

Joint Channel and Symbol Estimation for RIS-Assisted Fluid Antenna Systems

Josué V. de Araújo, Daniel C. Alcantara, Gilderlan T. de Araújo and André L. F. de Almeida

Abstract—This paper addresses joint channel and symbol estimation in reconfigurable intelligent surface (RIS)-aided multiuser uplink systems with fluid antennas (FAs) at the base station. We propose the Nested Tucker for Fluid Antenna Systems (NTFAS) protocol, in which FA port selection and user-dependent coding vary across blocks while the transmitted symbol matrix is shared across observations. This structure yields coupled Tucker models with common channel and data factors. A two-stage semi-blind bilinear alternating least squares (BALS) receiver is then developed to estimate the cascaded channel and symbols, and to separate the user-to-RIS and RIS-to-BS channels through the embedded PARAFAC structure. Simulations show that NTFAS improves cascaded-channel NMSE and spectral efficiency (SE) with respect to a competing semi-blind benchmark, while maintaining comparable BER performance.

Keywords—Fluid antenna system, reconfigurable intelligent surface, tensor decomposition, coupled Tucker model, semi-blind receiver, bilinear alternating least squares.

I. INTRODUCTION

Future sixth-generation (6G) wireless networks are expected to support stringent requirements on throughput, latency, reliability, connectivity, and energy efficiency [1]. Although massive multiple-input multiple-output (MIMO) and millimeter-wave massive MIMO have enabled large spatial gains, their scalability is constrained by hardware cost, power consumption, and the number of required RF chains [2]. Reconfigurable intelligent surfaces (RISs) provide a complementary solution by shaping the propagation environment via passive reflecting elements, thereby improving coverage and link quality while consuming low power [3].

Fluid antenna systems (FASs) provide another form of spatial adaptability by allowing the receiver to select favorable antenna ports within a compact aperture [4]. This port-selection capability exploits small-scale spatial channel variations without requiring a separate RF chain for each candidate port. However, the same spatial reconfigurability that provides diversity also complicates channel acquisition, since the receiver observes different effective channels as the selected port changes.

Josué V. de Araújo, Daniel C. Alcantara and André L. F. de Almeida are with the Teleinformatics Department, Federal University of Ceará, Fortaleza-CE, e-mails: danielchaves.josue.vas@alu.ufc.br; andre@gtel.ufc.br.

Gilderlan T. de Araújo is with the Federal Institute of Ceará, e-mail: gilderlan.tavares@ifce.edu.br.

This work is partially supported by the National Institute of Science and Technology (INCT-Signals) sponsored by Brazil's National Council for Scientific and Technological Development (CNPq) (Proc. 406517/2022-3), and FUNCAP (Proc. INCT-25255-82587.32.41/64). The research of André L. F. de Almeida is partially supported by CNPq (Proc. 303356/2025-1). The research of Gilderlan T. de Araújo is supported by CNPq (Proc. 151870/2025-0).

Channel estimation for FASs and related movable-antenna (MA) systems has gained attention, since both exploit reconfigurable spatial sampling. Existing methods include LMMSE for large-scale FASs [5], compressed sensing for MA channels [6], and Bayesian reconstruction for FASs [7], leveraging spatial correlation, sparsity, or prior structure. However, they mainly consider stand-alone FAS/MA links with dedicated pilots and do not capture the cascaded RIS-FAS setting, where RIS phases, FA port selection, user coding, and unknown multiuser symbols are jointly coupled.

Combining RISs and FASs is attractive because the RIS shapes the propagation environment while the FA receiver adapts its sampling points. Prior RIS-aided movable/fluid-antenna studies mainly optimize beamforming/positioning assuming CSI [8], but CSI acquisition is costly for RISs [9] and becomes harder with FA port selection and multiuser signaling. Recent FA-RIS works address index modulation [10] and localization-assisted reconstruction for ISAC [11], but not tensor-based semi-blind joint channel-and-symbol estimation in multiuser RIS-FAS uplinks. This motivates a low-overhead receiver that exploits the coupled observations to jointly estimate channels and data.

Tensor-based signal processing provides a natural framework for exploiting multilinear structures in wireless communication systems [12]. It has been successfully applied to blind and semi-blind estimation in MIMO and space-time-frequency coded systems [13], [14], as well as to tensor-based channel estimation in RIS-assisted links [15]–[17]. Building on our previous work on movable antenna systems [18] and RIS-aided FAS receivers [19], this paper addresses the above gap by proposing the NTFAS transmission strategy and a semi-blind receiver for joint channel and symbol estimation in RIS-aided multiuser FAS uplinks.¹

The main contributions are summarized as follows:

- We propose the NTFAS protocol, in which the FA port selection matrix and the user-dependent coding matrix vary across blocks, while the transmitted symbol matrix is shared by all observations.
- We show that the received signals can be represented as a set of coupled Tucker models sharing common symbol

