

Decoupled Delay-Doppler and Angle Estimation in BD-RIS Sensing via Nested Tucker Decomposition

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Abstract—We study single-target sensing in a group-connected beyond-diagonal reconfigurable intelligent surface (BD-RIS)-assisted monostatic system with K element groups. We propose a Nested Tensor Factorization and Estimation (NTFE) algorithm that models the received signal as a 3-rd-order nested Tucker tensor, thereby decoupling the delay-Doppler and angle domains. The resulting two-stage procedure estimates the target-bearing tensor factors and then extracts the other physical parameters using subspace and closed-form steps. We also analyze identifiability and uniqueness conditions. Simulations show that NTFE exploits the group-connected BD-RIS structure and outperforms state-of-the-art sensing benchmarks.

Index Terms—Beyond diagonal reconfigurable surfaces, target sensing, nested tensor decomposition, Tucker decomposition.

I. INTRODUCTION

Reconfigurable intelligent surface (RIS) technology shapes wireless propagation to improve coverage, spectral efficiency, and signal-to-noise ratio (SNR) [1], but passive RIS operation relies on cascaded channel estimation with suitable training [2]–[5]. For sensing at the sensing transmitter (ST), passive beyond-diagonal RIS (BD-RIS) architectures have emerged for integrated sensing and communications (ISAC) [6], using inter-element coupling to realize generally non-diagonal phase-shift matrices and extra degrees of freedom [7]. Following [8], we adopt a group-connected BD-RIS that preserves intra-group coupling while avoiding full interconnection, balancing flexibility and hardware complexity. Related estimation, beamforming, and ISAC applications appear in [9]–[12].

Tensor-based methods are well suited for this setting because they exploit the multilinear structure of the received signal rather than treating the channel as an unstructured matrix [13]–[16]. Our previous work used a two-stage nested tensor approach based on ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) for delay, Doppler, and angle estimation in diagonal-RIS-assisted multiple-input multiple-output (MIMO) sensing [17], [18], and later applied tensor processing to BD-RIS-assisted bistatic multi-target sensing [12]. As summarized in Table I, this

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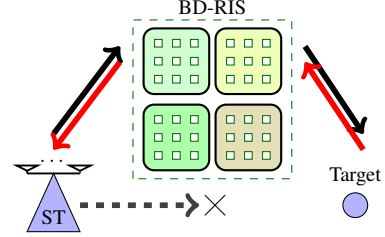


Fig. 1: Group-connected BD-RIS-assisted sensing.

paper bridges these efforts by studying single-target sensing with a group-connected BD-RIS while explicitly exploiting the block-wise structure induced by the element groups to recast the received sensing signal as a nested Tucker tensor, yielding a deterministic, low-complexity estimation pipeline that explicitly decouples the temporal and angular parameters.

We propose a nested Tucker factorization estimation (NTFE) algorithm for target sensing in a BD-RIS-assisted sensing. The received signal is modeled as a 3rd order nested Tucker tensor that separates the delay-Doppler and angle domains. The method estimates the tensor factors, extracts the parameters using subspace and closed-form steps, and admits identifiability and uniqueness conditions. Simulations against state-of-the-art benchmarks, including [19], show improved estimation accuracy in terms of root mean squared error (RMSE) and normalized mean squared error (NMSE).

Notation: Scalars, vectors, matrices, and tensors are denoted by a , $\mathbf{a}$, $\mathbf{A}$, and $\mathcal{A}$, respectively. For matrix $\mathbf{A}$, $\mathbf{A}^*$, $\mathbf{A}^T$, $\mathbf{A}^H$, and $\mathbf{A}^\dagger$ denote the conjugate, transpose, Hermitian transpose, and pseudoinverse. The j th column of $\mathbf{A}$ is denoted by $\mathbf{a}_j$, and $\mathbf{I}_N$ is the $N \times N$ identity. The operator $\text{D}(\cdot)$ maps a vector to a diagonal matrix. The n th mode product is $\mathcal{A} \times_n \mathbf{B} = \mathcal{B}[\mathcal{A}]_{(n)}$. Operators $\otimes$, $\diamond$, $\odot$, and $\oslash$ denote Kronecker, Khatri-Rao, Hadamard, and element-wise division, respectively.

