

Practical Limitations of the Known Waveform DoA Estimation Algorithm

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Abstract—Known-waveform (KW) direction-of-arrival estimators exploit a known reference signal to improve spatial selectivity, but their practical performance depends on operating conditions that have not been systematically quantified. This paper provides a rigorous simulation-backed characterization of three such conditions: interferer modulation format, synchronization error between the received signal and the local reference, and available training-sequence length. We adopt a packet-based observation model in which the known preamble occupies a finite portion of the received buffer so that synchronization errors shift the correlation window into pre-arrival noise or unknown payload, and we evaluate performance via Monte Carlo trials on a compact uniform circular array. Results show that the KW approach retains a clear advantage over classical blind methods under unstructured interference; it remains largely robust across common structured interferer modulations but exhibits measurable residual sensitivity dependent on symbol alphabet and pulse shaping; its performance degrades sharply outside a narrow synchronization tolerance around correct alignment; and a regime-dependent minimum training length is required before KW gains appear. Together, these results delineate the operating envelope for KW spatial processing and yield concrete guidelines: prefer KW estimators when synchronization can be maintained within the identified tolerance and sufficient preamble length is available; otherwise classical covariance-based approaches may be preferable.

Keywords—Direction-of-arrival estimation, known waveform, interference rejection, synchronization error, training sequence, software-defined radio.

I. INTRODUCTION

The deployment of antenna arrays in tactical and commercial receivers has become widespread, driven by the consolidation of Software-Defined Radios (SDRs) as the dominant signal-processing architecture for such systems [1]. Antenna arrays enable the exploitation of the spatial domain through beamforming and Direction-of-Arrival (DoA) estimation, enhancing the gain of a signal of interest (SOI) and suppressing interference [2], [3]. Modern digital communication standards typically include preambles or training sequences at the beginning of data packets [4], [5], providing *a priori* information that can be systematically exploited for spatial processing.

Despite these advancements, the integration of antenna arrays into portable platforms such as handheld tactical radios and unmanned aerial vehicles imposes severe constraints on the array size, often resulting in inter-element spacing well below the half-wavelength spacing assumed in most classical

array-processing derivations [6], [7]. In such compact configurations, electromagnetic coupling between closely spaced elements is no longer negligible, modifying both the amplitude and the phase of the signals collected at each port and producing a deviation between the actual array response and the ideal steering vector [8], [9]. Spatial-processing algorithms that rely on a precise model of the array manifold are particularly affected by this mismatch and may exhibit large estimation errors even at moderate noise levels [10].

Waveform-aided strategies offer a fundamentally different approach to coping with these impairments. Rather than inferring the spatial signature from the statistics of the received vector alone, a known-waveform estimator correlates the array output with the known reference and directly extracts the steering response associated with the desired source, without invoking the eigenstructure of the sample covariance matrix [11]–[14]. The reference waveform thus serves as a deterministic anchor, reducing dependence on the fidelity of the manifold model and yielding measurable robustness gains under severe interference and mutual coupling, as shown by the effects of mutual coupling on interference mitigation reported in [15].

The price for these advantages, however, is a stricter set of operating prerequisites: the interferer must be statistically uncorrelated with the local reference, the local copy of the waveform must be time-aligned with the received signal, and the training sequence must be long enough for the empirical correlations to converge to their statistical expectations.

These conditions are routinely taken for granted in the KW literature but are not guaranteed in operational SDR deployments. This paper examines each of them as a practical limitation, quantifying how far the KW estimator can be pushed before its advantage over classical alternatives disappears. The main contributions are:

- analysis of how different interferer modulation types affect KW performance;
- assessment of the impact of synchronization error on DoA accuracy;
- evaluation of the influence of training-sequence length on estimator robustness.

This paper is organized as follows. Section II establishes the system and signal models, including the preamble-based packet structure adopted for the synchronization analysis. Section III reviews the KW interferometric estimator and the classical DoA algorithms used for comparison. Section IV presents and discusses the simulation results for each of the three limitations. Section V concludes the paper.

