

Recursive Non-Data-Aided SNR Estimation for Coherent Free-Space Optical Systems

Humberto V. Q. Melo, Robson A. Colares and Darli A. A. Mello

Abstract—We evaluate recursive and non-data-aided (NDA) signal-to-noise ratio (SNR) estimators for coherent free-space optical (FSO) systems. By replacing the block-wise moment accumulations of the M2M4 and skewness-corrected signal-to-noise variance (SNV) estimators with exponentially weighted recursive updates, the proposed framework achieves symbol-rate SNR tracking without requiring large buffers. Simulation validations confirm successful SNR tracking and experimental validations confirm rapid convergence under real hardware impairments.

Keywords—SNR estimation, free-space optics.

I. INTRODUCTION

Free-space optical (FSO) communication has emerged as a compelling complement to radio frequency (RF) and fiber-based links, offering license-free operation, high spectral efficiency, and data rates compatible with those of backbone optical networks [1, 2]. By exploiting coherent detection and advanced digital signal processing (DSP), modern FSO transceivers can achieve high bit rate transmission over multi-kilometer links [2, 3]. Despite these advantages, FSO links are inherently vulnerable to atmospheric turbulence, which induces random intensity fluctuations known as scintillation. Under strong turbulence, the instantaneous received power can drop by tens of decibels over time scales on the order of milliseconds, a phenomenon commonly referred to as deep fading [4, 1]. The Gamma-Gamma distribution has been widely adopted to model the irradiance statistics of FSO channels under a broad range of turbulence conditions, from moderate to strong regimes [4, 5]. These rapid and severe fluctuations impose strict requirements on the receiver's ability to continuously monitor the instantaneous link quality, since an inaccurate or delayed estimate of the channel state can render rate adaptive mechanisms ineffective [3].

Adaptive coding modulation (ACM) is a natural countermeasure against turbulence-induced fading. By dynamically adjusting the modulation order and the forward error correction (FEC) code rate in response to the estimated channel state information (CSI), ACM can substantially improve the average spectral efficiency of an FSO link while keeping the bit error rate (BER) below a prescribed threshold [2, 3]. Effective ACM, however, requires accurate and low-latency estimates of a link quality metric such as the instantaneous signal-to-noise ratio (SNR) or the pre-FEC BER.

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Various non-data-aided (NDA) SNR estimation techniques have been proposed in the literature [6, 7, 8, 9]. Among the NDA approaches, the M2M4 estimator exploits the second and fourth-order moments of the received signal to estimate the SNR without requiring pilot symbols or knowledge of the transmitted data [6]. Pauluzzi and Beaulieu show that the M2M4 method approaches the Cramér-Rao bound (CRB) at high SNR for quadrature phase-shift keying (QPSK) signals in additive white Gaussian noise (AWGN) [6, 10] channels. Moreover, Harris and Dick propose a skewness-based correction applied to the signal-to-noise variance (SNV) estimator, which yields nearly unbiased SNR estimates even at negative SNR values [11]. Hereinafter, this method is referred to simply as the Correction Method. Both the M2M4 method and the Correction Method are inherently insensitive to residual carrier phase noise, a property of particular value in FSO receivers where phase recovery may be severely degraded during deep fades. Moreover, both methods are conventionally implemented in a block-wise fashion, whereby moments are accumulated over a fixed window of N received symbols before a single SNR estimate is produced. This batch processing requires large buffers for sample storage.

In a previous experimental study, we show that the M2M4 estimator reliably tracks the SNR profile in FSO systems, outperforming the error vector magnitude (EVM)-based SNR estimator particularly at low SNR values [12]. In this paper, we propose replacing the block-wise moment accumulations with exponentially weighted recursive updates applied symbol by symbol using a forgetting factor. The resulting recursive estimators produce a new SNR estimate at every symbol period while retaining a geometrically weighted memory of past observations. We show, through simulations, that both recursive estimators (based on the M2M4 method and Correction Method) approach the performance of their block-wise counterparts in normalized mean squared error (NMSE) across the entire evaluated SNR range. Additionally, the simulations show that both recursive estimators are able to continuously track SNR fluctuations due to scintillation. Experimental validation on a 4-GBaud coherent testbed confirms the convergence behavior in the presence of real hardware impairments.

