# Performance Analysis of a 2×512 19-PSK MAP Wavelet-Coding Scheme for Multipath Varying Channels

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***Abstract*— In this work we analyze the performance of a new scheme combining 2x512/19-phase shift keying Wavelet Coding with a maximum a posteriori receiver on varying multipath channels in terms of bit-error rate. Considering COST 207-based power delay profiles for urban environments, we find that a maximum a posteriori receiver can provide a gain of nearly 1.5 dB in relation to Euclidean based Wavelet Coded systems on fading channels with low frequency selectivity. We also investigate in which extent a state-of-the-art equation for error probability of Wavelet-Coded systems is valid for varying multipath channels. Our findings show moderate mismatch between those equations and Monte-Carlo simulations, indicating that channel memory should be considered for the formulation of upper-bound solutions. Nevertheless, the results suggest that the wavelet coding shows considerable resilience to channels with memory, making it a suitable candidate for the use of mm-wave frequencies in future generation mobile communications.**

***Keywords*— Wavelet Coding, Multipath, Frequency Selectivity, Radio over Fiber, mm-Waves**

## I. Introduction

As the fifth generation (5G) of mobile communication technology is developed and implemented, worldwide solutions tend to propose the use of much higher frequencies, including the mm-wave range [1], [2]. Such a requirement imposes a critical demand for transmissions that are robust to both varying fading and frequency selectivity, implying the use of diversity schemes that are either highly power hungry, or somewhat compromising to the system's throughput [3]. The wavelet coding (WC) scheme has been proposed in [4] as a diversity strategy with low complex receiver that, most notably, does not add redundancy to the transmitted bit stream.

In this work we evaluate the performance of a 2x512 extended-Haar matrix for WC, not yet investigated in combination with a new 19-PSK (i.e. phase shift keying) modulation scheme using a maximum a posteriori (MAP) receiver. As a channel model, we take into account COST 207-based power delay profiles [5] implemented as a varying tap delay line with Rayleigh statistics. These models suit well with the channel response characteristics of single-input-single-output (SISO) line-of-sight (LoS) mm-wave based radio-over-fiber (RoF) links, as observed in [6]. We also seek to validate the mathematical analysis of the wavelet coding performance that has been proposed in [7]. We find that the state-of-the-art equations for the theoretical evaluation on the BER performance of WC mismatch the simulated results for channels with intense memory.

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## II. Wavelet Coding

Wavelet-Coding enables diversity gains without necessarily diminishing the system's efficiency through the use WCMs. WCMs have arbitrarily long rows, which are orthogonal to each other even when shifted or added. This work focuses on the use of integer flat WCMs. The matrix $A = (a_k^j)$ with dimensions $m \times mg$ (where $m = 2$ and $g_2 = 256$), whose coefficients $a_k^j$ take value in the integer set $\{+1, -1\}$ and satisfies the *modified wavelet scaling conditions* [4].

Let a discrete source generate statistically independent and identically distributed (i.i.d.) bits $x \in \{+1, -1\}$. The wavelet symbol produced by wavelet coding at time $n = pm + q$ is expressed by

$$y_{pm+q} = \sum_{j=0}^{m-1} \sum_{l=0}^{g-1} a_{lm+g}^{j} x_{(p-l)m+j}, \tag{1}$$

and take values in the set $\{-mg, -mg+2, \ldots, 0, \ldots, mg-2, mg\}$ with probability

$$Pr(y_n = 2k - mg) = \binom{mg}{k} 0.5^{mg}, 0 \leq k \leq mg. \tag{2}$$

Within this process, the information represented by an information bit is spread along the transmitted sequence, causing a small part of the signal to contain information about an entire block of data. This process can be used for improving robustness against the combined effects of varying fading and noise bursts [7], [8]. It can be verified that $m$ information bits are encoded in $m$ wavelet symbols and sent during $m$ signaling intervals, thus allowing a spectral efficiency of 1 bit/s/Hz.

At the receiver, the decoder must decide between $m \times mg+1$ levels. In [9] a solution has been proposed for integrating wavelet coding with power-limited systems, resulting in probabilistically shaped, non-uniform constellation diagrams. In order to avoid performance degradation due to constellation crowding, a many-to-one mapping $\mathcal{P}(\cdot)$ of the wavelet symbols generated by Equation (1) is performed onto an 19-PSK signal space, according to Figure 1, as proposed in [8].