II. SYSTEM AND SIGNAL MODEL

The antenna array considered in this work is a Uniform Circular Array (UCA) composed of M antennas arranged equidistantly on a circumference of radius r . The angular position of the m -th element is given by

$$\beta_m = \frac{2\pi(m-1)}{M}, \quad m = 1, \dots, M. \quad (1)$$

A single SOI, whose waveform is partially known to the receiver, and one interferer impinge on the array as narrowband plane waves. The complex baseband snapshot vector at sample instant k is modeled as

$$\mathbf{x}_{\text{ideal}}(k) = s(k) \mathbf{a}(\theta_s, \phi_s) + i(k) \mathbf{a}(\theta_i, \phi_i) + \mathbf{n}(k), \quad (2)$$

where $s(k)$ is the desired-signal waveform, $i(k)$ is the interferer, $\mathbf{n}(k)$ is the additive complex Gaussian noise vector, and the UCA steering vector is given by

$$\mathbf{a}(\theta, \phi) = \begin{bmatrix} e^{-j(2\pi/\lambda_0) r \sin \theta \cos(\phi - \beta_1)} \\ e^{-j(2\pi/\lambda_0) r \sin \theta \cos(\phi - \beta_2)} \\ \vdots \\ e^{-j(2\pi/\lambda_0) r \sin \theta \cos(\phi - \beta_M)} \end{bmatrix}, \quad (3)$$

where λ_0 is the operating wavelength. The zenith and azimuth angles of the SOI and the interferer are denoted by (θ_s, ϕ_s) and (θ_i, ϕ_i) , respectively.

A. Mutual Coupling

In compact UCAs, where the inter-element spacing is appreciably smaller than half a wavelength, the electromagnetic interaction between closely spaced radiating elements is no longer negligible. This phenomenon, known as mutual coupling, alters both amplitudes and phases of the signals captured at each port and introduces a deviation between the actual array response and the ideal steering vector defined in (3). The coupling matrix $\mathbf{C}_M \in \mathbb{C}^{M \times M}$ used in this work is obtained from a full-wave electromagnetic simulation of the UCA performed with the MATLAB® Antenna Array Designer Toolbox, using vertically oriented half-wavelength dipoles as array elements. The coupled snapshot vector observed by the receiver is then modeled as

$$\mathbf{x}(k) = \mathbf{C}_M \mathbf{x}_{\text{ideal}}(k), \quad (4)$$

where $\mathbf{x}_{\text{ideal}}(k)$ denotes the snapshot vector that would be observed in the absence of mutual coupling. Throughout this work, all DoA estimators operate on the coupled vector $\mathbf{x}(k)$, so the resulting manifold mismatch is included in the conditions under which the estimators are evaluated.

B. Packet model with finite preamble

The KW literature traditionally assumes that the desired waveform $s(k)$ is fully known at the receiver and is observed over the entire processing window [11], [12], [14]. This assumption is convenient for the derivation of the estimator but does not reflect the typical structure of a digital communication packet, in which only the preamble, or training sequence, is known and is preceded by an asynchronous arrival interval

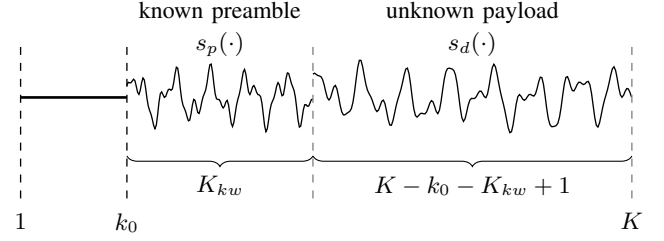


Fig. 1: Structure of the received SOI within the observation window of K samples. The known preamble $s_p(\cdot)$ with K_{kw} samples starts at $k = k_0$ and is followed by the unknown payload $s_d(\cdot)$.