The remainder of this paper is organized as follows. Section II presents the conventional M2M4 method and the Correction Method and introduces the proposed recursive framework. Section III describes the simulation setup and simulation results. Section IV details the experimental testbed and experimental results. Lastly, Section V concludes the paper.

II. SNR ESTIMATION METHODS

A. M2M4 Method

The M2M4 estimator exploits the second- and fourth-order moments of the received signal to estimate SNR without requiring pilot symbols. The p -th order sample moment of the received sequence can be estimated in a block-wise manner as

$$\hat{M}_p \triangleq \frac{1}{N} \sum_{n=0}^{N-1} |y(n)|^p. \quad (1)$$

The M2M4 estimator uses the specific instances $p = 2$ and $p = 4$. For QPSK modulation, it can be computed as [6]

$$\widehat{\text{SNR}}_{\text{M2M4}} = \frac{\sqrt{2\hat{M}_2^2 - \hat{M}_4}}{\hat{M}_2 - \sqrt{2\hat{M}_2^2 - \hat{M}_4}}. \quad (2)$$

Because Eq. (2) depends exclusively on the moments of $|y(n)|$, the estimator is inherently insensitive to residual carrier phase noise, a property of particular value in FSO receivers. At high SNR, the M2M4 estimator approaches the Cramér–Rao bound (CRB) for QPSK signals in AWGN. However, its accuracy degrades significantly below approximately 0 dB. In this regime, statistical fluctuations of $\hat{M}_2$ and $\hat{M}_4$ can drive the argument of the square root in Eq. (2) towards zero or negative values, destabilizing the estimate and inducing a large positive bias [6]. This limitation motivates the evaluation of SNR estimation methods with bias-correction approaches.

B. Correction Method

At low SNR, the absolute-value operation on $y(n)$ causes a statistical distortion to the conditional probability density of $|y(n)|$, inflating $\hat{M}_1$ and compressing the apparent variance of the magnitude sequence [11]. As a result, the conventional SNV estimator

$$\text{SNR}_{\text{SNV}} = \frac{\hat{M}_1^2}{2(\hat{M}_2 - \hat{M}_1^2)}, \quad (3)$$

systematically overestimates the true SNR for $\text{SNR} \lesssim 8$ dB with a smooth and monotonic estimation bias.

In [11], *Harris et al.* propose the use of the sample skewness of $|y(n)|$ to correct the estimation bias of Eq. (3). The sample skewness of $|y(n)|$ can be expressed in terms of the p -th order sample central moment,

$$\hat{\mu}_p \triangleq \frac{1}{N} \sum_{n=0}^{N-1} (|y(n)| - \hat{M}_1)^p, \quad (4)$$

as

$$\hat{\gamma}_y = \frac{\hat{\mu}_3}{\hat{\mu}_2^{3/2}}. \quad (5)$$

At high SNR, the distribution of $|y(n)|$ is nearly symmetric and $\hat{\gamma}_y \approx 0$. As SNR decreases, the folding effect increases the asymmetry of the distribution, so $\hat{\gamma}_y$ rises monotonically. This behavior allows the estimation bias in Eq. (3) to be approximated by a correction factor based on the polynomial function of the sample skewness

$$\Delta(\hat{\gamma}_y) = a_1 \hat{\gamma}_y + a_2 \hat{\gamma}_y^2. \quad (6)$$

The bias-corrected SNR estimate in logarithmic scale is then given by

$$\widehat{\text{SNR}}_{\text{dB}} = 10 \log_{10}[\text{SNR}_{\text{SNV}}] - \Delta(\hat{\gamma}_y). \quad (7)$$

The coefficients a_1 and a_2 in Eq. (6) can be determined by a least squares regression to minimize the mean squared error between $\widehat{\text{SNR}}_{\text{dB}}$ and the true SNR for a data set, in which the true SNR is known. The Correction Method retains the non-data-aided character of the SNV estimator and introduces negligible computational overhead, consisting solely of the third-order central moment $\hat{\mu}_3$ and a two-term polynomial evaluation per block. Its principal requirement is an offline calibration stage to determine the regression coefficients.

