

Direction Finding with MUSIC using Circularly Polarized Fields

Lucas S. Pereira, Juner M. Vieira, Marcos V. T. Heckler and Felix Antreich

Abstract—This paper presents the analysis and implementation of the MUSIC algorithm for direction finding using a 2×2 antenna array with circularly polarized fields, applied to a GNSS-R scenario. The proposed approach incorporates the simulated active fields of each sensor considering dual-circularly polarized patterns, i.e., co-pol and cross-pol fields are used in the direction-of-arrival estimation. The results demonstrate good resolution and scanning accuracy in both azimuth and elevation angles.

Keywords—Direction finding, GNSS, GNSS-R, MUSIC, circular polarized fields.

I. INTRODUCTION

Direction Finding (DF) refers to the process of determining the direction of arrival (DoA) of a radio frequency (RF) signal. This capability plays a crucial role in several applications, including navigation, telecommunications, military operations, and search-and-rescue systems. In the military field, DF is particularly important for electronic intelligence, enabling the detection, identification, and tracking of hostile communication sources. Over the past decades, a wide range of techniques and methodologies have been developed to improve the accuracy, resolution, and robustness of DoA estimation systems, as well as to mitigate the adverse effects of multipath propagation [1], [2].

In the literature, DoA estimation methods can be broadly classified into three main categories: classical, maximum likelihood (ML), and subspace-based methods. Classical methods rely on beamforming techniques, such as Delay-and-Sum (DS) and Minimum Variance Distortionless Response (MVDR), in which the array beam is steered across different directions and the received power is evaluated to estimate the DoAs.

ML methods exploit the statistical model of the received data, typically providing high accuracy at the expense of increased computational complexity. Subspace-based methods, in turn, achieve high-resolution DoA estimation by exploiting the separation between signal and noise subspaces. Well-known algorithms in this category include Multiple Signal Classifier (MUSIC), Minimum Norm (MN), and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [3], [4].

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This work addresses the DF problem through the implementation of the MUSIC algorithm applied to a 2×2 planar antenna array. The proposed system, comprising both the array antenna and the developed signal processing algorithm, is designed to receive and process signals from global navigation satellite system (GNSS) and GNSS-reflectometry (GNSS-R) applications. Furthermore, the system is capable of handling both right-hand circular polarization (RHCP) and left-hand circular polarization (LHCP), enabling the exploitation of different polarization characteristics in the received signals. The cross-polar patterns are included in the analysis, since there will be crosstalk between co and cross-polar channels. Some studies address the DF problem primarily from a theoretical and computational perspective, often assuming a uniform linear array (ULA) or a planar array (PA) composed of isotropic radiating elements. In many cases, these approaches rely on a large number of sensors, sometimes reaching dozens of elements [5]–[11].

Other works investigate non-uniform array configurations (i.e., non-ULA or non-PA), while still assuming isotropic radiators [12]. In [13], [14] LP-CP antenna arrays with isotropic sensors are applied in DF using traditional MUSIC method. More realistic approaches, however, account for the actual radiation patterns and polarization characteristics of the antenna elements. In [15], the authors use a dipole array for DF, enabling polarization estimation. Although dipoles inherently exhibit linear polarization (LP), the combination of vertically (V) and horizontally (H) oriented elements allows the identification of the incoming signal polarization, including both linear and circular components. A sparse four-element ULA based on microstrip technology is proposed in [16] for DF applications, considering linearly polarized radiators. In [19], for example, both conventional and circularly polarized (CP) MUSIC-based methods are employed in a passive DF system for radar signal detection. In [20] a 2×2 antenna array was applied to interference mitigation in a real environment, using a two-step blind adaptive beamforming algorithm to search the directions of the interferences.

In this context, this work investigates the use of a planar antenna array based on microstrip radiators, taking into account the effects of mutual coupling among the array elements. These effects are incorporated into the analysis through the active-element patterns and the antenna array scattering parameters (S-parameters), providing a realistic representation of the system, especially of the array response. Electromagnetic simulations were performed using Ansys HFSS (High-Frequency Structure Simulator), while the DoA estimation algorithm, the MUSIC algorithm for dual-polarization DoA estimation was implemented in MATLAB and its performance was evaluated

TABLE I
 COMPARISON WITH RELATED LITERATURE.