and followed by unknown payload symbols modulated by the same source. To capture this structure, the SOI contribution to (2) is rewritten as $s(k) \mathbf{a}(\theta_s, \phi_s)$ with

$$s(k) = \begin{cases} 0, & 1 \leq k < k_0, \\ s_p(k - k_0 + 1), & k_0 \leq k < k_0 + K_{kw}, \\ s_d(k - k_0 - K_{kw} + 1), & k_0 + K_{kw} \leq k \leq K, \end{cases} \quad (5)$$

where K is the length of the received buffer, K_{kw} is the length of the known preamble, k_0 is its true arrival index within the buffer, $s_p(\cdot)$ is the reference preamble waveform known by the receiver, and $s_d(\cdot)$ is the unknown payload, which shares the carrier frequency and direction of arrival of $s_p(\cdot)$. Figure 1 illustrates the resulting structure of the received SOI within the observation window. Note that the interference and noise terms are present along the entire window.

A synchronization mismatch is modeled by an offset Δ , expressed in samples, between the true and the estimated arrival indices: the KW estimator extracts a K_{kw} -sample window starting at $\hat{k}_0 = k_0 + \Delta$. For $\Delta = 0$ the window is aligned with the preamble and the estimator operates under its nominal assumptions. For $\Delta < 0$ the window overlaps the zero-padded interval that precedes the packet, reducing the effective signal energy available to the cross-correlation. For $\Delta > 0$ the window overlaps the unknown payload $s_d(\cdot)$, so that the cross-correlation accumulates contributions from random data of the same source, sharing direction of arrival but uncorrelated with the local reference. The two failure modes are qualitatively distinct, and Section IV quantifies the practical consequences of each.

III. ESTIMATORS UNDER EVALUATION

The KW interferometric estimator for UCAs employed in this work was proposed in [14] and is summarized below. Given the K_{kw} -sample window starting at $\hat{k}_0$, the algorithm forms the sample cross-correlation vector and auto-correlation scalar

$$\hat{\mathbf{r}}_{xy} = \frac{1}{K_{kw}} \sum_{k=\hat{k}_0}^{\hat{k}_0 + K_{kw} - 1} \mathbf{x}(k) s_p^*(k - \hat{k}_0 + 1), \text{ and} \quad (6)$$

$$\hat{r}_{yy} = \frac{1}{K_{kw}} \sum_{k=1}^{K_{kw}} |s_p(k)|^2. \quad (7)$$

from which the maximum-likelihood estimate of the spatial signature is obtained as $\hat{\mathbf{b}} = \hat{\mathbf{r}}_{xy}/\hat{r}_{yy}$. Under the ideal signal model, the m -th entry of $\hat{\mathbf{b}}$, denoted $\hat{b}_m$, is asymptotically colinear with the m -th entry of the steering vector, $a_m(\theta_s, \phi_s)$, differing only by an unknown complex scalar that cancels in any phase-difference between two elements of $\hat{\mathbf{b}}$. The phase difference associated with the sensor pair (p, q) is therefore

$$\hat{\eta}_{p,q} = \arg\{\hat{b}_p \hat{b}_q^*\}, \quad (8)$$

and, from the UCA geometry, it relates linearly to the unknown angles through

$$\hat{\eta}_{p,q} = -\frac{2\pi r}{\lambda_0} [\cos \beta_p - \cos \beta_q, \sin \beta_p - \sin \beta_q] \mathbf{g}, \quad (9)$$

with $\mathbf{g} = [\sin \theta_s \cos \phi_s, \sin \theta_s \sin \phi_s]^T$. Stacking (9) for all non-redundant baselines yields the linear system $\hat{\boldsymbol{\eta}} = \mathbf{C} \mathbf{g}$, where the rows of $\mathbf{C}$ encode the array geometry, each pair (p, q) contributing one row

$$-\frac{2\pi r}{\lambda_0} [\cos \beta_p - \cos \beta_q, \sin \beta_p - \sin \beta_q], \quad (10)$$

such that $\mathbf{C}$ depends only on the array geometry and the operating wavelength, and can be precomputed. The least-squares solution $\hat{\mathbf{g}} = (\mathbf{C}^T \mathbf{C})^{-1} \mathbf{C}^T \hat{\boldsymbol{\eta}}$ then provides azimuth and zenith in closed form, without eigendecomposition or spectral search.

