

Error-Based Accuracy and Convergence Assessment of the DMFT-SSPE Radiopropagation Algorithm

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Abstract—This paper analyzes the influence of the vertical discretization step on the numerical behavior of the DMFT-SSPE algorithm for propagation scenarios at 1 GHz, 10 GHz and 26 GHz. The analysis considers statistical error metrics, numerical stability, and path loss responses for flat terrain and wedge configurations. The results show that similar statistical error levels may produce different propagation behaviors and that instability regions become more frequent at higher frequencies. The study also highlights the sensitivity of the algorithm to discretization variations and to instability regions associated with the “bad alpha” phenomenon.

Keywords—DMFT-SSPE, radio wave propagation, numerical discretization, numerical stability, path loss prediction

I. INTRODUCTION

Reliable prediction of electromagnetic wave propagation remains an important challenge in computational electromagnetics, especially in scenarios influenced by terrain irregularities, atmospheric variations and surface interactions. The increasing complexity of modern wireless communication systems has intensified the need for propagation models capable of providing accurate coverage estimation while maintaining acceptable computational cost. In this context, methods based on the Parabolic Equation (PE) have become attractive alternatives for radiowave propagation analysis due to their balance between computational efficiency and modeling capability [1] [2].

The PE method is derived from an approximation of the scalar wave equation assuming a preferential propagation direction, allowing the field to be evaluated progressively along the path and significantly reducing computational effort compared to full-wave methods. This has made PE-based techniques widely used in long-range propagation, terrain modeling and atmospheric propagation studies [3] [4].

Different numerical approaches may be used to solve the PE formulation. Among them, the Split-Step Parabolic Equation

(SSPE) method stands out for its efficiency in large propagation domains, computing the solution through successive steps in the spatial and spectral domains using Fourier-based operators [4] [5]. The Discrete Mixed Fourier Transform (DMFT) is incorporated to enforce the impedance boundary condition at the propagation surface, improving the modeling of lossy ground effects while keeping computational cost low for large-scale problems [1] [6], and has been successfully applied in several propagation studies due to its ability to represent surface effects within the SSPE framework [5]. This work focuses on the numerical behavior of the DMFT implementation, particularly the influence of the vertical discretization step Δz on accuracy and stability.

The selection of discretization values is commonly based on theoretical criteria related to wavelength and propagation conditions [6]. However, even when similar criteria are satisfied, the resulting path loss curves may differ significantly: some Δz values yield similar statistical error indicators while producing distinct propagation responses, and some configurations remain more stable and physically consistent than others. This indicates that statistical metrics alone may not fully describe the quality of the numerical solution.

Previous studies have reported numerical instability regions in DMFT-SSPE, commonly associated with the so-called “bad alpha” problem [7]. These instabilities arise from specific combinations of discretization and boundary conditions, leading to nonphysical oscillations in the solution.

The need for robust radiopropagation tools capable of accurately modeling current wireless communication channels in Brazil has become more significant in recent years. According to [8], the adoption of higher frequency bands and the expansion of broadband communication systems have become strategic topics for the national telecommunications scenario. In this context, frequencies around 1 GHz remain important for wide-area coverage and rural IoT applications, while higher frequency bands, such as 26 GHz, are directly associated with millimeter-wave and 5G communication systems. Intermediate frequency ranges, such as 10 GHz, also represent relevant scenarios for broadband wireless communication and high-capacity propagation studies. As wireless systems continue to evolve toward higher frequencies and more heterogeneous coverage conditions, the use of robust numerical propagation models capable of providing stable and physically consistent solutions becomes essential.

This work investigates the relationship between the vertical discretization step and the statistical error obtained in DMFT-SSPE simulations, evaluating whether similar error levels are consistently associated with similar path loss behavior across

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different propagation conditions and operating frequencies. To this end, the algorithm is evaluated at 1 GHz, 10 GHz and 26 GHz under successive variations of Δz , identifying stable and unstable regions and their influence on the resulting path loss curves, and providing practical insight into discretization conditions that yield more accurate and physically consistent responses.

Additionally, analytical and deterministic approaches, such as the Two-Ray model and Ray Tracing (RT), are employed as reference solutions for validation and comparison purposes. The robustness of the discretization criterion is assessed through statistical error evaluation and qualitative analysis of the corresponding path loss curves obtained with these reference models.