Ref.	Sensors	Geometry	Fields	Pol.
[5]– [8]	>8	ULA	iso.	LP
[9]– [11]	>8 to 100	PA	iso.	LP
[12]	3	non-ULA	iso.	LP
[13]– [14]	>10	ULA/PA	iso.	LP
[15]	2	ULA	real	LP
[19]	6	non-PA	real	LP
[20]	4	PA	real	CP
this work	4	PA	real	CP

by computer simulations.

Table I presents a comparison between the proposed approach and related works in the literature. It is worth noting that many papers address the MUSIC algorithm considering linearly polarized fields and idealized isotropic radiating elements, assuming perfect cross-polar isolation. However, practical antennas do not exhibit uniform radiation in all directions. Furthermore, depending on the operating frequency band, antenna arrays with a large number of elements may become impractical in terms of physical size and implementation cost. Additionally, for each array sensor we need 2 channels to be built, this reinforces the choice of a 2×2 geometry.

This paper is organized as follows. Section II presents the signal model adopted for the reception of dual-circular polarized fields (RHCP and LHCP). Section III describes the conventional MUSIC algorithm, along with the necessary adaptations for DoA estimation with dual-pol fields. Section IV describes the geometry of the 2×2 antenna array. The results obtained for the DoA estimation of one or more independent sources are presented in Section V, and Section VI concludes the paper with final remarks.

II. DATA MODEL

The signal model considered in this work is based on a scenario involving the reception of GNSS signals in line of sight (LoS) and signals reflected by the ground (GNSS-R). This model was originally proposed in [21] for an array with isotropic radiators and accounts for cross-polarized field components arising from limited, angle-dependent cross-polarization isolation. In this case, the satellite transmits an RHCP signal $c(t) \in \mathbb{R}$ with bandwidth B .

The baseband signal received by an antenna array with M elements and $2M$ outputs, M for RHCP and M for LHCP, from D incident sources, indexed by $d = 1, 2, \dots, D$, corresponding to circularly polarized (CP) GNSS signals, including both LoS and ground-reflected (GNSS-R) components, is given by

$$\begin{aligned} \mathbf{x}(t) &= \begin{bmatrix} \mathbf{x}_R(t) \\ \mathbf{x}_L(t) \end{bmatrix} \\ &= \mathbf{b}_1(t) + \sum_{d=2}^D \mathbf{b}_{R,d}(t) + \sum_{d=2}^D \mathbf{b}_{L,d}(t) + \boldsymbol{\eta}(t), \end{aligned} \quad (1)$$

where $\boldsymbol{\eta}(t)$ denotes spatially and temporally white Gaussian noise with $\mathcal{N}(0, \sigma_n^2)$ and the index $d = 1$ is related to the

LoS and the indexes $d = 2, \dots, D$ are related to the reflected signals. $\mathbf{x}_R(t) \in \mathbb{C}^{M \times 1}$ denotes the M RHCP outputs and $\mathbf{x}_L(t) \in \mathbb{C}^{M \times 1}$ the M LHCP outputs of the array. The terms $\mathbf{b}_0(t) \in \mathbb{C}^{2M \times 1}$, $\mathbf{b}_{R,d}(t) \in \mathbb{C}^{2M \times 1}$ and $\mathbf{b}_{L,d}(t) \in \mathbb{C}^{2M \times 1}$ represent the superimposed signal replicas associated with the source, considering both RHCP and LHCP, and can be expressed by

$$\mathbf{b}_1(t) = \gamma_1 \begin{bmatrix} \mathbf{s}_{R,co}(\phi_1, \theta_1) \\ \mathbf{s}_{L,cross}(\phi_1, \theta_1) \end{bmatrix} c(t - \tau_1), \quad (2)$$

$$\mathbf{b}_{R,d}(t) = \gamma_d \alpha_{R,d} \begin{bmatrix} \mathbf{s}_{R,co}(\phi_d, \theta_d) \\ \mathbf{s}_{L,cross}(\phi_d, \theta_d) \end{bmatrix} c(t - \tau_d), \quad (3)$$