For comparison purposes, three classical algorithms are also evaluated: Delay-and-Sum (D&S), Capon, and MUSIC [3]. These estimators rely exclusively on the sample covariance matrix and do not exploit the structure of $s_p(\cdot)$. To eliminate any asymmetry in sample support, all four estimators operate on the same K_{kw} -sample processing window extracted from $\mathbf{x}(k)$ starting at $\hat{k}_0$. The classical estimators are thus implicitly using a window that may overlap zeros, preamble, or payload depending on Δ ; their dependence on Δ is therefore expected to follow a different mechanism from that of the KW estimator, as discussed in Section IV-B.

IV. SIMULATION RESULTS

The simulation setup adopts a UCA configuration with: $M = 8$ half-wavelength dipoles arranged on a circumference of radius $r = 0.2\lambda_0$ at carrier frequency $f_c = 500$ MHz, with the mutual coupling matrix $\mathbf{C}_M$ computed from a full-wave electromagnetic simulation of the array. The SOI is a FSK2 waveform with symbol rate $R_s = 9600$ sym/s, frequency deviation of 4.8 kHz, and $N_{\text{sym}} = 30$ samples per symbol. Sources lie on the horizontal plane ($\theta = 90^\circ$), and their azimuth angles are independently drawn from $[-180^\circ, +180^\circ]$ at each realization. The reported accuracy is the average Root Mean Square Error (RMSE) of the SOI azimuth estimate, computed over 1000 Monte Carlo realizations in which bits, symbol streams, interferer realizations, noise, and source directions are regenerated independently.

A. Effect of Interferer Modulation Type

This experiment isolates the effect of the interferer waveform on the four estimators. The desired signal occupies the

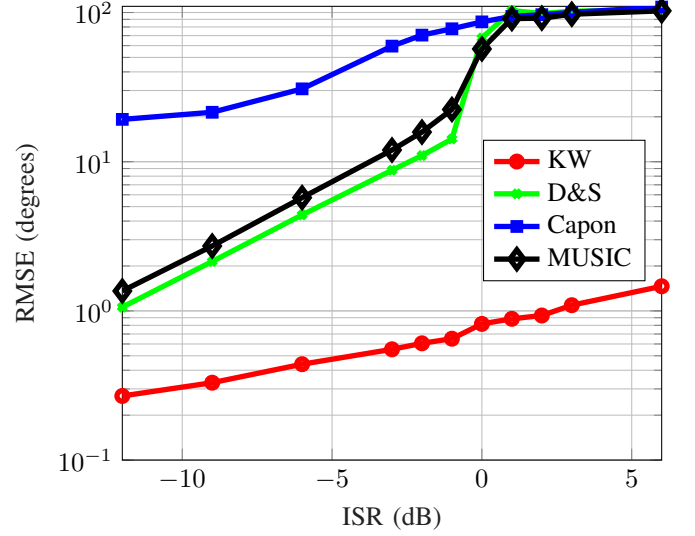


Fig. 2: Average RMSE as a function of ISR for a noise-like interferer (SNR = 10 dB, $K = 2000$ snapshots, $\Delta = 0$).

entire observation window of $K = 2000$ samples and the synchronization offset is set to $\Delta = 0$. Five representative interferer types are considered: AM, FM, FSK2, QAM64, and complex Gaussian noise. The classical estimators are evaluated for the two extreme cases (FSK2 and noise-like), since their behavior depends weakly on the modulation format of the interferer, while the KW estimator is evaluated across all five cases.

Figure 2 presents a comparison of the four estimators against a noise-like interferer. The KW estimator maintains the lowest RMSE across the entire ISR range. Among the classical methods, Capon exhibits the fastest degradation as the interferer power increases. At the same time, D&S and MUSIC degrade jointly and approach the saturation level associated with random azimuth assignment as ISR approaches the equal-power regime.