¹*Notation:* Vectors are denoted by boldface lowercase letters ($\mathbf{a}$), matrices by boldface capital letters ($\mathbf{A}$), and tensors by calligraphic letters ($\mathcal{A}$). The transpose, Hermitian transpose, and pseudo-inverse of $\mathbf{A}$ are denoted by $\mathbf{A}^T$, $\mathbf{A}^H$, and $\mathbf{A}^\dagger$, respectively. The matrix $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. The operator $D_i(\mathbf{A})$ forms a diagonal matrix from the i -th row of $\mathbf{A}$, while $\text{diag}(\mathbf{a})$ forms a diagonal matrix from the vector $\mathbf{a}$. The operator $\text{vec}(\cdot)$ vectorizes a matrix, whereas $\text{unvec}_{I \times J}(\cdot)$ denotes the inverse vectorization operator, satisfying $\text{unvec}_{I \times J}(\text{vec}(\mathbf{A})) = \mathbf{A}$ for $\mathbf{A} \in \mathbb{C}^{I \times J}$. The symbols $\otimes$, $\circ$, and $\oslash$ denote the Kronecker product, Khatri-Rao product, and element-wise division, respectively.

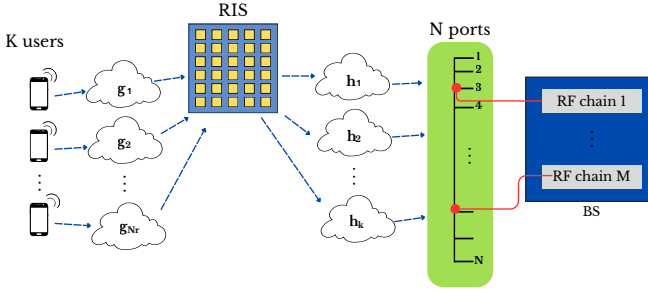


Fig. 1: RIS-assisted FA-based uplink multi-user (MU) scenario.

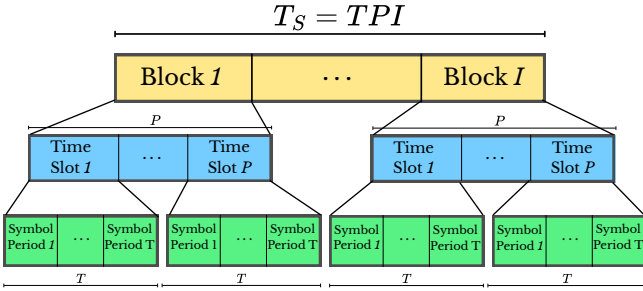


Fig. 2: Transmission time structure.

and cascaded-channel matrices.

- We derive a two-stage semi-blind receiver based on bilinear alternating least squares (BALS). The first stage jointly estimates the transmitted symbols and the cascaded RIS-FA channel, whereas the second one exploits the embedded PARAFAC structure to separate the user-to-RIS and RIS-to-BS channels.
- We use the estimated CSI to perform joint FA port selection and RIS phase-shift optimization, and evaluate the resulting BER, NMSE, and SE performances against a competing benchmark previously introduced in [19].

In this paper, we make use of the following identities:

$$\text{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A}) \text{vec}(\mathbf{B}), \quad (1)$$

$$(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = (\mathbf{AC}) \otimes (\mathbf{BD}), \quad (2)$$

$$(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \diamond \mathbf{D}) = (\mathbf{AC} \diamond \mathbf{BD}), \quad (3)$$

$$\text{vec}(\mathbf{A} \text{diag}(\mathbf{x})\mathbf{B}) = (\mathbf{B}^T \diamond \mathbf{A})\mathbf{x}. \quad (4)$$

II. SYSTEM MODEL AND TENSOR FORMULATION

We consider a multiuser uplink communication system comprising K single-antenna users, an RIS with N_r elements, and a base station equipped with M active RF chains connected to N FA ports, similarly to the architecture described in [18]. To significantly enrich temporal diversity and improve channel identifiability, we propose NTFAS. In this hierarchical frame structure, the FA port selection matrix $\mathbf{S}_i \in \{0, 1\}^{M \times N}$ and the temporal coding matrix $\mathbf{D}_i(\mathbf{C}) \in \mathbb{C}^{K \times K}$ are fixed during the i -th block. Conversely, the RIS phase-shift matrix $\mathbf{D}_p(\boldsymbol{\Theta}) = \text{diag}(\boldsymbol{\theta}_p) \in \mathbb{C}^{N_r \times N_r}$ varies dynamically at each

p -th time slot, where $\boldsymbol{\theta}_p = \boldsymbol{\Theta}(p, :)^T$ is the corresponding RIS configuration vector and $|\boldsymbol{\theta}_p|_{n_r} = 1$, $n_r = 1, \dots, N_r$. Furthermore, the transmitted symbol matrix $\mathbf{X} \in \mathbb{C}^{K \times T}$ remains invariant across the I blocks, allowing the same data block to be observed under multiple FA selection, RIS phase-shift, and temporal coding configurations. The (i, p) -th component of the received signal is formulated as

$$\mathbf{Y}_{i,p} = \mathbf{S}_i \mathbf{H} \mathbf{D}_p(\boldsymbol{\Theta}) \mathbf{G} \mathbf{D}_i(\mathbf{C}) \mathbf{X} + \mathbf{Z}_{i,p} \in \mathbb{C}^{M \times T}. \quad (5)$$