The transmitted information bit sequence $x$ at the moment $i = m(g+p)-1$ can be estimated from the received wavelet symbols sequence $y$ by using a bank of $m$ correlators of length $mg$ matched with the $m$ rows of the WCM as

$$z_i^j = \sum_{k=0}^{mg-1} a^j_{(mg-1)-k}(y_{i-k} + e_{i-k}), \tag{3}$$

where $e_{i-k}$ is an integer random variable that denotes the *demodulation noise*. According to [7], the conditional probability of $e_n$ can be expressed as

$$P_r(e_n = e|x_{j+i-(mg-1)})) = \sum_{\epsilon(e)} P_r(\tilde{y}_n|y_n) \times P_r(y_n|x_{j+i-(mg-1)}), \tag{4}$$

in which the original transmitted bits can be estimated as $\hat{x}_{j+1-(mg-1)} = sgn(z_i^j)$. Such a decoding simplicity is stated as one of the main features of the wavelet coding [4]. Further on, according to [7], the bit error rate at the wavelet decoder output can be expressed from Equation 4 as

$$\begin{aligned} P_r(e) =& 0.5mg(\nu_i^j = mg|x_{j+i-(mg-1)=-1}) \\ &+ \sum_{k=1}^{\frac{mg(2mg-1)}{2}} P_r(\nu_i^j = mg + 2k|x_{j+i-(mg-1)=-1}), \end{aligned} \tag{5}$$

considering that: the information bits are equiprobable; the distribution of the channel noise is symmetric; and the interleaving is ideal.

## III. Methodology

Simulations are performed by considering blocks of a million ($10^6$) i.i.d. bits per execution. After wavelet coding according to Equation 1 and further mapping $\mathcal{P}(\cdot)$ (obtained by exhaustive search) as illustrated by Figure 1, the resulted sequence $s$ of values within the $\mathbb{C}$ domain is ideally interleaved

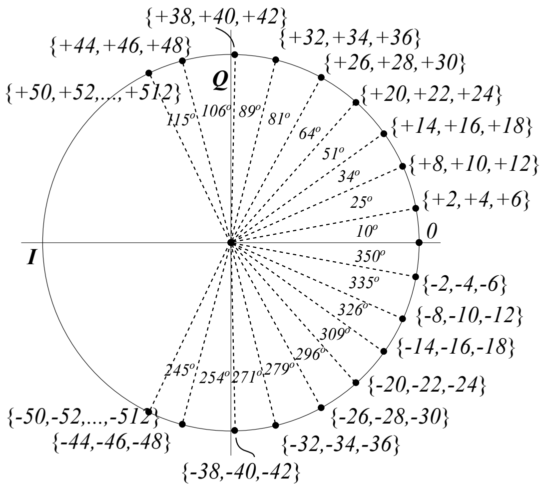


Fig. 1
Signal constellation after mapping $\mathcal{P}(\cdot)$ of 513 wavelet symbols into a 19-PSK constellation.

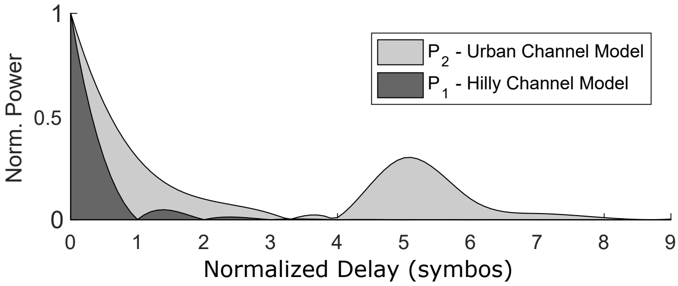


Fig. 2
Normalized power vs. normalized delay (in symbols) for two COST207-based channel models. The darker area represents the hilly scenario ($PDP_1$) whereas the lighter area represents the urban scenario ($PDP_2$).