Figure 3 restricts the comparison to the KW estimator across the five interferer types. Two distinct behaviors emerge. The four structured modulations (AM, FM, FSK2, and QAM64) produce nearly coincident curves, indicating that, within the structured class, the specific modulation has only a marginal effect on the estimator. In contrast, the noise-like interferer yields a markedly lower RMSE over the entire ISR range.

This ordering, although initially counter-intuitive, has a clear interpretation. The KW estimator relies on the cross-correlation between the array output and the reference $s_p(\cdot)$ to extract spatial information. Structured interferers may yield larger finite-sample cross-correlation residuals with the reference waveform, thereby biasing the estimate. The complex Gaussian interferer, by contrast, is uncorrelated with any deterministic waveform and its contribution vanishes in expectation. The KW estimator is therefore essentially immune to unstructured interference, but retains a residual sensitivity to interferers whose temporal structure overlaps that of the reference waveform.

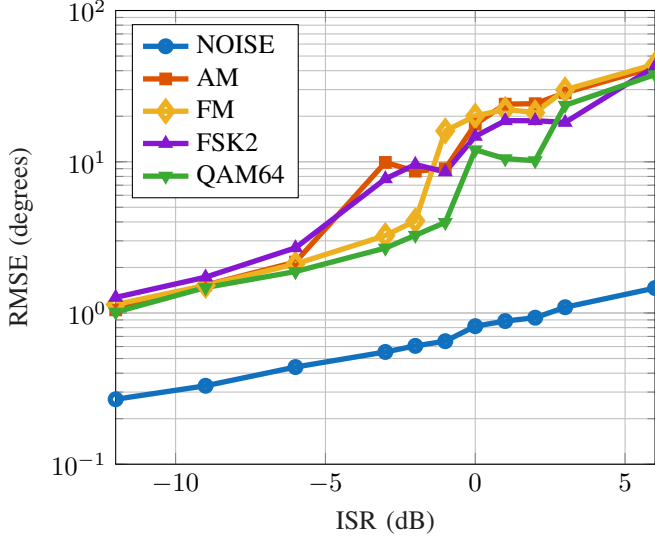


Fig. 3: Average RMSE of the KW estimator as a function of ISR for four structured interferer modulations and a noise-like interferer (SNR = 10 dB, $K = 2000$ snapshots, $\Delta = 0$).

B. Effect of Synchronization Error

The second experiment quantifies the impact of the synchronization offset Δ under the packet model of Section II-B. The received buffer comprises $K = 30\,000$ samples; the preamble length is fixed at $K_{kw} = 3000$ samples (equivalently, 100 symbol periods), and the preamble is centered within the buffer so that the available range of valid offsets exceeds $\pm 50 N_{\text{sym}}$ in either direction. The interferer is set to FM. Two operating points are considered, with $\text{SNR} \in \{0, +10\}$ dB and $\text{ISR} = -6$ dB.

Figure 4 reports the KW estimator across the two operating points. The estimator exhibits a sharp RMSE minimum at $\Delta = 0$ and rapidly transitions, over approximately one symbol period, to the high-error regime corresponding to random azimuth assignment. This narrow recovery region reflects the autocorrelation profile of the reference waveform: outside its support, the cross-correlation ceases to align with the steering vector of the SOI, and the spatial signature collapses.

Figure 5 contrasts the four estimators. The KW estimator achieves the lowest RMSE at $\Delta = 0$ but becomes the worst-performing estimator once the magnitude of the synchronization offset exceeds approximately one symbol period: its RMSE saturates near the upper bound, whereas the classical estimators recover to a broad plateau of moderate error. The classical estimators do not rely on alignment with the local reference; their dependence on Δ arises only through the contents of the K_{kw} -sample window: for large negative Δ the window falls on the zero-padded interval preceding the packet and the SOI is no longer present, whereas for positive Δ it captures the unknown payload, which shares the direction of arrival of the preamble and is therefore equivalent to additional SOI samples from the classical perspective. This explains the asymmetric response observed in Figure 5. The crossover around one symbol period identifies the synchronization range within which waveform-aided processing is preferable.