II. SYSTEM MODEL

We consider the monostatic setup in Fig. 1, where a multi-antenna ST is assisted by a group-connected BD-RIS with $N = N_z N_y$ elements split into K groups, with $\sum_{k=1}^K N_k = \bar{N}$. The direct line-of-sight (LOS) path is blocked, so sensing occurs through the BD-RIS-assisted echo link. The ST uses an $L = L_z L_y$ -element uniform planar array and transmits an orthogonal frequency division multiplexing (OFDM) pulse with Q subcarriers, M symbols, and pilots $\mathbf{x}_{q,m} \in \mathbb{C}^{L \times 1}$. The

TABLE I: Comparison of the proposed method with related works

Reference	Architecture	Setup	Targets	Algorithm	Key Distinguishing Feature(s)
[12]	Fully-connected BD-RIS	Bistatic	Single/Multi	Nested PARAFAC	Bistatic scenarios and CRLB analysis
[17]	Standard diagonal RIS	Monostatic	Single	PARAFAC	Exploits rank-one structure
[18]	Standard diagonal RIS	Monostatic	Single	Nested Tucker	SVD-based parameter extraction
[19]	Standard diagonal RIS	Monostatic	Single	ML / OMP	High-complexity sequential search
Proposed	Group-connected BD-RIS	Monostatic	Single	Nested Tucker	Decouples temporal and spatial domains via tensor modeling for enhanced single target sensing

received echo is

$$\begin{aligned}
 \mathbf{y}_{q,m,t} = & \sum_{k=1}^K \alpha_k \underbrace{\mathbf{a}(\phi_{st}, \theta_{st}) \mathbf{b}^T(\phi_{ris_A}, \theta_{ris_A}) \mathbf{S}_{k,t}^T \mathbf{p}(\phi_{ris_D}, \theta_{ris_D})}_{\text{Target-BD-RIS-ST path via group } k} \\
 & \times \underbrace{\mathbf{p}^T(\phi_{ris_D}, \theta_{ris_D}) \mathbf{S}_{k,t} \mathbf{b}(\phi_{ris_A}, \theta_{ris_A}) \mathbf{a}^T(\phi_{st}, \theta_{st})}_{\text{ST-RIS-Target path via group } k} \\
 & \times \mathbf{x}_{q,m}[\mathbf{c}(\tau)]_q [\mathbf{d}(\nu)]_m + \mathbf{z}_{q,m,t},
 \end{aligned}$$

where $\mathbf{a}(\phi_{st}, \theta_{st})$ is the ST response, and $\mathbf{b}(\phi_{ris_A}, \theta_{ris_A})$ and $\mathbf{p}(\phi_{ris_D}, \theta_{ris_D})$ are the RIS steering vectors for the ST and target, respectively. We assume half-wavelength spacing with the ST and BD-RIS responses defined according to [12]. The phase-shift matrix $\mathbf{S}_{k,t} \in \mathbb{C}^{N_k \times N_k}$ of the k th group satisfies $\mathbf{S}_{k,t}^H \mathbf{S}_{k,t} = \mathbf{I}_{N_k}$, and $\mathbf{S}_t = \text{blkdiag}(\mathbf{S}_{1,t}, \dots, \mathbf{S}_{K,t})$ [20]. The delay and Doppler steering vectors are $\mathbf{c}(\tau) = [1, \dots, e^{-j2\pi(Q-1)\Delta f\tau}]^T$ and $\mathbf{d}(\nu) = [1, \dots, e^{j2\pi(M-1)T_s\nu}]^T$, while $\mathbf{z}_{q,m,t}$ is additive white Gaussian noise (AWGN) and α_k is the group-dependent complex gain [12]. Stacking all subcarriers and symbols gives

$$\begin{aligned}
 \mathbf{Y}_t = & \sum_{k=1}^K \mathbf{G}_k \mathbf{S}_{k,t}^T \mathbf{p}(\phi_{ris_D}, \theta_{ris_D}) \mathbf{p}^T(\phi_{ris_D}, \theta_{ris_D}) \mathbf{S}_{k,t} \mathbf{G}_k^T \\
 & \times \mathbf{X} \mathbf{D}(\mathbf{c}(\tau) \otimes \mathbf{d}(\nu)) + \mathbf{Z}_t \in \mathbb{C}^{L \times MQ},
 \end{aligned}$$