Vectorizing this expression while using property (4) yields

$$\mathbf{y}_{i,p} = ((\mathbf{G} \mathbf{D}_i(\mathbf{C}) \mathbf{X})^T \diamond \mathbf{S}_i \mathbf{H}) \boldsymbol{\theta}_p + \mathbf{z}_{i,p} \in \mathbb{C}^{TM \times 1}, \quad (6)$$

where $\mathbf{y}_{i,p} \triangleq \text{vec}(\mathbf{Y}_{i,p})$ and $\mathbf{z}_{i,p} \triangleq \text{vec}(\mathbf{Z}_{i,p})$. By concatenating these vectorized observations across all P time slots, we construct the aggregated signal matrix $\mathbf{Y}_i \triangleq [\mathbf{y}_{i,1}, \dots, \mathbf{y}_{i,P}]$ for the i -th block:

$$\mathbf{Y}_i = ((\mathbf{G} \mathbf{D}_i(\mathbf{C}) \mathbf{X})^T \diamond \mathbf{S}_i \mathbf{H}) \boldsymbol{\Theta}^T + \mathbf{Z}_i \in \mathbb{C}^{TM \times P}, \quad (7)$$

where $\boldsymbol{\Theta} \in \mathbb{C}^{P \times N_r}$ collects the RIS phase-shift vectors across the P time slots, with its p -th row given by $\boldsymbol{\theta}_p^T$, and $\mathbf{Z}_i \in \mathbb{C}^{TM \times P}$ is the aggregated noise matrix.

$$\mathbf{Y}_i = ((\mathbf{G} \mathbf{D}_i(\mathbf{C}) \mathbf{X})^T \diamond \mathbf{S}_i \mathbf{H}) \boldsymbol{\Theta}^T + \mathbf{Z}_i \in \mathbb{C}^{TM \times P}, \quad (8)$$

where $\boldsymbol{\Theta} \in \mathbb{C}^{P \times N_r}$ collects the complete set of RIS phase shifts, and $\mathbf{Z}_i \in \mathbb{C}^{TM \times P}$ is the aggregated noise matrix. Using the property (3), the received signal in 8 can be rewritten as

$$\mathbf{Y}_i = (\mathbf{X}^T \otimes \mathbf{I}_M)(\mathbf{D}_i(\mathbf{C}) \otimes \mathbf{S}_i) \boldsymbol{\Phi}^T + \mathbf{Z}_i \in \mathbb{C}^{TM \times P}, \quad (9)$$

where

$$\boldsymbol{\Phi} \triangleq \boldsymbol{\Theta}(\mathbf{G}^T \diamond \mathbf{H})^T \in \mathbb{C}^{P \times KN} \quad (10)$$

denotes the composite cascaded channel matrix.

Finally, to fully decouple the symbol matrix $\mathbf{X}$ from the spatial variables, we define a third-order core tensor $\mathcal{A}_i \in \mathbb{C}^{KN \times M \times K}$, whose mode-1 unfolding encapsulates the combined coding and port selection terms, i.e., $[\mathcal{A}_i]_{(1)} = \mathbf{D}_i(\mathbf{C}) \otimes \mathbf{S}_i^T$, $i = 1, \dots, I$. In tensor notation, the noiseless observation tensor $\bar{\mathbf{Y}}_i \in \mathbb{C}^{P \times M \times T}$ is compactly formulated as

$$\bar{\mathbf{Y}}_i = \mathcal{A}_i \times_1 \boldsymbol{\Phi} \times_2 \mathbf{I}_M \times_3 \mathbf{X}^T. \quad (11)$$

By applying standard tensor matricization rules, the mode-1 unfolding directly isolates the cascaded channel matrix $\boldsymbol{\Phi}$:

$$[\bar{\mathbf{Y}}_i]_{(1)} = \boldsymbol{\Phi} [\mathcal{A}_i]_{(1)} (\mathbf{X}^T \otimes \mathbf{I}_M)^T \in \mathbb{C}^{P \times MT}. \quad (12)$$

while the mode-3 unfolding decouples the symbol matrix $\mathbf{X}$:

$$[\bar{\mathbf{Y}}_i]_{(3)} = \mathbf{X}^T [\mathcal{A}_i]_{(3)} (\mathbf{I}_M \otimes \boldsymbol{\Phi})^T \in \mathbb{C}^{T \times PM}. \quad (13)$$

These structured unfoldings serve as the basis for the subsequent estimation stages of the alternating least squares (ALS) algorithm [20].

III. PROPOSED TWO-STAGE SEMI-BLIND RECEIVER

In the first stage, the Tucker structure is exploited to jointly estimate the global cascaded channel matrix $\boldsymbol{\Phi}$ and the common symbol matrix $\mathbf{X}$. In the second stage, the embedded PARAFAC structure of $\boldsymbol{\Phi}$ is used to decouple the individual channel matrices $\mathbf{G}$ and $\mathbf{H}$.