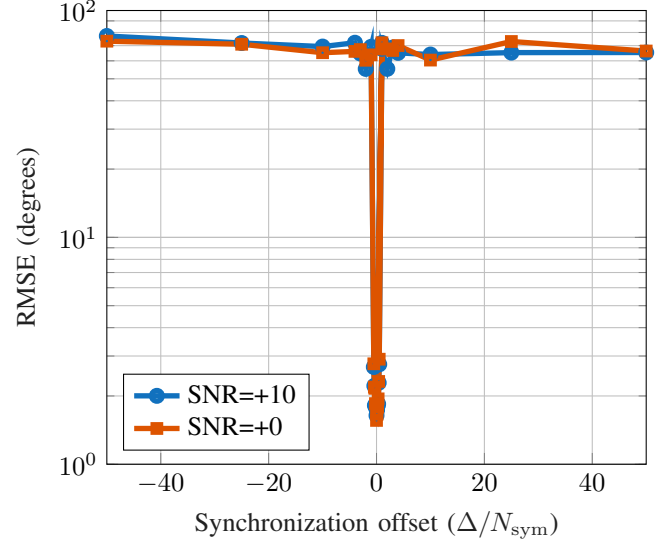


Fig. 4: Average RMSE of the KW estimator as a function of the synchronization offset Δ/N_{sym} for two operating points (FM interferer, $K = 30\,000$, $K_{kw} = 3\,000$ and $N_{\text{sym}} = 30$).

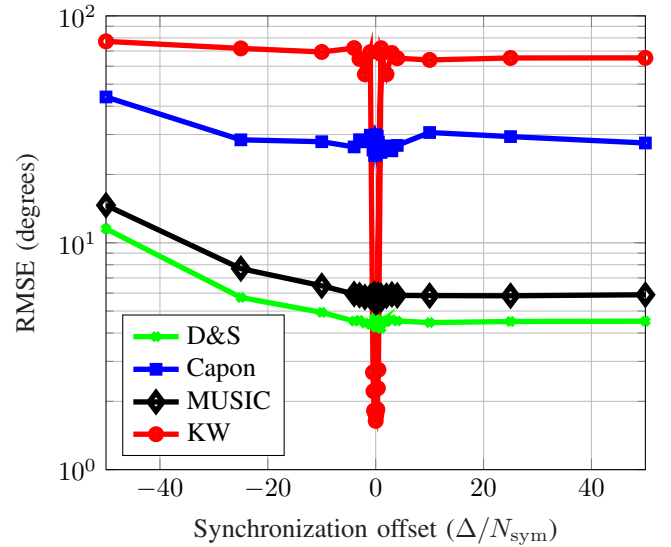


Fig. 5: Average RMSE as a function of Δ/N_{sym} for SNR = +10 dB and $\text{ISR} = -6$ dB, FM interferer ($K = 30\,000$, $K_{kw} = 3\,000$). The KW estimator is the best at $\Delta = 0$ but the worst for any non-zero offset beyond the symbol period.

C. Effect of Training Sequence Length

The third experiment characterizes the dependence of the four estimators on the number of available training samples. The interferer is set to FM, synchronization is perfect ($\Delta = 0$), and four operating points are considered, combining low and moderate SNR with moderate and severe ISR. The training-sequence length K_{kw} is varied between 50 and 8000 samples.