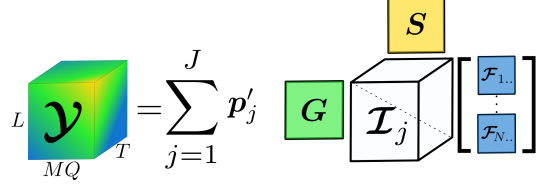
where $\mathbf{G}_k = \alpha_k \mathbf{a}(\phi_{st}, \theta_{st}) \mathbf{b}^T(\phi_{ris_A}, \theta_{ris_A}) \in \mathbb{C}^{L \times N_k}$ is the geometric channel between the ST and the k th BD-RIS group, $\mathbf{X} \in \mathbb{C}^{L \times MQ}$ is the combined transmitted pilot matrix across MQ OFDM symbols, and $\mathbf{Z}_t$ is AWGN at the t th time-slot. For ease of notation, we omit the noise terms in the following derivations, and consider their impact later in the numerical evaluations. Since both nodes are at fixed locations, we assume $\{\mathbf{G}_k\}_{k=1}^K$ is known *a priori* via standard channel estimation procedures, e.g., channel sounding, which is a common assumption in the literature [17]–[19]. Applying the property $\text{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A}) \text{vec}(\mathbf{B})$ and defining $\mathbf{F}_{\tau\nu,k} = \mathbf{G}_k^T \mathbf{X} \mathbf{D}(\mathbf{c}(\tau) \otimes \mathbf{d}(\nu)) \in \mathbb{C}^{N_k \times MQ}$ as the group-wise matrix containing the delay and Doppler information leads to

$$\mathbf{y}_t = \sum_{k=1}^K \{\mathbf{p}' \otimes (\mathbf{F}_{\tau\nu,k}^T \otimes \mathbf{G}_k)\} \text{vec}((\mathbf{S}_{k,t} \otimes \mathbf{S}_{k,t})^T),$$

where $\mathbf{p}' = \text{vec}(\mathbf{p} \mathbf{p}^T) \in \mathbb{C}^{1 \times N_k^2}$ and $\mathbf{y}_t = \text{vec}(\mathbf{Y}_t) \in \mathbb{C}^{LMQ \times 1}$. Assuming a specific group of size N_k and collecting all T samples of the pilot signal in a column, we obtain

$$\mathbf{Y} = \sum_{k=1}^K \{\mathbf{p} \otimes \mathbf{F}_{\tau\nu,k}^T \otimes \mathbf{G}_k\} \mathbf{S}_k^T, \quad (1)$$

where $\mathbf{S}_k = [\text{vec}(\mathbf{S}_{k,1}^T \otimes \mathbf{S}_{k,1}^T), \dots, \text{vec}(\mathbf{S}_{k,T}^T \otimes \mathbf{S}_{k,T}^T)]^T$.


 Fig. 2: Noiseless nested Tucker decomposition of $\mathcal{Y}$.

The preceding expressions describe the generic group-connected architecture before fixing a particular processing block. In the remainder of the derivation, we focus on an arbitrary group k and perform all processing within this selected group. Thus the group index k is suppressed from this point onward, so that N , $\mathbf{G}$, $\mathbf{S}$, and $\mathbf{F}_{\tau\nu}$ refer to the corresponding quantities of the selected group. This convention does not restrict the model to a fully-connected BD-RIS, since the same processing can be applied to any group, and the overall formulation remains valid. Thus, for the k th group, (1) corresponds to the 3rd mode transpose of a tensor $\mathcal{Y} \in \mathbb{C}^{L \times MQ \times T}$, which is expressed as a 3rd order Tucker decomposition multiplied by the vector containing the angle information of the 4th mode. This yields the tensor $\mathcal{Y} \in \mathbb{C}^{L \times MQ \times T}$ expressed as

$$\mathcal{Y} = \mathcal{I} \times_1 \mathbf{G} \times_2 [\mathcal{F}]_{(1)}^T \times_3 \mathbf{S} \times_4 \mathbf{p}', \quad (2)$$

where $\mathcal{I} \in \mathbb{C}^{N \times N \times N^4 \times N^2}$ is the known core tensor built from the identity $[\mathcal{I}]_{(3)} = \mathbf{I}_{N^4}$ and $[\mathcal{F}]_{(1)} = \mathbf{F}_{\tau\nu}^T \in \mathbb{C}^{N \times MQ}$ is the mode-1 unfolding of a 3rd order Tucker tensor. The unfoldings of $\mathcal{Y}$ are then expressed as