A. First Stage: BALS Estimation of Φ and $\mathbf{X}$

By exploiting the mode-1 unfolding in (12), the global channel is estimated by solving the following LS problem:

$$\hat{\Phi} = \arg \min_{\Phi} \sum_{i=1}^I \|\mathcal{Y}_i^{(1)} - \Phi \mathbf{V}_i\|_F^2, \quad (14)$$

where $\mathbf{V}_i \triangleq [\mathcal{A}_i]_{(1)}(\mathbf{X}^T \otimes \mathbf{I}_M)^T = [\mathcal{A}_i]_{(1)}(\mathbf{X} \otimes \mathbf{I}_M) \in \mathbb{C}^{KN \times MT}$. The global problem can be compactly written as

$$\hat{\Phi} = \arg \min_{\Phi} \|\mathbf{Y}_{(1)} - \Phi \mathbf{V}\|_F^2, \quad (15)$$

where $\mathbf{Y}_{(1)} \triangleq [\mathcal{Y}_1]_{(1)} \cdots [\mathcal{Y}_I]_{(1)} \in \mathbb{C}^{P \times IMT}$, and $\mathbf{V} \triangleq [\mathbf{V}_1 \cdots \mathbf{V}_I] \in \mathbb{C}^{KN \times IMT}$. The estimate for the cascaded channel is obtained as $\hat{\Phi} = \mathbf{Y}_{(1)} \mathbf{V}^\dagger$.

From (13), $\mathbf{X}$ is estimated by solving the following problem:

$$\hat{\mathbf{X}}^T = \arg \min_{\mathbf{X}^T} \sum_{i=1}^I \|\mathcal{Y}_i^{(3)} - \mathbf{X}^T \mathbf{U}_i\|_F^2, \quad (16)$$

where $\mathbf{U}_i \triangleq [\mathcal{A}_i]_{(3)}(\mathbf{I}_M \otimes \Phi)^T \in \mathbb{C}^{K \times PM}$. By horizontally concatenating the 3-mode unfoldings and the corresponding regression matrices, we obtain

$$\mathbf{Y}_{(3)} \triangleq [\mathcal{Y}_1]_{(3)} \cdots [\mathcal{Y}_I]_{(3)} \in \mathbb{C}^{T \times IPM}, \quad (17)$$

and $\mathbf{U} \triangleq [\mathbf{U}_1 \cdots \mathbf{U}_I] \in \mathbb{C}^{K \times IPM}$, then $\mathbf{X}$ is found as

$$\hat{\mathbf{X}}^T = \mathbf{Y}_{(3)} \mathbf{U}^\dagger. \quad (18)$$

B. Second Stage: BALS Estimation of $\mathbf{G}$ and $\mathbf{H}$

The first stage provides the cascaded-channel estimate $\hat{\Phi} \in \mathbb{C}^{P \times KN}$, whose transpose follows the PARAFAC structure, as follows

$$\Phi^T \approx (\mathbf{G}^T \diamond \mathbf{H}) \Theta^T \in \mathbb{C}^{KN \times P}, \quad (19)$$

Let $\hat{\varphi}_p \in \mathbb{C}^{KN \times 1}$ denote the p -th column of $\hat{\Phi}^T$. Then, the corresponding matrix slice is obtained as $\hat{\Phi}_p = \text{unvec}_{N \times K}(\hat{\varphi}_p) = \mathbf{H} \mathbf{D}_p(\Theta) \mathbf{G} \in \mathbb{C}^{N \times K}$.

1) *Estimation of the RIS-to-BS Channel $\mathbf{H}$* : Let us define $\mathbf{R}_p \triangleq \mathbf{D}_p(\Theta) \mathbf{G} \in \mathbb{C}^{N_r \times K}$. By horizontally concatenating the local observations across all P time slots, we have

$$\underbrace{[\hat{\Phi}_1 \cdots \hat{\Phi}_P]}_{\hat{\Phi}_{(H)}} = \mathbf{H} \underbrace{[\mathbf{R}_1 \cdots \mathbf{R}_P]}_{\mathbf{A}_H}, \quad (20)$$

then the RIS-to-BS channel can be estimated as

$$\hat{\mathbf{H}} = \hat{\Phi}_{(H)} \mathbf{A}_H^\dagger. \quad (21)$$

2) *Estimation of the User-to-RIS Channel $\mathbf{G}$* : Let us define $\mathbf{Q}_p \triangleq \mathbf{D}_p(\Theta) \mathbf{H}^T \in \mathbb{C}^{N_r \times N}$, we establish the parallel concatenated system

$$\underbrace{[\hat{\Phi}_1^T \cdots \hat{\Phi}_P^T]}_{\hat{\Phi}_{(G)}} = \mathbf{G}^T \underbrace{[\mathbf{Q}_1 \cdots \mathbf{Q}_P]}_{\mathbf{A}_G}, \quad (22)$$

which gives the following estimate for the User-to-RIS channel

$$\hat{\mathbf{G}}^T = \hat{\Phi}_{(G)} \mathbf{A}_G^\dagger. \quad (23)$$

Algorithm 1: Proposed two-stage BALS semi-blind receiver

Input: $\{\mathcal{Y}_i\}_{i=1}^I$, $\{\mathcal{A}_i\}_{i=1}^I$, Θ , ref. column $\mathbf{x}_1$, tolerance δ