$$\mathbf{b}_{L,d}(t) = \gamma_d \alpha_{L,d} \begin{bmatrix} \mathbf{s}_{R,cross}(\phi_d, \theta_d) \\ \mathbf{s}_{L,co}(\phi_d, \theta_d) \end{bmatrix} c(t - \tau_d), \quad (4)$$

where $\mathbf{s}_{R,co}(\phi_d, \theta_d)$ and $\mathbf{s}_{L,co}(\phi_d, \theta_d) \in \mathbb{C}^{M \times 1}$ denote the co-polar embedded sensor patterns and $\mathbf{s}_{R,cross}(\phi_d, \theta_d)$ and $\mathbf{s}_{L,cross}(\phi_d, \theta_d) \in \mathbb{C}^{M \times 1}$ denote the cross-polar embedded patterns of the array. $c(t - \tau_d)$ represents the transmitted pseudorandom sequence of the received satellite signals with propagation delay τ_d . The parameters (ϕ_d, θ_d) represent the azimuth and elevation angles and $\gamma_d \in \mathbb{C}$ denotes the complex amplitude due to the path loss and propagation of the LoS and reflected signals. $\alpha_{R,d} \in \mathbb{C}$ and $\alpha_{L,d} \in \mathbb{C}$ are the RHCP and LHCP reflection coefficients, respectively.

For the k th snapshot, corresponding to one period of the pseudorandom sequence $c(t)$, with $k = 1, 2, \dots, K$, the output signals of the array are given by

$$\mathbf{x}[(k-1)N + n] = \mathbf{x}(((k-1)N + n)T_s), \quad (5)$$

where the sampling rate $f_s = \frac{1}{T_s}$, and $n = 1, 2, \dots, N$. Thus, the signal model in (1) can be written in a matrix notation

$$\mathbf{X}[k] = \sum_{d=1}^D \gamma_d \mathbf{S}(\phi_d, \theta_d) \boldsymbol{\alpha}_d \mathbf{c}^T(\tau_d) + \mathbf{E}[k], \quad (6)$$

where

$$\begin{aligned} \mathbf{c}[k; \tau_d] &= \mathbf{c}(\tau_d) = [c(((k-1)N + 1)T_s - \tau_d), \\ &\quad \dots, c(((k-1)N + N)T_s - \tau_d)]^T \end{aligned} \quad (7)$$

represents one period of the pseudorandom sequence with the delay τ_d , $\mathbf{E}[k] \in \mathbb{C}^{2M \times N}$ is spatially and temporally white Gaussian noise,

$$\mathbf{S}(\phi_d, \theta_d) = \begin{bmatrix} \mathbf{s}_{R,co}(\phi_d, \theta_d) & \mathbf{s}_{R,cross}(\phi_d, \theta_d) \\ \mathbf{s}_{L,cross}(\phi_d, \theta_d) & \mathbf{s}_{L,co}(\phi_d, \theta_d) \end{bmatrix} \in \mathbb{C}^{2M \times 2}, \quad (8)$$

and

$$\boldsymbol{\alpha}_d = [\alpha_{R,d} \quad \alpha_{L,d}]^T \in \mathbb{C}^{2 \times 1}. \quad (9)$$

In order to preserve the multiple access properties of the DS-CDMA system, i.e. enabling DoA estimation of the received signals for each satellite of the GNSS separately, and to reduce the large data amount of samples N to a smaller dimension, the received signal $\mathbf{X}[n]$ is compressed in the temporal domain by multiplying it by a compression matrix $\mathbf{Q} \in \mathbb{C}^{N \times Q}$ with the property $\mathbf{Q}^H \mathbf{Q} = \mathbf{I}_Q$, where $\mathbf{I}_Q$ is the identity matrix of dimension $Q \times Q$. The compression matrix $\mathbf{Q}$ is derived

by using a correlator bank that is concentrated around τ_1 , as described in [21]. Thus, we can write