$$[\mathcal{Y}]_{(1)} = \mathbf{G} [\mathcal{I}]_{(1)} (\mathbf{p}' \otimes \mathbf{S} \otimes [\mathcal{F}]_{(1)}^T) \in \mathbb{C}^{L \times MQT}, \quad (3)$$

$$[\mathcal{Y}]_{(2)} = [\mathcal{F}]_{(1)}^T [\mathcal{I}]_{(2)} (\mathbf{p}' \otimes \mathbf{S} \otimes \mathbf{G})^T \in \mathbb{C}^{MQ \times LT}, \quad (4)$$

$$[\mathcal{Y}]_{(3)} = \mathbf{S} [\mathcal{I}]_{(3)} (\mathbf{p}' \otimes [\mathcal{F}]_{(1)}^T \otimes \mathbf{G})^T \in \mathbb{C}^{T \times LMQ}, \quad (5)$$

$$[\mathcal{Y}]_{(4)} = \mathbf{p}' [\mathcal{I}]_{(4)} (\mathbf{S} \otimes [\mathcal{F}]_{(1)}^T \otimes \mathbf{G})^T \in \mathbb{C}^{1 \times LMQT}. \quad (6)$$

The delay-Doppler factors are further isolated through

$$\hat{\mathcal{F}} = \mathcal{X} \times_1 \mathbf{G}^T \times_2 \mathbf{D}(\mathbf{d}(\nu)) \times_3 \mathbf{D}(\mathbf{c}(\tau)) \in \mathbb{C}^{N \times M \times Q}, \quad (7)$$

where $\mathcal{X} \in \mathbb{C}^{L \times M \times Q}$ satisfies $[\mathcal{X}]_{(1)} = \mathbf{X}$, with unfoldings

$$[\hat{\mathcal{F}}]_{(1)} = \mathbf{G}^T [\mathcal{X}]_{(1)} (\mathbf{D}(\mathbf{c}(\tau)) \otimes \mathbf{D}(\mathbf{d}(\nu))) \in \mathbb{C}^{N \times MQ}, \quad (8)$$

$$[\hat{\mathcal{F}}]_{(2)} = \mathbf{D}(\mathbf{d}(\nu)) [\mathcal{X}]_{(2)} (\mathbf{D}(\mathbf{c}(\tau)) \otimes \mathbf{G}) \in \mathbb{C}^{M \times NQ}, \quad (9)$$

$$[\hat{\mathcal{F}}]_{(3)} = \mathbf{D}(\mathbf{c}(\tau)) [\mathcal{X}]_{(3)} (\mathbf{D}(\mathbf{d}(\nu)) \otimes \mathbf{G}) \in \mathbb{C}^{Q \times NM}. \quad (10)$$

Equations (2) and (7) form the nested Tucker model, separating the delay-Doppler and angle parameters for structured extraction, as illustrated in Fig. 2.

III. NESTED TUCKER TENSOR TARGET SENSING

We present an iterative solution for the nested Tucker tensor model (2) and (7) that relies on the alternating least squares (ALS) method [21]–[23]. It yields subspace estimates of $[\mathcal{F}]_{(1)}$, and $\mathbf{p}'$. The parameters of interest are then extracted via subspace techniques, and the complex gain α is obtained afterward through a closed-form least squares (LS) step [24].

A. Decoupling the Angle Information

Starting from the Tucker model in (2), the ST estimates the factor matrices that contain the target information $[\mathcal{F}]_{(1)}$ and $\mathbf{p}'$, by solving the following LS fitting problem:

$$\{[\hat{\mathcal{F}}]_{(1)}, \hat{\mathbf{p}}'\} = \arg \min_{[\mathcal{F}]_{(1)}, \mathbf{p}'} \left\| \begin{bmatrix} \mathcal{Y} - \mathcal{I} \times_1 \mathbf{G} \times_2 \\ [\mathcal{F}]_{(1)}^T \times_3 \mathbf{S} \times_4 \mathbf{p}' \end{bmatrix} \right\|_F^2, \quad (11)$$