C. Recursive moment estimation

The block-wise definitions in (1) and (4) require accumulating N samples before producing an estimate, limiting temporal resolution to one update per block duration, or recording an entire sample block for moving average computation. To achieve symbol-rate estimation without extensive buffering, we propose replacing block-wise accumulations by a recursive estimation of the p -th order moments, M_p , and central moments, μ_p , applied symbol by symbol. At each received symbol $y(n)$, the observation $|y(n)|^p$ serves as an instantaneous measurement of M_p . The p -th order moment is then updated as

$$\widetilde{M}_p(n) = \lambda \widetilde{M}_p(n-1) + (1-\lambda) |y(n)|^p, \quad (8)$$

where $\lambda \in (0, 1)$ is the forgetting factor.

Furthermore, the envelope instantaneous deviation from the current mean estimate is defined as

$$d(n) \triangleq |y(n)| - \widetilde{M}_1(n). \quad (9)$$

The central moments are then updated as

$$\widetilde{\mu}_p(n) = \lambda \widetilde{\mu}_p(n-1) + (1-\lambda) d^p(n). \quad (10)$$

Thus, the recursive SNR estimates $\widehat{\text{SNR}}(n)$ are obtained by substituting the recursively updated moments and central-moments in Eq. (8) and Eq. (10) in place of their block-wise counterparts Eq. (1) and Eq. (4) in the estimators of Sections II-A and II-B.

The effective memory of the recursive estimators is

$$N_{\text{eff}} = \frac{1}{\alpha} \quad [\text{symbols}], \quad (11)$$

where $\alpha \in (0, 1)$ is the tracking rate, defined as $\alpha \triangleq 1 - \lambda$, which controls the trade-off between estimation variance and channel tracking speed. A larger α reduces N_{eff} , enabling faster adaptation to turbulence-induced fading at the cost of increased estimation noise, while a smaller α improves statistical averaging at the cost of slower tracking. Consequently, the optimal α depends on the turbulence coherence time and minimizes the estimation error over fading realizations.

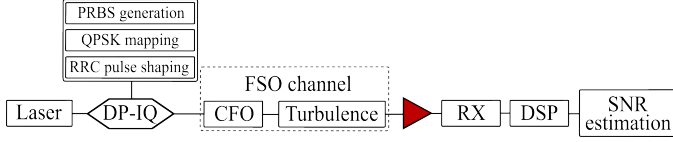


Fig. 1. Simulation setup block diagram for the evaluation of the SNR estimators.

III. SIMULATION SETUP AND RESULTS

A. Simulation setup

To evaluate the performance of the proposed recursive estimators, we implemented the simulation setup illustrated in Fig. 1. A QPSK signal is generated from a pseudo-random binary sequence (PRBS), pulse-shaped with a root raised cosine (RRC) filter with roll-off 0.1, and modulated onto an optical carrier at symbol rate $R_s = 4$ Gbaud via a dual-polarization in-phase and quadrature modulator (DP-IQ).

A sinusoidal carrier frequency offset (CFO) is applied to account for Doppler in the FSO channel and laser frequency offsets. The turbulence of the FSO channel is modeled by a Gamma–Gamma fading process corresponding to a strong turbulence regime, and coherence time $T_c = 100 \mu s$. The fading realizations are generated using the stochastic differential equation method according to [5]. The amplified spontaneous emission (ASE) noise is added by an erbium-doped fiber amplifier (EDFA) positioned immediately before the receiver. At the receiver, coherent detection is followed by a DSP chain consisting solely of a constant modulus algorithm (CMA) adaptive equalizer. The SNR estimators are applied to the post-DSP received symbols.

B. Simulation results

The proposed recursive estimation framework is evaluated using numerical simulations with the previously described simulation setup. The estimator performance is quantified using the NMSE, defined as

$$\text{NMSE} = \mathbb{E} \left[\left(\frac{\widehat{\text{SNR}} - \text{SNR}}{\text{SNR}} \right)^2 \right]. \quad (12)$$

First, we compare the performance of the recursive and the block-wise estimators. Figure 2 compares the NMSE as a function of the pre-DSP SNR for the block-wise and recursive variants of both estimators. The channel is assumed static, which is justified by the fact that the memory lengths of the estimators are much shorter than the channel coherence time T_c . To enable a fair comparison, the block-wise estimators use observation windows of $N = 1000$ symbols and the recursive methods use $\alpha = 10^{-3}$, yielding $N_{\text{eff}} = 1000$. Also, we consider only the estimations after convergence of the recursive methods.