The remainder of this paper is organized as follows. Section II presents the mathematical formulation adopted in this work. Section III describes the proposed DMFT-SSPE algorithm and its numerical implementation. Section IV discusses the simulation scenarios and the obtained results. Finally, Section V presents the conclusions of this work.

II. FORMULATION

The DMFT is used to introduce the impedance boundary condition into the PE formulation, representing the discrete implementation of the continuous Mixed Fourier Transform (MFT), expressed as [4]:

$$F(p) = \int_0^\infty f(z) [\alpha \sin(pz) - p \cos(pz)] dz. \quad (1)$$

where α represents the impedance parameter associated with the boundary condition at the propagation surface.

The discrete formulation is applied to the auxiliary field $u(x, m\Delta z)$ along the vertical discretization grid, leading to the spectral-domain representation

$$U(x, l\Delta p) = \sum_{m=0}^{N'} u(x, m\Delta z) \left[\alpha \sin\left(\frac{\pi l m}{N}\right) - \frac{\sin\left(\frac{\pi l}{N}\right) \Delta z \cos\left(\frac{\pi l m}{N}\right)}{\Delta z} \right] \quad (2)$$

where the discretization parameters satisfy the relation $\Delta p \Delta z = \pi/N$. The choice of the vertical discretization step Δz is directly associated with the stability and accuracy of the PE solution.

The spatial-domain field is recovered through the inverse DMFT, given by

$$u(x, m\Delta z) = \frac{2}{N} \sum_{l=0}^{N'} U(x, l\Delta p) \frac{\alpha \sin\left(\frac{\pi l m}{N}\right) - \sin\left(\frac{\pi l}{N}\right) \Delta z \cos\left(\frac{\pi l m}{N}\right)}{\alpha^2 + \left(\sin\left(\frac{\pi l}{N}\right) \Delta z\right)^2} \quad (3)$$

For propagation angles smaller than 15° , the Split-Step solution associated with the *Narrow Angle Parabolic Equation* (NAPE) is expressed as

$$u(x + \Delta x, z) = e^{[ik(n^2-1)\frac{\Delta x}{2}]} \mathcal{F}^{-1} \left\{ e^{(-ip^2 \frac{\Delta x}{2k})} \mathcal{F}\{u(x, z)\} \right\}, \quad (4)$$

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ denote the Fourier Transform and the Inverse Fourier Transform, respectively. The spectral variable is defined by

$$p = k \sin \theta \quad (5)$$

with θ corresponding to the propagation angle relative to the horizontal axis.

For larger propagation angles, the *Wide Angle Parabolic Equation* (WAPE) is adopted, whose Split-Step solution is written as

$$u(x + \Delta x, z) = e^{[ik(n-1)\frac{\Delta x}{2}]} \mathcal{F}^{-1} \left\{ e^{\left[\frac{-ip^2 \Delta x}{k} \left(\sqrt{1 - \left(\frac{p}{k}\right)^2} + 1\right)^{-1}\right]} \mathcal{F}\{u(x, z)\} \right\} \quad (6)$$

III. DMFT-SSPE ALGORITHM

The algorithm combines the SSPE method with the DMFT technique to model electromagnetic wave propagation over different propagation scenarios. The overall processing sequence is illustrated in Fig. 1.

Initially, the input parameters required by the model are defined, including the operating frequency, antenna height, terrain properties and discretization parameters. The terrain profile is then discretized along the vertical grid, allowing the numerical representation of both flat and irregular surfaces. Fig. 2 illustrates the spatial discretization adopted in the SSPE formulation, highlighting the propagation steps along the horizontal direction (Δx) and the vertical discretization interval (Δz). For propagation scenarios involving irregular terrain profiles, a coordinate transformation is applied in order to incorporate the surface variations into the numerical propagation model [6].

After the discretization stage, the initial field $u(x, m\Delta z)$ is generated from the antenna radiation pattern. In this work, the source is represented by a Gaussian beam model due to its suitability for two-dimensional propagation simulations.

The propagation procedure is performed iteratively along the range direction. At each propagation step, the field is transformed to the spectral domain through the forward DMFT, where the propagation operator associated with the Parabolic Equation is applied. Subsequently, the inverse DMFT is employed to reconstruct the field in the spatial domain while enforcing the impedance boundary condition at the surface. This procedure is repeated until the maximum propagation distance is reached [6] [9].