and transmitted through a channel $h$, to produce a received signal

$$r[n] = \sum_{j=0}^{L-1} p_j c_j[n] s[n-\tau_j] + \eta[n] \tag{6}$$

where $L$ corresponds to the channel's time spreading, $p_j$ and $\tau_j$ are the j-th channel's gain and delay, respectively, and $\eta[n]$ is an additive white Gaussian noise sample. We assume that $h$ is composed of uncorrelated fading samples $c_j[n]$, and that each $c_j$ is a Gaussian stochastic variable modeled as a wide-sense stationary process (WSSUS). In this work we consider two power delay profiles (PDP) derived from the COST207 norm for broadband wireless applications [5]. One of them describes a hilly terrain ($PDP_1$) and the other one describes an urban environment ($PDP_2$), as shown in Figure 2. These models suit well with the channel response characteristics of SISO LoS mm-wave based RoF links, as observed in [6].

The channel state is assumed known for equalization through infinite impulse response (IIR) filtering. The equalizer's output $\bar{s}$ is described as

$$\bar{s}[n] = \frac{1}{p_0 c_0[n]} r[n] - \sum_{j=1}^{L-1} \frac{p_j c_j[n]}{p_0 c_0[n]} \bar{s}[n-\tau_j] \tag{7}$$

where $p_0 c_0[n]$ is the product between the channel's PDP and the n-th fading sample at the moment 0. The complex values $\bar{s}$ are then uninterleaved and delivered to the demodulator for estimation of wavelet levels by either a ML (i.e. maximum likelihood) or a MAP algorithm. Each symbol of the produced demodulated sequence is mapped back into its corresponding wavelet symbol according to Figure1. These symbols are finally decoded by the wavelet receiver according to Equation 3 for computation of bit-error rate (BER) until 100 errors can be accounted. Simulated results are contrasted with Equation 5, in which the conditional probabilities $P_r(\tilde{y}_n|y_n)$ are obtained by Monte Carlo method.

## IV. Results

Figure 3 shows the performance of the studied cases considering a 2×512-19 PSK WC system in terms of BER for levels of $E_b/N_0$ varying from 0 dB to 25 dB. The first thing one

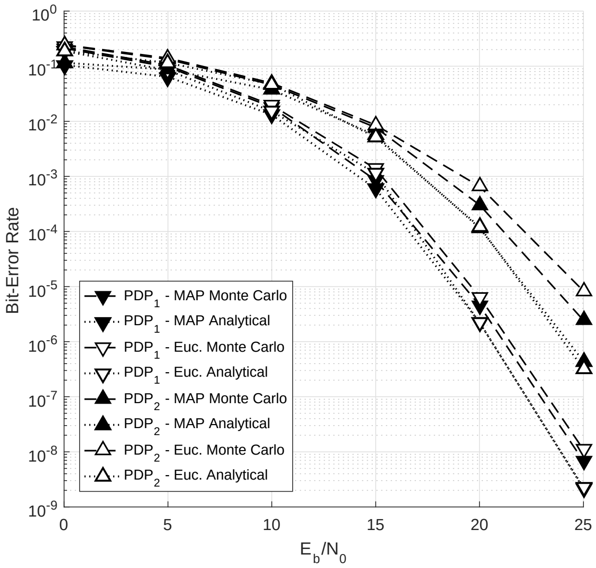


Fig. 3

BER VS. $E_b/N_0$ FOR THE PERFORMANCE OF 2×512-19 PSK WC SYSTEMS. DASHED LINES (--) REPRESENT THE SIMULATED RESULTS, WHEREAS POINTED LINES (··) REPRESENT ANALYTICAL CURVES. FILLED MARKERS (▼/▲) REPRESENT THE USE OF MAP METRICS AT THE RECEIVER, WHEREAS UNFILLED MARKERS (▽/ △) REPRESENT THE USE OF EUCLIDEAN METRICS AT THE RECEIVER. DOWN-TRIANGLES (▼/▽) REPRESENT THE $PDP_1$ SCENARIO, WHEREAS UP-TRIANGLES (▲/ △) REPRESENT THE $PDP_2$ SCENARIO.

should notice here is that, at a BER of $10^{-5}$, the MAP receiver provides a gain of nearly 0.5 dB in relation to Euclidean based systems on fading channels with low frequency selectivity (i.e. $PDP_1$). This is an expected outcome, according to [7], in which 2×128-11 PSK WC systems were evaluated on flat fading channels. The results then revel that higher coding gains can be provided to WC systems operating under superior levels of frequency selectivity regarded that, for the same system, now operating in the $PDP_2$ scenario, an improvement of nearly 1.5 dB within the same BER threshold can be observed on the comparison between MAP based and Euclidean based systems.