Figure 6 shows the KW estimator across the four operating points. For all regimes, the RMSE generally decreases as K_{kw} increases, in agreement with the consistency of the sample cross-correlation in (6). The rate of decay, however, depends on the interference regime: with moderate interference

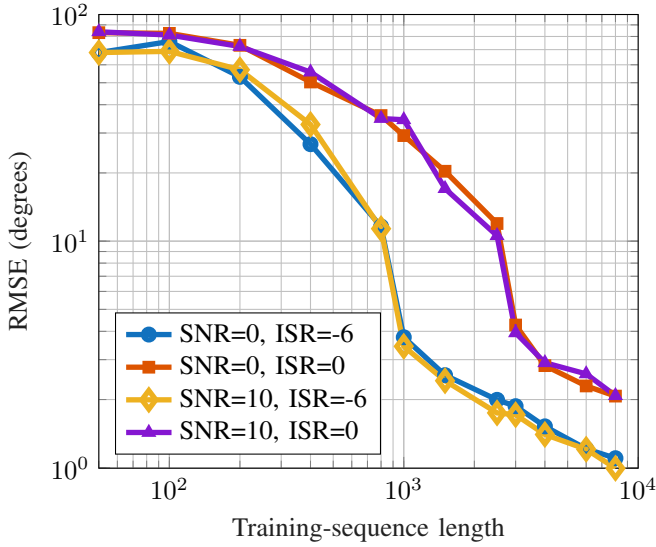


Fig. 6: Average RMSE of the KW estimator as a function of the training-sequence length K_{kw} for four operating points (FM interferer, $\Delta = 0$).

($\text{ISR} = -6$ dB), the estimator reaches sub-degree RMSE with relatively short training sequences, while under strong interference ($\text{ISR} = 0$ dB) reaching a comparable accuracy requires approximately an order of magnitude more samples.

Figure 7 compares the four estimators at a representative operating point ($\text{SNR} = +10$ dB, $\text{ISR} = -6$ dB). For short training sequences ($K_{kw} < 500$), the classical estimators outperform the KW, since the sample correlation matrices are poorly estimated. As K_{kw} increases, the KW RMSE decreases without saturating, while the classical estimators converge to a residual error floor imposed by the array-manifold mismatch caused by mutual coupling. The crossover near $K_{kw} \approx 800$ samples identifies a regime-dependent minimum preamble length below which the KW approach is not advantageous over the classical alternatives.

V. CONCLUSION

This paper characterized three practical limitations of the known-waveform DoA estimator that were left unaddressed by previous evaluations conducted under idealized assumptions. The results show that the estimator is essentially immune to unstructured (noise-like) interference but retains a residual sensitivity to interferers whose structure overlaps that of the reference waveform; that its advantage over classical methods is conditional on a synchronization error of less than approximately one symbol period; and that a regime-dependent minimum training-sequence length exists, below which the classical alternatives outperform the KW approach. Together, these findings delineate the operating envelope within which waveform-aided spatial processing should be preferred, and indicate concrete requirements on the synchronization stage and on the preamble length of any SDR system intended to exploit the KW principle.

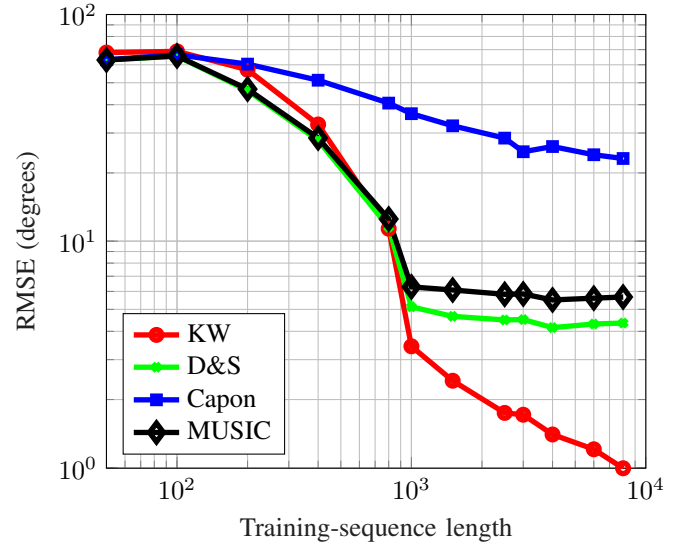


Fig. 7: Average RMSE as a function of K_{kw} for $\text{SNR} = 10$ dB and $\text{ISR} = -6$ dB, FM interferer ($\Delta = 0$). The classical estimators saturate at the manifold-mismatch error floor.

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