which can be solved by employing the bilinear alternating least squares (BALS) procedure, which consists of alternately solving the following optimization problems until a stopping criterion is reached¹:

$$[\hat{\mathcal{F}}]_{(1)} = \arg \min_{[\mathcal{F}]_{(1)}} \left\| [\mathcal{Y}]_{(2)} - [\mathcal{F}]_{(1)}^T [\mathcal{I}]_{(2)} (\mathbf{p}' \otimes \mathbf{S} \otimes \mathbf{G})^T \right\|_F^2, \quad (12)$$

$$\begin{aligned} \hat{\mathbf{p}}' &= \arg \min_{\mathbf{p}'} \left\| [\mathcal{Y}]_{(4)} - \mathbf{p}' [\mathcal{I}]_{(4)} (\mathbf{S} \otimes [\hat{\mathcal{F}}]_{(1)}^T \otimes \mathbf{G})^T \right\|_F^2, \\ &= [\mathcal{Y}]_{(4)} [\mathcal{I}]_{(4)} ([\hat{\mathcal{F}}]_{(1)}^T \otimes \mathbf{G} \otimes \mathbf{S})^T. \end{aligned} \quad (13)$$

The solutions above exist as long as $LT \geq N$ and $LMQT \geq N^2$, respectively. With the above, we have estimates of the mode-1 unfoldings of the Tucker tensor $\hat{\mathcal{F}}$ that contain the target delay and Doppler information.

B. Decoupled Delay and Doppler Information Estimation

Analogously to the first stage, we estimate the Tucker model in (7) containing the delay and Doppler information $\mathbf{d}(\nu)$ and $\mathbf{c}(\tau)$, by solving the following LS fitting problem

$$\{\hat{\mathbf{d}}(\nu), \hat{\mathbf{c}}(\tau)\} = \arg \min_{\mathbf{d}(\nu), \mathbf{c}(\tau)} \left\| \hat{\mathcal{F}} - \mathcal{X} \times_1 \mathbf{G}^T \times_2 \begin{bmatrix} \mathbf{D}(\mathbf{d}(\nu)) \times_3 \mathbf{D}(\mathbf{c}(\tau)) \end{bmatrix} \right\|_F^2. \quad (14)$$

Applying the property $\text{vec}(\mathbf{AD}(\mathbf{b})\mathbf{C}) = (\mathbf{C}^T \diamond \mathbf{A})\mathbf{b}$ to (9) and (10), which structurally enforces diagonalization and prevents noise from inducing erroneous non-zero off-diagonal elements, we have

$$\text{vec}([\hat{\mathcal{F}}]_{(2)}) = [([\mathcal{X}]_{(2)} \mathbf{D}(\mathbf{c}(\tau)) \otimes \mathbf{G})^T \diamond \mathbf{I}_M] \mathbf{d}(\tau), \quad (15)$$

$$\text{vec}([\hat{\mathcal{F}}]_{(3)}) = [([\mathcal{X}]_{(3)} \mathbf{D}(\mathbf{d}(\nu)) \otimes \mathbf{G})^T \diamond \mathbf{I}_Q] \mathbf{c}(\tau). \quad (16)$$

Employing BALS, we alternately solve the following:

$$\begin{aligned} \hat{\mathbf{d}}(\nu) &= \arg \min_{\mathbf{d}(\nu)} \left\| \begin{bmatrix} \text{vec}([\hat{\mathcal{F}}]_{(2)}) \\ - [([\mathcal{X}]_{(2)} \mathbf{D}(\mathbf{c}(\tau)) \otimes \mathbf{G})^T \diamond \mathbf{I}_M] \mathbf{d}(\nu) \end{bmatrix} \right\|_F^2 \\ &= [([\mathcal{X}]_{(2)} \mathbf{D}(\mathbf{c}(\tau)) \otimes \mathbf{G})^T \diamond \mathbf{I}_M]^\dagger \text{vec}([\hat{\mathcal{F}}]_{(2)}), \end{aligned} \quad (17)$$

$$\begin{aligned} \hat{\mathbf{c}}(\tau) &= \arg \min_{\mathbf{c}(\tau)} \left\| \begin{bmatrix} \text{vec}([\hat{\mathcal{F}}]_{(3)}) \\ - [([\mathcal{X}]_{(3)} \mathbf{D}(\mathbf{d}(\nu)) \otimes \mathbf{G})^T \diamond \mathbf{I}_Q] \mathbf{c}(\tau) \end{bmatrix} \right\|_F^2 \\ &= [([\mathcal{X}]_{(3)} \mathbf{D}(\mathbf{d}(\nu)) \otimes \mathbf{G})^T \diamond \mathbf{I}_Q]^\dagger \text{vec}([\hat{\mathcal{F}}]_{(3)}). \end{aligned} \quad (18)$$