Output: $\hat{\mathbf{H}}$, $\hat{\mathbf{G}}$, and $\hat{\mathbf{X}}$

Initialize $\hat{\mathbf{X}}$ randomly; set $\ell = 0$, $\epsilon_{\text{old}} = \infty$, $\Delta\epsilon = \infty$ **while**

$\Delta\epsilon \geq \delta$ **do**
 Build $\mathbf{V} = [\mathbf{V}_1, \dots, \mathbf{V}_I]$ from $\hat{\mathbf{X}}$ and $\{\mathcal{A}_i\}_{i=1}^I$
 Update $\hat{\Phi} \leftarrow \mathbf{Y}_{(1)} \mathbf{V}^\dagger$
 Build $\mathbf{U} = [\mathbf{U}_1, \dots, \mathbf{U}_I]$ from $\hat{\Phi}$ and $\{\mathcal{A}_i\}_{i=1}^I$
 Update $\hat{\mathbf{X}}^T \leftarrow \mathbf{Y}_{(3)} \mathbf{U}^\dagger$
 Compute ϵ_{new} ; set $\Delta\epsilon = |\epsilon_{\text{new}} - \epsilon_{\text{old}}|$, $\epsilon_{\text{old}} \leftarrow \epsilon_{\text{new}}$,
 $\ell \leftarrow \ell + 1$

$\mathbf{D}_\alpha \leftarrow \text{diag}(\mathbf{x}_1 \otimes \hat{\mathbf{x}}_1)$

Correct scaling: $\hat{\mathbf{X}} \leftarrow \mathbf{D}_\alpha \hat{\mathbf{X}}$, $\hat{\Phi}_p \leftarrow \hat{\Phi}_p \mathbf{D}_\alpha^{-1}$, $p = 1, \dots, P$

Second stage: $\hat{\mathbf{H}} \leftarrow \hat{\Phi}_{(H)} \mathbf{A}_H^\dagger$, $\hat{\mathbf{G}}^T \leftarrow \hat{\Phi}_{(G)} \mathbf{A}_G^\dagger$

IV. JOINT FA PORT SELECTION AND RIS OPTIMIZATION

After estimating $\hat{\mathbf{H}}$ and $\hat{\mathbf{G}}$, we use the estimated CSI to jointly select the FA ports and design the RIS phase shifts. For a given FA selection matrix $\mathbf{S}$ and RIS phase vector $\theta = [\theta_1, \dots, \theta_{N_r}]^T$, the estimated equivalent channel is

$$\hat{\mathbf{H}}_{\text{eq}}(\mathbf{S}, \theta) = \mathbf{S} \hat{\mathbf{H}} \text{diag}(\theta) \hat{\mathbf{G}} \in \mathbb{C}^{M \times K}. \quad (24)$$

Assuming equal power allocation, the corresponding SE is

$$R(\mathbf{S}, \theta) = \log_2 \det \left(\mathbf{I}_M + \frac{\rho}{K} \hat{\mathbf{H}}_{\text{eq}} \hat{\mathbf{H}}_{\text{eq}}^H \right), \quad (25)$$

where ρ denotes the SNR. The joint design problem is therefore

$$\begin{aligned} (\mathbf{S}^*, \theta^*) &= \arg \max_{\mathbf{S} \in \mathcal{S}, \theta} R(\mathbf{S}, \theta) \\ \text{s.t. } &|\theta_{n_r}| = 1, \quad n_r = 1, \dots, N_r, \end{aligned} \quad (26)$$

where $\mathcal{S} \in \{0, 1\}^{M \times N}$ denotes the feasible set of FA port selection matrices, such that activate M distinct ports among the N available FA ports.

Due to the combinatorial search over $\mathcal{S}$ and the unit-modulus RIS constraint, we adopt an exhaustive search over the FA port combinations combined with an SVD-based RIS phase design. For each candidate $\mathbf{S} \in \mathcal{S}$, let $\bar{\mathbf{H}} = \mathbf{S} \hat{\mathbf{H}}$. The equivalent channel can be written as

$$\hat{\mathbf{H}}_{\text{eq}} = \sum_{n_r=1}^{N_r} \theta_{n_r} \bar{\mathbf{h}}_{n_r} \hat{\mathbf{g}}_{n_r, :}, \quad (27)$$

where $\bar{\mathbf{h}}_{n_r}$ is the n_r -th column of $\bar{\mathbf{H}}$ and $\hat{\mathbf{g}}_{n_r, :}$ is the n_r -th row of $\hat{\mathbf{G}}$. We form the tensor $\mathcal{Z} \in \mathbb{C}^{M \times K \times N_r}$ with slices

$$\mathbf{Z}_{n_r} = \bar{\mathbf{h}}_{n_r} \hat{\mathbf{g}}_{n_r, :}. \quad (28)$$

Let $\mathbf{u}_{3,1}$ denote the first column of $\mathbf{U}_3$ obtained from the SVD $[\mathcal{Z}]_{(3)} = \mathbf{U}_3 \mathbf{\Sigma}_3 \mathbf{V}_3^H$, i.e., the dominant left singular vector of the mode-3 unfolding. The RIS phase vector is then chosen as

$$\theta(\mathbf{S}) = \exp(-j \angle \mathbf{u}_{3,1}). \quad (29)$$

Finally, the pair $(\mathbf{S}^*, \theta^*)$ is selected as the candidate that provides the largest estimated SE.