Furthermore, the mismatch between analytical and simulated curves drops by 18 % in favor of MAP based systems from the scenario $PDP_1$ to the scenario $PDP_2$, still at a BER of $10^{-5}$. It appears that such a mismatch is caused by the assumption of "wavelet errors being uncorrelated from each other at successive instants" on the formulation of Equation 5, which does not seem to hold anymore in a multipath-rich environment. This behavior can be explained by the increase in the statistical dependence between the random variables $e$, caused by the *memory* of the channel. Thus, it is important to stress that further mismatches should be noticed inasmuch as the propagating channels present more severe levels of frequency selectivity.

## V. CONCLUSIONS

Recently, a strategy for integrating the Wavelet Coding with power-limited systems has been reported, resulting in probabilistically shaped, non-uniform constellation diagrams. Based on that, the formulation of an expression for the error probability of Wavelet Coded systems becomes of extreme importance for the design of optimum constellations by minimization algorithms, as recently demonstrated. In this work we investigate the combination of 2×512-19 phase shift keying Wavelet Coded systems with a maximum a posteriori demodulator, that shows performance gains of 1.5 dB in relation to conventional Euclidean based receivers for channels with high levels of multipath fading. Our findings also show that the state-of-the art equations for theoretical analysis of the Wavelet Coded systems, as proposed in [7], cannot serve as a upper bound for the bit error rate of wavelet systems operating in multipath varying channels due to not considering channel memory on its formulation. Considerable mismatch reduction can be achieved by the use of maximum a posteriori receivers. Yet, future work should investigate how to re-address the hypothesis of a memoryless channel in the formulation of the equations of error probability of Wavelet Coded systems.

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## REFERENCES

[1] Rappaport, T., Sun, S., Mayzus, R., Zhao, H., Azar, Y., Wang, K., Wong, G., Schulz, J., Samimi, M., Gutierrez, F., 'Millimeter Wave Mobile Communications for 5G Cellular: It Will Work!', *IEEE Access*, 2013, p. 335-349.

[2] Cavalcante, L., Silveira, L.F., Rommel, S., Olmos, J.J., Monroy, I., 'On the capacity of radio-over-fiber links at the W-band', *Springer Optical and Quantum Electronics*, 2016, **48(5)**, p. 1-10.

[3] Cho, Y.S., Kim, J., Yang, W.Y., Kang, C.G., 'MIMO-OFDM Wireless Communications with MATLAB', *Chichester, UK, John Wiley & Sons, Ltd.*, 2010.

[4] Tzannes, M. 'Bit-by-bit Channel Coding Using Wavelets', *Proceedings of IEEE GLOBECOM*, 1992, **2**, p. 684-688.

[5] COST207: 'Digital Land Mobile Radio Communications', *Commission of the EC, Final Report*, 1989.

[6] Rommel, S., Cavalcante, L., Quintero, A.G., Mishra, A.K., Olmos, J.J., Monroy, I.T., 'W-band Photonic-Wireless Link with a Schottky Diode Envelope Detector and Bend Insensitive Fiber', *OSA Optics Express*, 2016, **24(11)**, p. 11312-11322.

[7] Silveira, L.F., Silveira, L.G., Assis, F.M., Pinto, E.L., 'Analysis and Optimization of Wavlet-Coded Communication Systems', *IEEE Transactions on Wireless Communications*, 2009, **8(2)**, p. 563-567.

[8] Cavalcante, L., Silveira, L.F., Rommel, S., Olmos, J.J., Monroy, I., 'Performance Analysis of Wavelet Channel Coding in COST207-based Channel Models on Simulated Radio-over-Fiber Systems at the W-Band', *Springer Optical and Quantum Electronics*, 2016, **48(1)**, p. 1-9.

[9] Silveira, L.G., Silveira, L.F., Assis, F.M., Pinto, E.L., 'Soft-Decision Demodulation of Wavelet-Coded PSK Signals over Flat Rayleigh Fading Channels', *IEEE International Telecommunications Symposium*, 2006, p. 758-763.