The solutions above require $NQ \geq 1$ and $NM \geq 1$, respectively. After obtaining $\hat{\mathbf{d}}(\nu)$ and $\hat{\mathbf{c}}(\tau)$, we apply ESPRIT to acquire the delay and Doppler estimates, τ and ν , respectively.

C. Angle Estimation

Having estimated the delay $\hat{\tau}$ and Doppler $\hat{\nu}$ parameters, we parametrically reconstruct their respective steering vectors to refine the estimates, yielding

$$\mathbf{c}(\hat{\tau}) = [1, \dots, e^{-j2\pi(Q-1)\Delta f \hat{\tau}}]^T, \quad (19)$$

$$\mathbf{d}(\hat{\nu}) = [1, \dots, e^{j2\pi(M-1)T_s \hat{\nu}}]^T, \quad (20)$$

and reconstruct the mode-1 unfolding of the tensor $\hat{\mathcal{F}}$:

$$[\hat{\mathcal{F}}']_{(1)} = \mathbf{G}^T [\mathcal{X}]_{(1)} (\mathbf{D}(\mathbf{c}(\hat{\tau})) \otimes \mathbf{D}(\mathbf{d}(\hat{\nu}))). \quad (21)$$

Finally, the target's angle information matrix $\hat{\mathbf{P}}$ is extracted by substituting these refined tensor factors into the mode-4 unfolding of $\mathcal{Y}$, which yields

$$\hat{\mathbf{P}} = \text{unvec}_{N \times N} \left([\mathcal{Y}]_{(4)} [[\mathcal{I}]_{(4)} ([\hat{\mathcal{F}}']_{(1)} \otimes \mathbf{G} \otimes \mathbf{S})^T]^\dagger \right). \quad (22)$$

From this matrix, we apply the 2D ESPRIT algorithm to estimate the azimuth and elevation angles, ϕ_{risd} and θ_{risd} , respectively. Once these angles are obtained, we parametrically reconstruct the angle matrix containing the target's information to yield the refined estimate

$$\hat{\mathbf{P}}' = \mathbf{p}(\hat{\phi}_{\text{risd}}, \hat{\theta}_{\text{risd}}) \mathbf{p}^T(\hat{\phi}_{\text{risd}}, \hat{\theta}_{\text{risd}}). \quad (23)$$

D. Closed-form Complex Channel Coefficient Estimation

In the final step, we estimate the complex channel gain coefficient α . Like other tensor algorithms, the parametric reconstructions above eliminate arbitrary scaling factors, and the only scalar ambiguity remaining in the reconstructed tensor signal is the channel coefficient itself. We first reconstruct the mode-1 unfolding of the tensor using the refined parameters:

$$[\mathcal{Y}']_{(1)} = \mathbf{G} [\mathcal{I}]_{(1)} (\text{vec}(\hat{\mathbf{P}}') \otimes \mathbf{S} \otimes [\hat{\mathcal{F}}']_{(1)})^T. \quad (24)$$

The coefficient α is then extracted via a low-complexity element-wise division step, given by

$$\hat{\alpha} = \mathbb{E} \{ [\mathcal{Y}]_{(1)} \oslash [\mathcal{Y}']_{(1)} \}. \quad (25)$$

We take the sample mean (denoted here by $\mathbb{E}\{\cdot\}$) over all $LMQT$ available elements in the unfolded tensor, effectively exploiting the full multidimensional dataset to mitigate noise. A summary of the proposed solution is given in Alg. 1.

IV. IDENTIFIABILITY AND UNIQUENESS

Unlike a standard Tucker decomposition with an unknown dense core, the proposed model uses a deterministic known

¹The convergence criteria for the proposed algorithms are specified in Alg. 1. Termination typically occurs upon reaching a predefined maximum number of iterations or when the objective function satisfies a convergence tolerance between consecutive iterations.