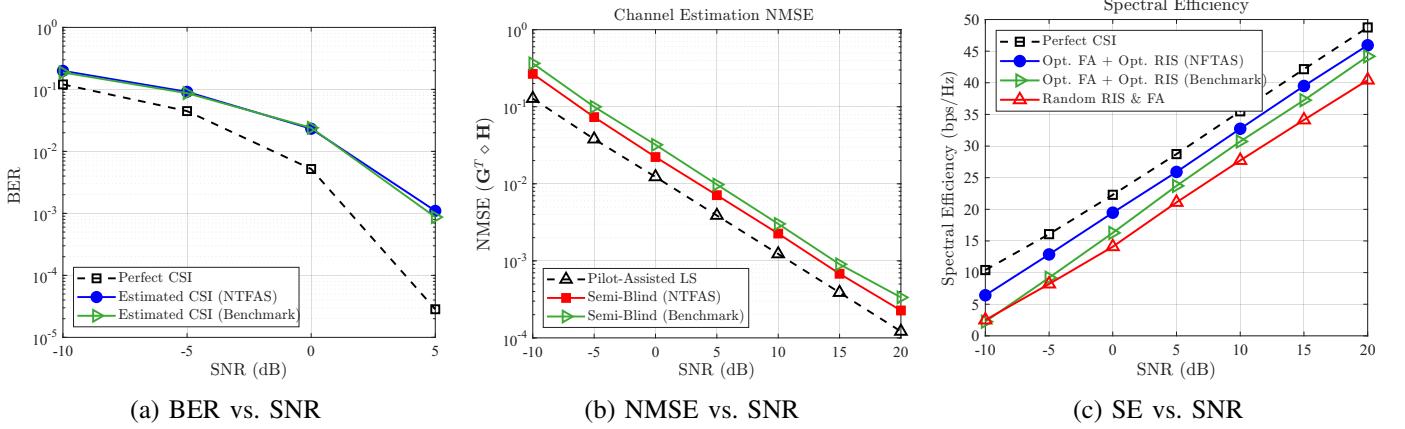


Fig. 3: Performance results versus SNR (dB): (a) BER, (b) NMSE, and (c) SE.

V. IDENTIFIABILITY AND COMPUTATIONAL COMPLEXITY

LS identifiability of the proposed two-stage BALS receiver requires full row rank in the regression matrices used in the pseudo-inverse updates. Stage 1 needs $\text{rank}(\mathbf{V}) = KN$ and $\text{rank}(\mathbf{U}) = K$, i.e., $IMT \geq KN$ and $IPM \geq K$. Stage 2 needs $\text{rank}(\mathbf{A}_H) = \text{rank}(\mathbf{A}_G) = N_r$, i.e., $PK \geq N_r$ and $PN \geq N_r$. Hence, $K \leq \min(IMT/N, IPM)$ and $N_r \leq P \min(K, N)$, which guide the choice of K, N, P , blocks, and time slots; large RISs may require larger P, K , or N . These are necessary conditions—the coding matrix, RIS phases, and FA port selection must also avoid rank loss.

The complexity is dominated by SVD-based pseudo-inverses in the LS updates of $\Phi, \mathbf{X}, \mathbf{H}$, and $\mathbf{G}$. Using $\mathcal{O}(PQ \min(P, Q))$ for the SVD of a $P \times Q$ matrix, the receiver has approximate complexity.

$$\mathcal{O}(IMT(KN)^2 + IPMK^2 + PKN_r^2 + PNN_r^2). \quad (30)$$

VI. SIMULATION RESULTS

In this section, we assess the proposed semi-blind receiver for NTFAS in terms of bit error rate (BER), normalized mean square error (NMSE) of the cascaded channel factor $\mathbf{G}^T \diamond \mathbf{H}$, and SE after joint FA port selection and RIS phase-shift optimization. A previously proposed semi-blind receiver from [19] is used as the competing benchmark under comparable transmission-overhead conditions. The Perfect CSI receiver provides a reference bound for BER and SE, such that All BER curves are obtained with 16-PSK modulation.

Fig. 3 compares NTFAS with the competing semi-blind benchmark. In Fig. 3a, both semi-blind receivers exhibit nearly identical BER behavior over the simulated SNR range, and both remain separated from the Perfect CSI curve. This is consistent with the proposed model: after the reference-symbol scaling correction, the symbol estimates are primarily limited by noise and residual channel-estimation error. Hence, although NTFAS improves the cascaded-channel estimate, the moderate NMSE difference between NTFAS and the benchmark is not sufficient to yield a visible BER gain at the considered operating points. This is consistent with the role of NTFAS as a complementary tensor protocol: by varying FA selection and user-dependent coding across blocks,