$$\begin{aligned} \mathbf{Z}[k] &= \sum_{d=1}^D \gamma_d \mathbf{S}(\phi_d, \theta_d) \boldsymbol{\alpha}_d \mathbf{c}^T(\tau_d) \mathbf{Q} + \mathbf{E}[k] \mathbf{Q} \\ &= \sum_{d=1}^D \gamma_d \mathbf{S}(\phi_d, \theta_d) \boldsymbol{\alpha}_d \mathbf{q}^T(\tau_d) + \mathbf{E}_Q[k], \end{aligned} \quad (10)$$

where $\mathbf{q}(\tau_d)$ represents the compressed temporal signature associated with the delay τ_d and $\mathbf{E}_Q[k]$ denotes the compressed noise with covariance $\mathbb{E}[\mathbf{E}_Q[k] \mathbf{E}_Q^H[k]] = \sigma_n^2 \mathbf{I}_{2M}$.

The embedded patterns of (8) must have the gains and phases of the active pattern of each element of the array. Thus, using the commercial electromagnetic software HFSS, the amplitudes (in V/m) and phases (in rad/m) of the active electric fields, as well as gain patterns (in dBi), were obtained for each radiator. Then, the electric fields were normalized and the gain in each direction (ϕ, θ) was used individually so that a gain pattern with amplitude (in dB) and phase (in rad) could be obtained.

In the next section, the conventional MUSIC algorithm is adapted to process the proposed data model, and the corresponding spatial spectrum is computed.

III. THE MUSIC ALGORITHM

The conventional MUSIC algorithm was proposed by Ralph Schmidt in 1986. The acronym Multiple Signal Classification (MUSIC) refers to a technique used to estimate the parameters of multiple wavefronts impinging on an antenna array by processing the signals received at each array element [22].

Considering an antenna array with M dual-polarization elements, the spatial covariance matrix $\mathbf{R}_{\mathbf{ZZ}} = \mathbb{E}[\mathbf{Z}[k] \mathbf{Z}^H[k]]$ has $2M$ eigenvalues $\boldsymbol{\lambda} = [\lambda_1, \dots, \lambda_m, \dots, \lambda_{2M}]$ and $2M$ corresponding eigenvectors $\mathbf{U} = [\mathbf{u}_1, \dots, \mathbf{u}_{2M}]$. An estimate of the spatial covariance matrix considering K snapshots can be given by

$$\hat{\mathbf{R}}_{\mathbf{ZZ}} = \frac{1}{K} \sum_{k=1}^K \mathbf{Z}[k] \mathbf{Z}^H[k]. \quad (11)$$

The eigen-decomposition of $\hat{\mathbf{R}}_{\mathbf{ZZ}}$ can be written as

$$\hat{\mathbf{R}}_{\mathbf{ZZ}} = \hat{\mathbf{U}}_s \hat{\boldsymbol{\Lambda}}_s \hat{\mathbf{U}}_s^H + \hat{\mathbf{U}}_n \hat{\boldsymbol{\Lambda}}_n \hat{\mathbf{U}}_n^H. \quad (12)$$

If D uncorrelated signals impinge on the antenna array, an estimate of the signal subspace is $\hat{\mathbf{U}}_s = [\hat{\mathbf{u}}_1, \dots, \hat{\mathbf{u}}_D] \in \mathbb{C}^{2M \times D}$, where $\hat{\mathbf{u}}_d$ are the eigenvectors associated with the D eigenvalues $\lambda_m > \sigma_n^2$. An estimate of the noise subspace is given by $\hat{\mathbf{U}}_n \in \mathbb{C}^{2M \times (2M-D)}$ including the remaining $2M - D$ eigenvectors. The corresponding eigenvalue matrices are defined as $\hat{\boldsymbol{\Lambda}}_s = \text{diag}(\lambda_1, \dots, \lambda_D)$ and $\hat{\boldsymbol{\Lambda}}_n = \text{diag}(\lambda_{D+1}, \dots, \lambda_{2M})$, associated with the signal and noise subspaces, respectively.