Finally, the algorithm generates the electromagnetic field distribution over the propagation domain, allowing the analysis of the behavior associated with different discretization values.

IV. SIMULATION AND RESULTS

The discretization analysis is performed by evaluating different vertical discretization values (Δz) around the theoretical criterion adopted for the DMFT-SSPE implementation. The objective is to investigate the sensitivity of the numerical solution to variations in the discretization step, emphasizing the relationship between statistical error metrics, numerical stability, and the resulting path loss behavior.

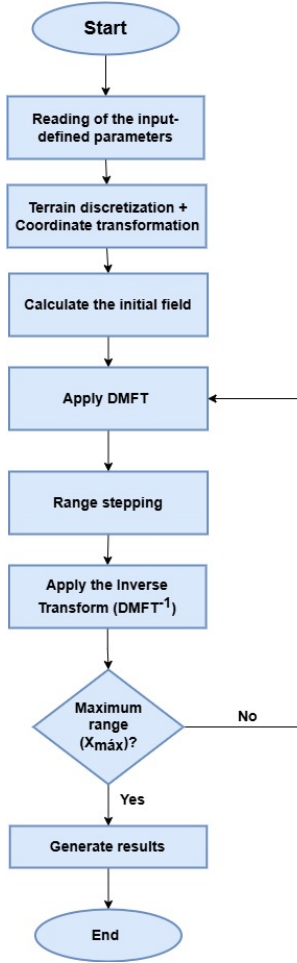


Fig. 1. Flowchart of the algorithm, based on [4].

The vertical discretization criterion is defined as [4]

$$\Delta z \leq \frac{\lambda}{2 \sin(\theta_{\max})}, \quad (7)$$

where λ is the operating wavelength and $\theta_{\max}$ represents the maximum propagation angle considered in the simulation.

The ground impedance conditions adopted in all scenarios correspond to a lossy standard surface with relative permittivity $\epsilon_r = 15$ and conductivity $\sigma = 0.012 \text{ S/m}$. The statistical error between the DMFT-SSPE and the reference solution is quantified by the Mean Absolute Difference (MAD) and the Standard Deviation (SD), both expressed in dB and reported in Tables I, II and III for propagation distances between 500 m and 5000 m. For each frequency, two stable Δz values are compared in Figs. 3–5, chosen for whichever contrast is more informative: values with comparable MAD/SD but visibly different curves, or values with markedly different MAD/SD but similar curves.

A. Lossy Flat Terrain at 1 GHz

In this scenario, the DMFT-SSPE implementation is evaluated at 1 GHz under a flat lossy ground configuration.

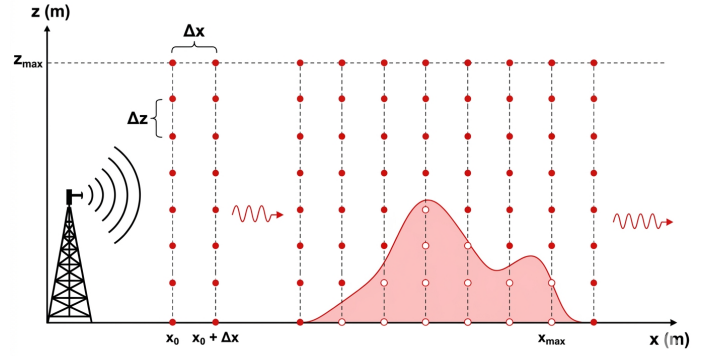


Fig. 2. Spatial discretization adopted in the SSPE propagation model.

The transmitter and receiver heights are fixed at 35 m and 10 m, respectively, and the maximum propagation angle adopted in the simulation is approximately 13° .

The Two-Ray model is employed as the analytical reference for both the statistical error analysis and the comparison of the generated path loss curves. The discretization results for this scenario are summarized in Table I.

TABLE I
DISCRETIZATION ANALYSIS FOR THE 1 GHz LOSSY FLAT TERRAIN
SCENARIO CONSIDERING PROPAGATION DISTANCES BETWEEN 500
METERS AND 5000 METERS.