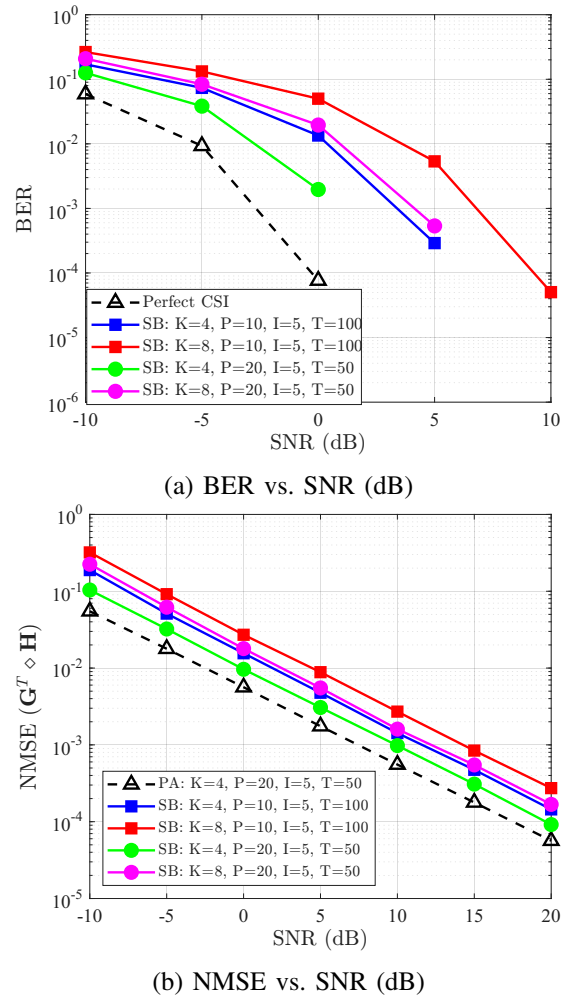


Fig. 4: Simulation results for the second scenario: (a) BER vs. SNR and (b) NMSE vs. SNR.

varying RIS phases across time slots, and keeping the symbol matrix common across observations, it preserves the detection capability of the benchmark while providing more accurate CSI for FA/RIS optimization.

Fig. 3b highlights the NMSE differences. NTFAS consistently improves the cascaded-channel estimate relative to

the competing semi-blind benchmark. As expected, the Pilot-Assisted LS estimator yields the lowest NMSE since it exploits known training symbols to estimate the cascaded channel. The NTFAS gain stems from combining multiple blocks with different FA selections and coding matrices while keeping a common symbol matrix, which increases the number of coupled equations to estimate Φ . For a fair comparison, all methods use the same physical dimensions (M, N, N_r, K) and the same total observation budget IPT (possibly redistributed across I , P , and T). The proximity of NTFAS to the Pilot-Assisted LS curve therefore shows that it can approach a training-assisted LS reference while requiring less prior knowledge of the transmitted symbols. The improvement is moderate, consistent with both semi-blind schemes exploiting tensor structure and meeting the LS identifiability conditions discussed earlier.

The SE results in Fig. 3c follow the NMSE trend. Since FA/RIS optimization is performed using the estimated CSI, a more accurate cascaded-channel estimate of NTFAS yields higher SE than the competing benchmark and keeps the curve closer to the Perfect CSI bound. The remaining gap to Perfect CSI is also expected because the optimization stage uses estimated, rather than true, channel matrices. Both optimized semi-blind schemes outperform the random RIS/antenna selection baseline, confirming that the estimated CSI remains useful for link adaptation.

Fig. 4 studies the impact of P , K , and T with fixed $I = 5$ and comparable temporal overhead. Because NTFAS reuses the same symbol matrix $\mathbf{X}$ across I blocks and P RIS configurations, increasing I or P adds observation diversity but reduces the effective number of new symbols (reuse factor $\propto 1/(IP)$). For $K = 4$, raising P from 10 to 20 improves BER and NMSE even when T is reduced from 100 to 50, confirming that extra RIS-domain diversity improves conditioning. When K increases from 4 to 8, both metrics degrade due to the larger coupled problem and the tighter LS requirement $IMT \geq KN$, increasing P to 20 partially mitigates this loss. Overall, larger P improves robustness, whereas larger K makes joint channel and symbol recovery more challenging.

VII. CONCLUSIONS

This paper proposed a two-stage semi-blind receiver for joint channel and symbol estimation in RIS-aided multiuser FAS uplinks. By varying the FA port selection and user-dependent coding across blocks while keeping the symbol matrix common, we obtain coupled Tucker models with shared symbol and cascaded-channel factors. We then exploit the embedded PARAFAC structure of the cascaded channel to separately recover the user-RIS and RIS-BS channels. Simulations show that NTFAS yields a moderate but consistent NMSE gain over the competing semi-blind benchmark, translating into higher SE after FA/RIS optimization, while achieving similar BER. The results also confirm the expected trade-off: more RIS phase configurations improve conditioning, whereas more users make joint recovery harder. Overall, NTFAS reduces pilot dependence while providing CSI suitable for FA port selection and RIS phase-shift design. Future work will

develop scalable FA/RIS optimization beyond exhaustive FA port search to reduce complexity for larger apertures and RIS sizes, and extend the model to hardware impairments, time-varying channels, and multi-antenna users.