Extending the classical MUSIC pseudospectrum to our dual-polarization case, we write

$$P_{MU}^{CP}(\phi, \theta) = \frac{1}{\lambda_{min}(\phi, \theta)}, \quad (13)$$

with

$$\lambda_{min}(\phi, \theta) = \min_{\boldsymbol{\alpha} \neq 0} \left\{ \frac{\boldsymbol{\alpha}^H \mathbf{S}^H(\phi, \theta) \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{S}(\phi, \theta) \boldsymbol{\alpha}}{\boldsymbol{\alpha}^H \mathbf{S}^H(\phi, \theta) \mathbf{S}(\phi, \theta) \boldsymbol{\alpha}} \right\} \quad (14)$$

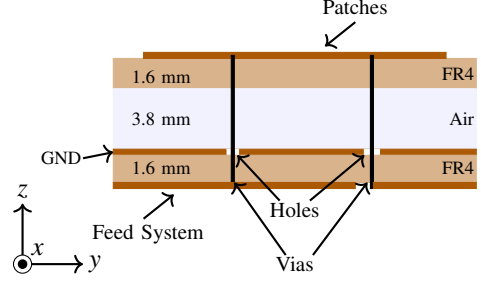


Fig. 1. Schematic cross-sectional view of the antenna array.

which corresponds to the minimum eigenvalue of the generalized eigenvalue problem

$$\mathbf{S}^H(\phi, \theta) \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{S}(\phi, \theta) \boldsymbol{\alpha} = \lambda(\phi, \theta) \mathbf{S}^H(\phi, \theta) \mathbf{S}(\phi, \theta) \boldsymbol{\alpha}, \quad (15)$$

where the generalized eigenvector $\boldsymbol{\alpha}$ associated with $\lambda_{min}(\phi, \theta)$ gives the reflection coefficients associated with each peak of the MUSIC spectrum, and thus of each received signal.

IV. ARRAY GEOMETRY

A 2×2 microstrip antenna array was designed and optimized using Ansys HFSS. Sequentially rotated square patches were employed, so as to improve the polarization purity, characterized by a low axial ratio (AR). The array must have reflection in the ports (S_{ii}) below -10 dB, $AR < 3$ dB, isolation between the ports (S_{ij}) lower than -15 dB and the possibility to operate with dual-polarization (RHCP and LHCP) [23].

The stack-up considered for the designed antenna array is shown in Fig. 1. The multilayer structure consists of two low-cost FR4 laminates with dielectric constant of 4.3 and loss tangent of 0.02. The top laminate supports the patches, which are tuned to resonate at 1575.42 MHz. The ground plane (GND) and the feeding network are implemented on the bottom laminate. An air layer is introduced between the FR4 layers to enhance the bandwidth. The patches are fed by independent pairs of feeding lines.

Figure 2 shows a schematic top view of the array and the port numbering, while Fig. 3 illustrates the feeding network of the array. In this configuration, the main polarization can be selected by properly configuring the input ports of the 90° hybrid couplers. Specifically, by selecting ports 1 to 4, RHCP fields are received as the main polarization. Alternatively, LHCP can be selected using ports 5 to 8.

The simulated gain patterns of each radiator in the array are shown in Fig. 4. Only the gain patterns corresponding to RHCP as the co-polarization are presented, since the array is reciprocal and symmetric. Therefore, the LHCP patterns exhibit the same behavior as the RHCP ones.

V. RESULTS

Initially, the LoS signal and only one ($D = 2$) multipath are impinging in a 2×2 planar microstrip array ($M = 4$), with a signal-to-noise ratio (SNR) of 10 dB and $Q = 11$ correlators. Figure 5 shows the normalized MUSIC spectrum calculated in