Δz (m)	Stability Condition	MAD (dB)	SD (dB)
0.40	Stable	7.0772	8.9181
0.41	Unstable	-	-
0.42	Stable	7.6841	9.4579
0.43	Unstable	-	-
0.44	Stable	8.6277	12.0347
0.45	Stable	8.6593	12.7453
0.46	Unstable	-	-
0.47	Unstable	-	-
0.48	Unstable	-	-
0.49	Stable	11.2093	14.2927
0.50	Stable	11.2746	14.2450
0.51	Stable	11.1288	13.8262
0.52	Unstable	-	-
0.53	Stable	10.8014	13.1137
0.54	Unstable	-	-
0.55	Stable	10.6368	12.8201
0.56	Unstable	-	-
0.57	Stable	9.8780	11.6709
0.58	Stable	9.7754	11.4577
0.59	Stable	8.5786	9.8252
0.60	Stable	8.2932	9.4254
0.61	Unstable	-	-
0.62	Stable	7.6874	8.5701
0.63	Unstable	-	-
0.64	Stable	7.2053	8.0052
0.65	Stable	7.4999	8.4114
0.66	Unstable	-	-

The alternation between stable and unstable Δz values in Table I is consistent with the numerical behavior reported in [7] for the central-difference formulation of the DMFT, which is the version implemented in this work. In this formulation, stability depends on the term $r = (1 - \alpha \Delta z/2)/(1 + \alpha \Delta z/2)$ remaining within a bounded region that shifts with Δz . Since the surface impedance α is fixed for a given scenario, only

certain Δz values place r inside this stable region, which explains why $\Delta z = 0.41$ m is unstable while the neighboring values 0.40 m and 0.42 m remain stable.

Among the stable values in Table I, $\Delta z = 0.40$ m and 0.64 m are compared in Fig. 3 for yielding the closest MAD and SD values within the stable set while lying near opposite ends of the tested range, isolating the effect of discretization on curve shape from that of error magnitude.

Although both configurations produce comparable statistical errors, the resulting propagation curves exhibit considerably different behaviors. The $\Delta z = 0.40$ m solution presents stronger oscillations and larger deviations throughout the propagation path, while the $\Delta z = 0.64$ m configuration produces a smoother response and better agreement with the Two-Ray model, especially at longer distances.

The results show that similar global error levels do not necessarily imply similar physical propagation behavior. Additionally, the alternation between stable and unstable solutions for closely spaced discretization values highlights the sensitivity of the DMFT-SSPE formulation to the vertical discretization step.

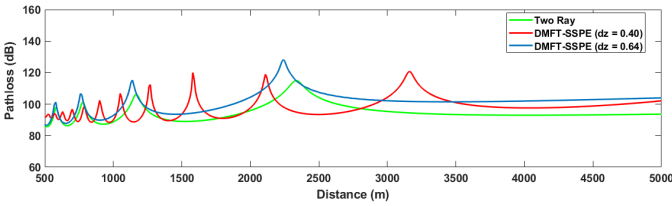


Fig. 3. Comparison of path loss curves obtained for discretization configurations with similar statistical error levels at 1 GHz, including comparison with the Two-Ray reference model.

B. Lossy Flat Terrain at 10 GHz

In this scenario, the DMFT-SSPE implementation is evaluated at 10 GHz under the same propagation configuration adopted in the previous 1 GHz lossy flat terrain case. However, the maximum propagation angle adopted in the simulation is adjusted to approximately 7° .

The Two-Ray model is again employed as the analytical reference, and the obtained discretization results are summarized in Table II.

Fig. 4 compares the path loss curves obtained for $\Delta z = 0.118$ m and $\Delta z = 0.121$ m. These values are adjacent stable points in Table II that nonetheless produce markedly different error levels, which allows the comparison to isolate the effect of a small discretization shift located near an instability boundary.

The $\Delta z = 0.121$ m configuration provides better agreement with the analytical solution over the entire propagation path, presenting smoother oscillatory behavior and reduced deviations at larger distances. In contrast, the $\Delta z = 0.118$ m solution exhibits stronger oscillations and more pronounced discrepancies, which is consistent with its larger MAD and SD values.

Compared to the 1 GHz scenario, a larger concentration of unstable discretization regions is observed at 10 GHz,

TABLE II
DISCRETIZATION ANALYSIS FOR THE 10 GHz LOSSY FLAT TERRAIN
SCENARIO CONSIDERING PROPAGATION DISTANCES BETWEEN 500
METERS AND 5000 METERS.