REFERENCES

- [1] H. Tataria, M. Shafi, A. F. Molisch, M. Dohler, H. Sjöland, and F. Tufvesson, "6g wireless systems: Vision, requirements, challenges, insights, and opportunities," *Proc. IEEE*, vol. 109, no. 7, pp. 1166–1199, 2021.
- [2] E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, "Massive mimo for next generation wireless systems," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 186–195, 2014.
- [3] M. Di Renzo, A. Zappone, M. Debbah, M.-S. Alouini, C. Yuen, J. de Rosny, and S. Tretyakov, "Smart radio environments empowered by reconfigurable intelligent surfaces: How it works, state of research, and the road ahead," *IEEE J. Sel. Areas Commun.*, vol. 38, no. 11, pp. 2450–2525, 2020.
- [4] K.-K. Wong, D. Morales-Jimenez, K.-F. Tong, and C.-B. Chae, "Slow fluid antenna multiple access," *IEEE Trans. Commun.*, vol. 71, no. 5, pp. 2831–2846, 2023.
- [5] C. Skouroumounis and I. Krikidis, "Fluid antenna with linear MMSE channel estimation for large-scale cellular networks," *IEEE Trans. Commun.*, vol. 71, no. 2, pp. 1112–1125, 2023.
- [6] W. Ma, L. Zhu, and R. Zhang, "Compressed sensing based channel estimation for movable antenna communications," *IEEE Commun. Lett.*, vol. 27, no. 10, pp. 2747–2751, 2023.
- [7] Z. Zhang, J. Zhu, L. Dai, and R. W. Heath, "Successive bayesian reconstructor for channel estimation in fluid antenna systems," *IEEE Trans. Wireless Commun.*, vol. 24, no. 3, pp. 1992–2006, 2025.
- [8] Y. Sun, H. Xu, B. Ning, Z. Cheng, C. Ouyang, and H. Yang, "Sum-rate optimization for ris-aided multiuser communications with movable antennas," *IEEE Wireless Commun. Lett.*, vol. 14, no. 2, pp. 450–454, 2025.
- [9] A. L. Swindlehurst, G. Zhou, R. Liu, C. Pan, and M. Li, "Channel estimation with reconfigurable intelligent surfaces: A general framework," *Proc. IEEE*, vol. 110, no. 9, pp. 1312–1338, 2022.
- [10] J. Zhu, Q. Luo, G. Chen, P. Xiao, Y. Xiao, and K.-K. Wong, "Fluid antenna empowered index modulation for RIS-aided mmWave transmissions," *IEEE Transactions on Wireless Communications*, vol. 24, no. 2, pp. 1635–1647, Feb. 2025.
- [11] L. Zhang, X. Lei, H. Niu, T. Ma, Y. Zhang, Y. Ge, G. K. Karagiannis, and H. Wymeersch, "Joint localization and channel estimation framework for FA-RIS assisted ISAC," *IEEE Transactions on Cognitive Communications and Networking*, 2026, in press.
- [12] N. D. Sidiropoulos, L. De Lathauwer, X. Fu, K. Huang, E. E. Papalexakis, and C. Faloutsos, "Tensor decomposition for signal processing and machine learning," *IEEE Trans. Signal Process.*, vol. 65, no. 13, pp. 3551–3582, 2017.
- [13] A. L. F. de Almeida, G. Favier, and J. C. M. Mota, "PARAFAC-based unified tensor modeling for wireless communication systems with application to blind multiuser equalization," *Signal Process.*, vol. 87, no. 2, pp. 337–351, 2007.
- [14] G. Favier and A. L. F. de Almeida, "Tensor space-time-frequency coding with semi-blind receivers for MIMO wireless communication systems," *IEEE Trans. Signal Process.*, vol. 62, no. 22, pp. 5987–6002, 2014.
- [15] G. T. de Araújo and A. L. F. de Almeida, "PARAFAC-based channel estimation for intelligent reflective surface assisted MIMO system," in *2020 IEEE 11th Sensor Array and Multichannel Signal Processing Workshop (SAM)*, 2020, pp. 1–5.
- [16] G. T. de Araújo, A. L. F. de Almeida, and R. Boyer, "Channel estimation for intelligent reflecting surface assisted MIMO systems: A tensor modeling approach," *IEEE J. Sel. Topics Signal Process.*, vol. 15, no. 3, pp. 789–802, 2021.
- [17] K. Ardah, S. Gharekhloo, A. L. F. de Almeida, and M. Haardt, "TRICE: A channel estimation framework for RIS-aided millimeter-wave MIMO systems," *IEEE Signal Process. Lett.*, vol. 28, pp. 513–517, 2021.
- [18] J. V. de Araújo, J. C. da Silva Filho, G. T. de Araújo, P. R. B. Gomes, and A. L. F. de Almeida, "Joint channel and symbol estimation for communication systems with movable antennas," in *SBrT*, Natal, RN, Brazil, Sept. 2025.
- [19] J. V. de Araújo, G. T. de Araújo, and A. L. F. de Almeida, "Semi-blind receivers for RIS-aided fluid antenna systems," Apr. 2026, arXiv preprint arXiv:2604.18138, [Online]. Available: <https://arxiv.org/abs/2604.18138>
- [20] P. Comon, X. Luciani, and A. L. F. de Almeida, "Tensor decompositions, alternating least squares and other tales," *J. Chemometrics*, vol. 23, no. 7–8, pp. 393–405, 2009.