The recursive estimators consistently outperform their block-wise counterparts across the entire evaluated SNR range. This gain stems from the exponentially weighted averaging in Eqs. (8) and (10): unlike block-wise estimators, which treat all N samples within a window equally and discard past

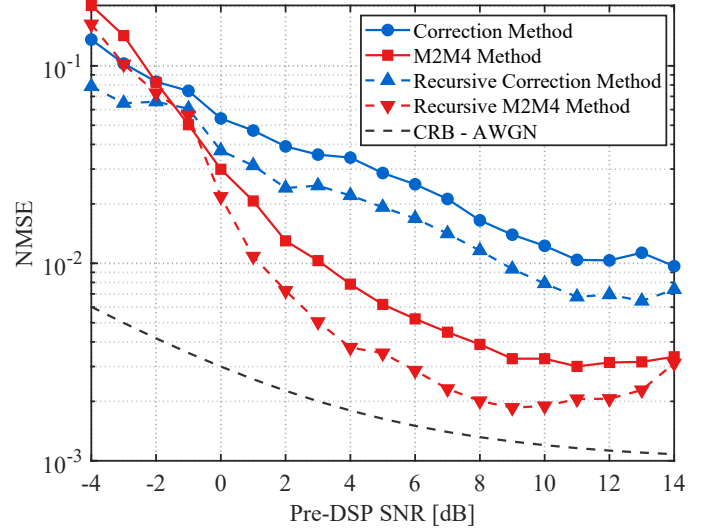


Fig. 2. NMSE versus pre-DSP SNR. The figure compares the block-wise and recursive variants of the M2M4 method and Correction Method. For the block-wise methods the sample averages are computed in blocks of 1000 symbols. The pre-DSP CRB for the AWGN channel is included as a reference lower bound.

observations at each update, the recursive approach retains a continuously updated, geometrically weighted history of all past observations, yielding lower moment-estimation variance.

Regarding the relative performance of the two methods, M2M4 achieves lower NMSE than the Correction Method across most of the evaluated range. However, below about 0 dB, the NMSE of the M2M4 estimator rises sharply, as the estimator becomes biased at low SNR [6]. The Correction Method partially mitigates this degradation through the skewness-based bias correction of Eq. (7), at the cost of higher variance at moderate and high SNR. Furthermore, the M2M4 method does not approach the CRB at high SNR as in [6]. This gap is attributable to the post-DSP SNR penalty introduced by the residual mean squared error (MSE) of the CMA equalizer, which degrades the effective signal quality seen by the estimator beyond what the CRB for an ideal AWGN channel accounts for.

The recursive estimators employ an exponential forgetting factor, which controls the trade-off between temporal smoothing and tracking capability. Smaller values of the tracking rate, α , increase the averaging window, reducing estimation variance, while larger values improve tracking responsiveness to rapid channel fluctuations. Therefore, the selection of α directly impacts the estimation accuracy.

Figure 3(a) shows the NMSE as a function of α for the recursive Correction and recursive M2M4 methods considering a Gamma–Gamma distribution with a channel coherence time of $T_c = 100 \mu s$. Both methods exhibit a well-defined optimal operating point, corresponding to the minimum NMSE. The recursive Correction Method achieves its best performance around $\alpha \approx 2 \times 10^{-3}$, whereas the recursive M2M4 method reaches its optimum near $\alpha \approx 10^{-3}$. Additionally, the recursive M2M4 estimator consistently achieves lower NMSE values over the entire evaluated range. Figure 3(b) presents the estimated SNR trajectories obtained using the optimal α