Δz (m)	Stability Condition	MAD (dB)	SD (dB)
0.103	Stable	6.7136	8.7828
0.104	Unstable	-	-
0.105	Unstable	-	-
0.106	Unstable	-	-
0.107	Unstable	-	-
0.108	Unstable	-	-
0.109	Stable	7.3272	9.5790
0.110	Stable	6.8477	9.0495
0.111	Unstable	-	-
0.112	Stable	6.5052	8.5810
0.113	Unstable	-	-
0.114	Unstable	-	-
0.115	Unstable	-	-
0.116	Stable	7.4365	9.7450
0.117	Unstable	-	-
0.118	Stable	8.2114	10.5526
0.119	Unstable	-	-
0.120	Unstable	-	-
0.121	Stable	4.3916	5.4645
0.122	Stable	5.5963	7.3831
0.123	Stable	6.6845	8.7323

including consecutive instability intervals for closely spaced Δz values. This behavior indicates an increased sensitivity of the DMFT-SSPE formulation at higher operating frequencies.

Since higher frequencies are associated with smaller wavelengths, the discretization criterion becomes more restrictive, favoring the occurrence of instability regions commonly related to the “bad alpha” phenomenon.

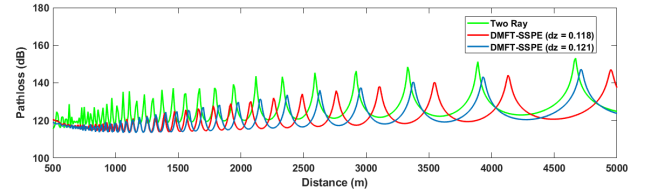


Fig. 4. Comparison of path loss curves obtained for discretization configurations with different statistical error levels at 10 GHz, including comparison with the Two-Ray reference model.

C. Wedge Scenario at 26 GHz

This scenario evaluates the DMFT-SSPE implementation in the presence of a wedge-shaped obstacle. The propagation configuration follows the *Inclusion of a Wedge* case presented in [1], with the transmitter and receiver heights fixed at 25 m and 10 m, respectively, and the same lossy ground parameters adopted in the previous scenarios. The wedge is located between 3 km and 4 km along the propagation path, with a height of 15 m and its peak positioned at 3.5 km. In this work, however, the maximum propagation angle adopted in the numerical simulations is adjusted to approximately 5.1° , since this value provides better agreement between the DMFT-SSPE and Ray Tracing propagation curves.

The RT solution is employed as the reference model for both the statistical error analysis and the comparison of the

generated path loss curves. The discretization results are summarized in Table III.

TABLE III
DISCRETIZATION ANALYSIS FOR THE 26 GHz WEDGE SCENARIO
CONSIDERING PROPAGATION DISTANCES BETWEEN 500 M AND 5000 M.

Δz (m)	Stability Condition	MAD (dB)	SD (dB)
0.0629	Stable	4.8033	6.4878
0.0630	Unstable	-	-
0.0631	Unstable	-	-
0.0632	Stable	4.3019	6.0070
0.0633	Unstable	-	-
0.0634	Stable	3.3779	4.9741
0.0635	Stable	3.3881	4.9638
0.0636	Unstable	-	-
0.0637	Stable	3.8921	5.6231
0.0638	Unstable	-	-
0.0639	Unstable	-	-
0.0640	Unstable	-	-
0.0641	Stable	4.7070	6.3851
0.0642	Stable	5.2692	7.1398
0.0643	Stable	5.1357	7.0470
0.0644	Unstable	-	-
0.0645	Unstable	-	-
0.0646	Unstable	-	-
0.0647	Unstable	-	-
0.0648	Stable	4.3645	6.2071
0.0649	Stable	4.3033	6.1087

Fig. 5 compares the path loss curves obtained for $\Delta z = 0.0635$ m and $\Delta z = 0.0642$ m, two stable points from Table III selected for presenting contrasting MAD and SD levels, in order to verify whether the pattern observed at 1 GHz and 10 GHz still holds at the highest operating frequency considered in this work.

Although the selected discretization values present different MAD and SD levels, both solutions produce qualitatively similar propagation responses and maintain good agreement with the RT reference model along the propagation path. The differences observed between the curves remain relatively small when compared to the overall propagation behavior.