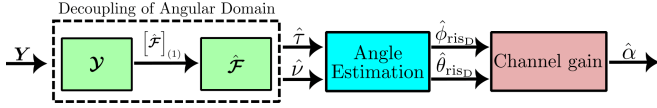


Fig. 3: Block diagram of the proposed NTFE algorithm. The nested Tucker model in (2) is solved to estimate delay and Doppler via ESPRIT, the tensor factors are refined to extract the angle parameters using (22), and the complex channel gain is obtained in closed form via (25).

Algorithm 1 Nested Tucker factorization estimation (NTFE)

Require: Tensor $\mathcal{Y}$, $\mathcal{G}$, $\mathcal{S}$, $\mathcal{X}$, maximum number of iterations $i_{\max}$, and convergence threshold δ .

- 1: Randomly initialize $[\hat{\mathcal{F}}]_{(1)}$ and $\hat{\mathbf{p}}'$ at iteration $i = 0$.
- 2: **while** $\|e(i) - e(i-1)\| \geq \delta$ and $i < i_{\max}$ **do**
- 3: Find the LS estimate of $\mathbf{F}_{\tau\nu}$: $[\hat{\mathcal{F}}]_{(1)} = [\mathcal{Y}]_{(2)}([\mathcal{Z}]_{(2)}(\hat{\mathbf{p}}' \otimes \mathcal{S} \otimes \mathcal{G})^T)^\dagger$.
- 4: Find the LS estimate of $\mathbf{p}'$: $\hat{\mathbf{p}}' = [\mathcal{Y}]_{(4)}([\mathcal{Z}]_{(4)}([\hat{\mathcal{F}}]_{(1)}^T \otimes \mathcal{G} \otimes \mathcal{S})^T)^\dagger$.
- 5: Update $e(i) = \|\hat{\mathcal{Y}} - \hat{\mathcal{Y}}(i)\|_F^2$.
- 6: **end while**
- 7: **return** $[\hat{\mathcal{F}}]_{(1)}$ and $\hat{\mathbf{p}}'$.
- 8: Randomly initialize $\hat{\mathbf{d}}(\nu)$ and $\hat{\mathbf{c}}(\tau)$ at iteration $i = 0$.
- 9: **while** $\|e(i) - e(i-1)\| \geq \delta$ and $i < i_{\max}$ **do**
- 10: Find the LS estimate of $\mathbf{d}(\nu)$: $\hat{\mathbf{d}}(\nu) = [([\mathcal{X}]_{(2)}\mathbf{D}(\mathbf{c}(\tau)) \otimes \mathcal{G})^T \diamond \mathbf{I}_M]^\dagger \text{vec}([\hat{\mathcal{F}}]_{(2)})$.
- 11: Find the LS estimate of $\mathbf{c}(\tau)$: $\hat{\mathbf{c}}(\tau) = [([\mathcal{X}]_{(3)}\mathbf{D}(\mathbf{d}(\nu)) \otimes \mathcal{G})^T \diamond \mathbf{I}_Q]^\dagger \text{vec}([\hat{\mathcal{F}}]_{(3)})$.
- 12: Update $e(i) = \|\hat{\mathcal{F}} - \hat{\mathcal{F}}(i)\|_F^2$.
- 13: **end while**
- 14: **return** $\hat{\mathbf{d}}(\nu)$ and $\hat{\mathbf{c}}(\tau)$.
- 15: Extract delay $\hat{\tau}$ and Doppler $\hat{\nu}$ via 1D ESPRIT applied to $\hat{\mathbf{c}}(\tau)$ and $\hat{\mathbf{d}}(\nu)$ and parametrically reconstruct the vectors $\mathbf{c}(\hat{\tau}) = [1, \dots, e^{-j2\pi(Q-1)\Delta f \hat{\tau}}]^T$ and $\mathbf{d}(\hat{\nu}) = [1, \dots, e^{j2\pi(M-1)T_s \hat{\nu}}]^T$.
- 16: Reconstruct tensor factor $[\hat{\mathcal{F}}']_{(1)} = \mathcal{G}^T[\mathcal{X}]_{(1)}(\mathbf{D}(\mathbf{c}(\hat{\tau})) \otimes \mathbf{D}(\mathbf{d}(\hat{\nu})))$ and estimate the angle matrix: $\hat{\mathbf{P}} = \text{unvec}_{N \times N}([\mathcal{Y}]_{(4)}([\hat{\mathcal{F}}']_{(1)} \otimes \mathcal{G} \otimes \mathcal{S})^T)^\dagger$.
- 17: Extract angles $\hat{\phi}_{\text{risD}}$ and $\hat{\theta}_{\text{risD}}$ via 2D ESPRIT applied to $\hat{\mathbf{P}}$ and parametrically reconstruct the angle matrix $\hat{\mathbf{P}}' = \mathbf{p}(\hat{\phi}_{\text{risD}}, \hat{\theta}_{\text{risD}})\mathbf{p}^T(\hat{\phi}_{\text{risD}}, \hat{\theta}_{\text{risD}})$.
- 18: Reconstruct received tensor signal $[\mathcal{Y}']_{(1)} = \mathcal{G}[\mathcal{Z}]_{(1)}(\text{vec}(\hat{\mathbf{P}}') \otimes \mathcal{S} \otimes [\hat{\mathcal{F}}']_{(1)})^T$ and estimate the complex channel coefficient: $\hat{\alpha} = \mathbb{E}\{[\mathcal{Y}]_{(1)} \otimes [\mathcal{Y}']_{(1)}\}$.
- 19: **return** $\hat{\tau}$, $\hat{\nu}$, $\hat{\phi}_{\text{risD}}$, $\hat{\theta}_{\text{risD}}$, and $\hat{\alpha}$.