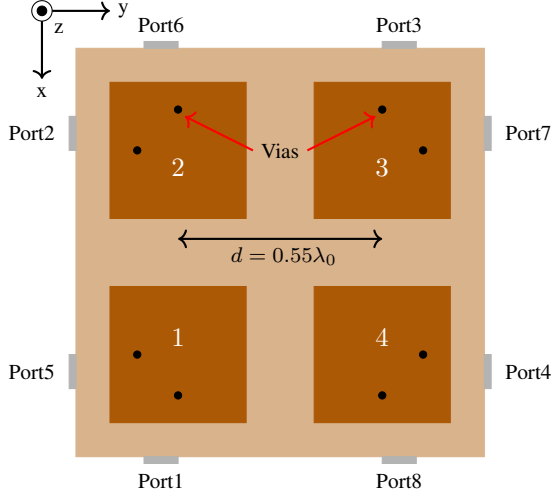
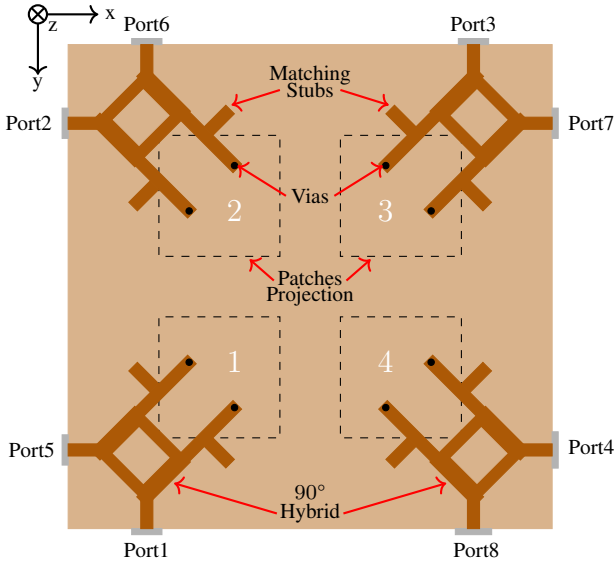

 Fig. 2. Schematic top view of the 2×2 antenna array.


Fig. 3. Schematic bottom view of the feed system configuration.

decibel scale. Figure 6 demonstrates the ability to detect LoS and two multipath signals. Finally, Figure 7 demonstrates the method's ability to detect two closely spaced multipath signals.

To investigate the performance of the proposed signal model, Table II presents the RMSE (Root-Mean Square Error) calculated for a LoS in $(15^\circ, 30^\circ)$ and $d = 2$ multipath in $(-30^\circ, 55^\circ)$. Absolute errors were estimated considering 2,046 samples for each snapshot ($K = 100$). Finally, the RMSE was obtained by performing 500 Monte Carlo simulations. Overall, a good agreement between the true and estimated DoAs can be observed, even with low SNR and $Q = 11$ correlators.

VI. CONCLUSION

This paper presented a DF formulation for antenna arrays that incorporate both the active co and cross-pol radiation patterns applied to the detection of GNSS-R signals. In contrast to most approaches found in the literature, which often

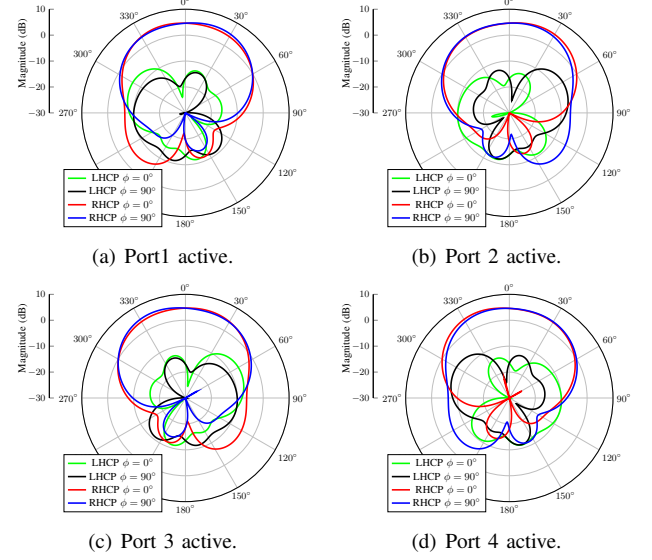
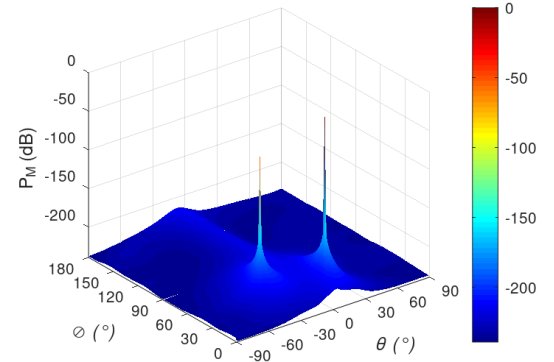
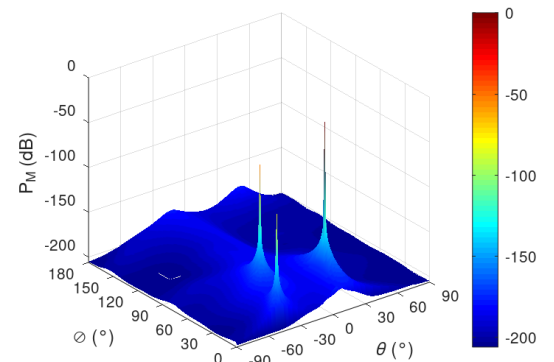