The results also show the persistence of alternating stable and unstable discretization regions, reinforcing the sensitivity of the DMFT-SSPE formulation to small variations in the vertical discretization step.

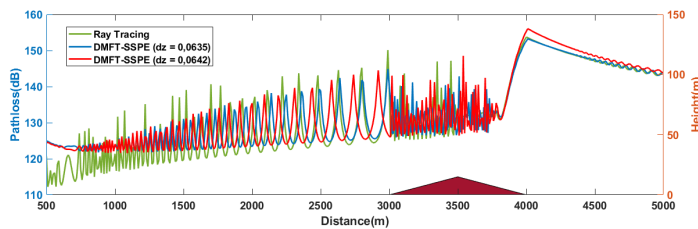


Fig. 5. Comparison of path loss curves obtained for different discretization values in the 26 GHz wedge scenario.

V. CONCLUSIONS

This work evaluated the influence of the vertical discretization step on the numerical behavior of the DMFT-SSPE formulation under different propagation scenarios and operating frequencies.

The obtained results showed that the discretization step directly affects both the numerical stability and the physical consistency of the generated propagation responses. The analyses also demonstrated that statistical error metrics alone are not always sufficient to fully describe the quality of the obtained solutions, since different discretization values may produce distinct propagation behaviors even for similar MAD and SD levels.

Furthermore, the results highlighted the increase of unstable discretization regions at higher frequencies, reinforcing the sensitivity of the formulation to small variations in the vertical sampling step and to instability regions associated with the “bad alpha” phenomenon.

The proposed analysis contributes to the development of more robust radio propagation tools capable of modeling different terrain configurations and operating frequencies. As future work, automated and interactive strategies for discretization selection will be investigated in order to reduce the dependence on manual parameter adjustment during simulations.

REFERENCES

- [1] M. G. S. Santos, D. G. Ibarra, D. C. T. López, C. G. do Rego, M. J. L. Urbina e D. A. Parada, “Millimeter-Wave Propagation Modeling with the DMFT-SSPE Numerical Algorithm,” in *Proc. 2026 IEEE 2nd Latin American Conference on Antennas and Propagation (LACAP)*, 2026.
- [2] D. A. P. Rozo, D. Guevara, A. Rocha, D. Tami, and C. G. Rego, “Ground-reflection analytical approach for horizontally variable refractive index condition,” *Journal of Communication and Information Systems*, vol. 40, no. 1, pp. 31–35, 2025.
- [3] J. Heo, J. Yang, J. Kim, D.-Y. Na, and Y. B. Park, “Propagation Analysis Using Discrete Mixed Fourier Transform-Based Parabolic Equation Method for Long Range Radar Detection Error Correction,” *IEEE Access*, vol. 11, pp. 119436–119445, 2023.
- [4] G. Apaydin and L. Sevgi, *Radio Wave Propagation and Parabolic Equation Modeling*. Wiley, 2017.
- [5] D. Parada, C. G. Rego, D. Guevara, A. Navarro, G. L. Ramos, and R. Oliveira, “A modified radiopropagation multipath model for constant refractivity gradient profiles,” *Journal of Microwaves, Optoelectronics and Electromagnetic Applications*, vol. 22, no. 2, p. 298–312, 2023.
- [6] L. Sevgi, C. Uluisik e F. Akleman, “A MATLAB-based two-dimensional parabolic equation radiowave propagation package,” *IEEE Antennas and Propagation Magazine*, vol. 47, no. 4, pp. 164–175, Aug. 2005.
- [7] J. R. Kuttler e R. Janaswamy, “Improved Fourier transform methods for solving the parabolic wave equation,” *Radio Science*, vol. 37, no. 2, pp. 1–11, 2002.
- [8] Agência Nacional de Telecomunicações (ANATEL), “Plano Estrutural de Redes de Telecomunicações (PERT) 2025–2029: Planejamento regulatório para a ampliação da Conectividade Universal e Significativa no Brasil,” 2025.
- [9] D. A. Parada, C. G. Rego, C. G. Batista e G. L. Ramos, “Algoritmos para a Solução de Equações Parabólicas em Problemas de Radiopropagação,” in *Proc. XXXVI Brazilian Symposium on Telecommunications and Signal Processing*, 2018.