continuation of this work. In the nonuniform TPD problem, one way to make it continuous is to assume precoding matrices whose entries are continuous values. Then, the metric in (5) would be a convex function. The solution provided by the min-sum algorithm would possibly be near the globally optima but reached in an iterative manner. However, one potential issue is the exchange of messages, now continuous functions. In this sense, solutions for continuous-message pass in factor graphs have been investigated. In [15] some remarks on continuous variables are addressed and have proved to be useful for message-passing algorithms.

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