Fig. 4. Gain patterns at 1.575 GHz for each array sensor.


 Fig. 5. Normalized MUSIC spectrum for CP fields 2×2 antenna array with LoS at $(\phi = 15^\circ, \theta = 30^\circ)$ and multipath signal at $(\phi = -30^\circ, \theta = 50^\circ)$.

 Fig. 6. Normalized MUSIC spectrum for CP fields 2×2 antenna array with LoS at $(\theta = 15^\circ, \phi = 30^\circ)$ and two NLoS at $(\theta = -30^\circ, \phi = 50^\circ)$ and $(\theta = -45^\circ, \phi = 10^\circ)$.

assume isotropic radiators or large arrays, the proposed model accounts for practical aspects such as direction-dependent gain and also considers reflected signals as a superposition of RHCP and LHCP waves.

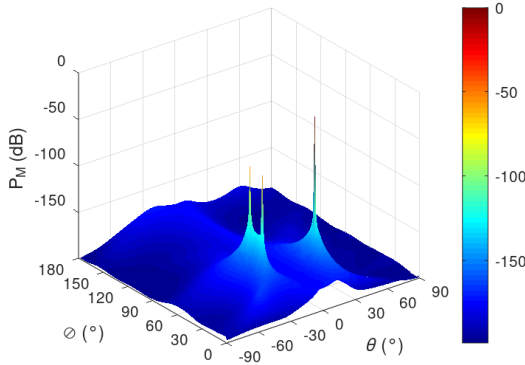


Fig. 7. Normalized MUSIC spectrum for CP fields 2x2 antenna array with LoS at $(\theta = 15^\circ, \phi = 300^\circ)$ and two closer NLoS at $(\theta = -30^\circ, \phi = 50^\circ)$ and $(\theta = -30^\circ, \phi = 35^\circ)$

TABLE II
RMSE CALCULATED FOR DIFFERENT SNR SCENARIOS.

Param.	SNR=1dB	SNR=2dB	SNR=3dB	SNR=4dB
RMSE (θ LoS)	1.31°	1.11°	1.01°	0.98°
RMSE (ϕ LoS)	2.32°	2.29°	2.12°	2.01°
RMSE (θ NLoS)	0.99°	0.97°	0.92°	0.84°
RMSE (ϕ NLoS)	1.92°	1.84°	1.82°	1.72°

For GNSS signal detection, a 2×2 planar antenna array with dual circular polarization and $2M$ ports was proposed. The array is modeled using low-cost laminates and designed to operate with RHCP for LoS signals, and RHCP+LHCP for ground-reflected signals in GNSS-R scenarios. The structure was simulated and optimized using Ansys HFSS and good impedance matching and isolation between the ports were achieved. The main polarization can be obtained by selecting the appropriate active ports. To achieve this, 90° hybrid couplers were used in the antenna feed system.

The conventional MUSIC algorithm was adapted to process dual-CP fields, enabling DoA estimation. The results demonstrated a good DoA estimation accuracy across several scenarios, even at a relatively low signal-to-noise ratio. The signal model and the adaptation of the traditional MUSIC algorithm seem adequate for GNSS-R applications, as the DoAs of several signals even with low SNR of 1 dB can be estimated with an RMSE of less than 2.32° .

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