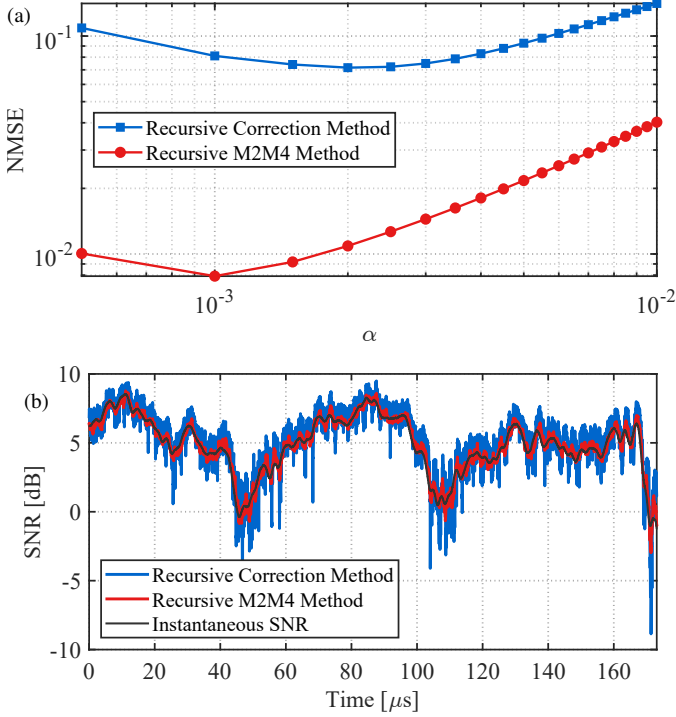


Fig. 3. Simulation results over a Gamma-Gamma FSO fading channel with coherence time $T_c = 100 \mu\text{s}$. (a) NMSE versus tracking rate α . (b) Estimated SNR versus time at the optimal α for each method.

values. Both recursive estimators are capable of tracking the channel fluctuations. However, the recursive M2M4 method provides smoother estimates while maintaining adequate responsiveness to fading variations. In contrast, the recursive Correction Method exhibits larger estimation fluctuations, indicating higher estimation variance.

IV. EXPERIMENTAL SETUP AND RESULTS

Experimental measurements were conducted to evaluate the recursive estimators under realistic hardware impairments using the setup shown in Fig. 4. Dual-polarization QPSK digital waveforms are generated by an arbitrary waveform generator (AWG, Keysight M8199A-ATO) operating at 128 GSa/s, with each channel driven by a 15th-order and an 11th-order PRBS, respectively. The signal is transmitted at a symbol rate of 4 GBaud.

The electrical signals produced by the AWG drive a dual-polarization in-phase/quadrature modulator (DP-IQ, IDPhotonics OMFT-C-01-FA), which modulates a narrow-linewidth external cavity laser (ECL). An optical isolator is placed immediately after the DP-IQ modulator to suppress back-reflections and prevent spurious optical feedback into the laser cavity. The modulated signal is subsequently amplified by an erbium-doped fiber amplifier (EDFA, Padtec LOAC211GAH) and combined with ASE noise through a 3-dB optical coupler. The ASE power level is controlled by a variable optical attenuator (VOA, Eigenlight 420 WDM) equipped with an integrated power meter, enabling adjustment of the optical SNR.

The combined signal is passed through a tunable bandpass filter (OTF-350), whose passband is selected to yield a spec-

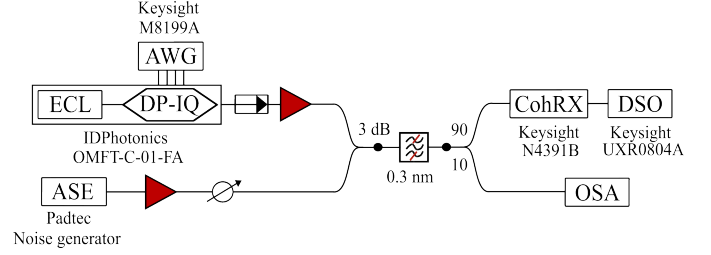


Fig. 4. Experimental setup for SNR estimation. ECL: external cavity laser; DP-IQ: dual-polarization in-phase/quadrature modulator; AWG: arbitrary waveform generator; ASE: amplified spontaneous emission; CohRX: coherent receiver; DSO: digital sampling oscilloscope; OSA: optical spectrum analyzer.

trally flat region around the signal while retaining all spectral content required for DSP. The filtered signal is then directed into a 90:10 optical coupler: 90% of the optical power is routed to the coherent receiver (CohRX), while the remaining 10% is monitored by an optical spectrum analyzer (OSA) with a 0.1-nm resolution bandwidth for independent SNR measurement. Coherent detection is performed using a free-running local oscillator (LO) laser integrated within an optical modulation analyzer (OMA, Keysight N4391B-059). The coherent receiver's electrical outputs are sampled at 32 GSa/s by a high-speed digital storage oscilloscope (DSO, Keysight UXR0594A) and subsequently processed offline.

Figure 5 shows the convergence of both recursive estimators for SNR levels of (a) 1 dB, (b) 6 dB, and (c) 11 dB. Both estimators exhibit an initial transient before converging to a steady-state value depicted by the long-term convergence reference. Both methods converge within 4×10^3 symbols across a wide range of SNR levels. The recursive M2M4 method converges slightly faster, with low steady-state variance. As expected from simulations, the recursive Correction Method shows higher steady-state variance than the M2M4 method.

V. CONCLUSION

We propose recursive and non-data-aided SNR estimation methods for coherent FSO systems. By replacing the block-wise moment accumulations of the M2M4 and SNV estimators with symbol-by-symbol recursive updates, the proposed framework achieves continuous SNR tracking without requiring large buffers. Simulation results over a Gamma-Gamma channel with coherence time $T_c = 100 \mu\text{s}$ show that both recursive estimators outperform their block-wise counterparts in NMSE across the entire evaluated SNR range, with the recursive M2M4 method achieving the best overall performance. Experimental validation on a 4 GBaud optical testbed shows that both methods converge within 4,000 symbols for a wide range of SNRs. As expected, the recursive Correction Method presents higher steady-state variance compared to the M2M4 method.

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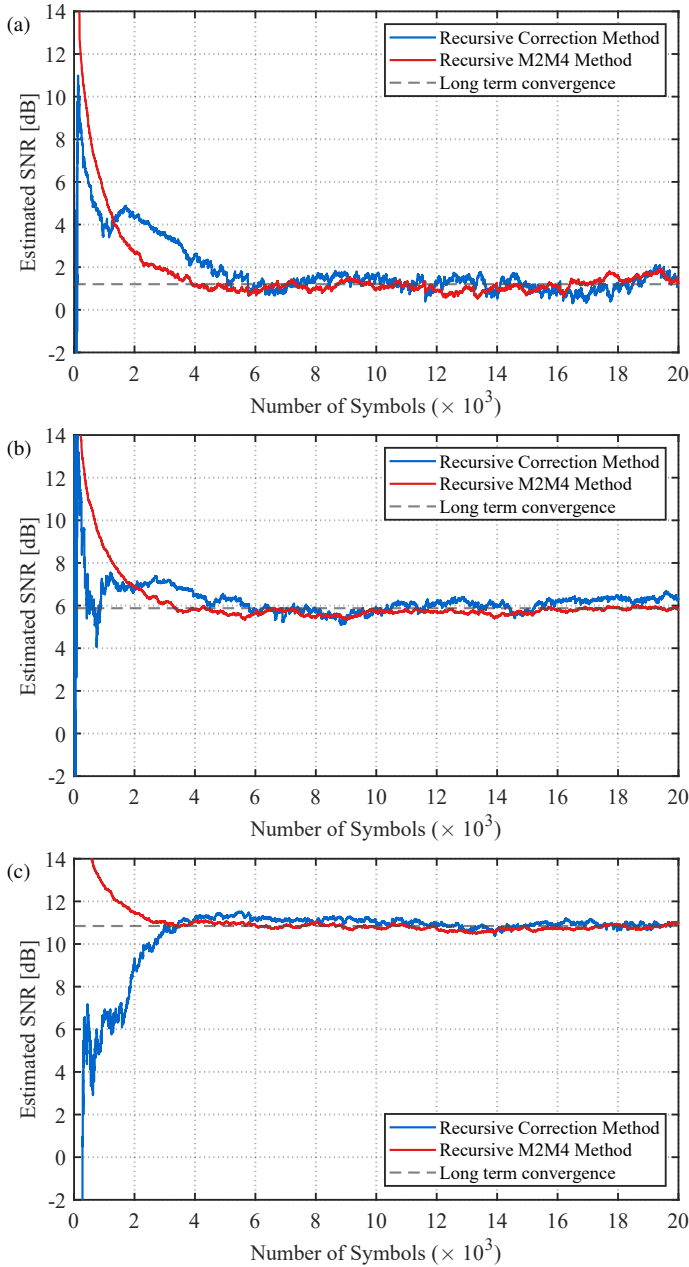


Fig. 5. Experimental convergence of the recursive SNR estimators for three SNR levels: (a) 1 dB, (b) 6 dB, and (c) 11 dB. The dashed line indicates the long-term convergence of both methods.

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