core $\mathcal{I}$ with the factor matrices corresponding to the 1st and 3rd modes, which define the part of the system geometry assumed to be known at the receiver. Hence, for (2), arbitrary rotations of the estimates do not occur and the remaining ambiguities reduce to trivial scalings. Thus, the BALS estimates satisfy $[\hat{\mathcal{F}}]_{(1)} = [\mathcal{F}]_{(1)}\mathbf{\Lambda}_1$ and $\hat{\mathbf{p}}' = \mathbf{p}'\mathbf{\Lambda}_2$, with $\mathbf{\Lambda}_1 \in \mathbb{C}^{MQ \times MQ}$ and $\mathbf{\Lambda}_2 \in \mathbb{C}^{N^2 \times N^2}$ such that $\mathbf{\Lambda}_2 \otimes \mathbf{\Lambda}_1 \otimes \mathbf{I}_N = \mathbf{I}_{N^4}$. These scalings cancel each other in the reconstructed tensor and preserve the column space of $[\hat{\mathcal{F}}]_{(1)}$, enabling delay-Doppler extraction. However, a non-uniform scaling may destroy the shift-invariance of $\hat{\mathbf{p}}'$, so (22) is used to refine the angle subspace estimate. For (7), the ambiguities reduce to $\hat{\mathbf{d}}(\nu) = \lambda_3 \mathbf{d}(\nu)$ and $\hat{\mathbf{c}}(\tau) = \lambda_4 \mathbf{c}(\tau)$, with $\lambda_3 \lambda_4 = 1$.

Identifiability follows from the invertibility of the LS systems in Alg. 1. For (2), define $\mathbf{P}_1 = [\mathcal{Z}]_{(2)}(\mathbf{p}' \otimes \mathcal{S} \otimes \mathcal{G})^T \in \mathbb{C}^{N \times LT}$ and $\mathbf{P}_2 = [\mathcal{Z}]_{(4)}([\hat{\mathcal{F}}]_{(1)} \otimes \mathcal{G} \otimes \mathcal{S})^T \in \mathbb{C}^{N^2 \times LMQT}$.

TABLE II: Simulation parameters.

RIS elements	$N_y N_z = 2 \times 2 = 4$
Antennas at the ST	$L_{\text{ST}_y} L_{\text{ST}_z} = 2 \times 2 = 4$
Arrays spacing	$\lambda/2$
Angles	$\mathcal{U}(0^\circ, 90^\circ)$
ST - RIS	$\mathcal{U}(10\text{m}, 250\text{m})$
RIS - Cluster	$\mathcal{U}(10\text{m}, 250\text{m})$
Velocity	$\mathcal{U}(-25\text{m/s}, +25\text{m/s})$
RCS	2m^2
Symbol duration	$1/\Delta f$
Carrier frequency	28 GHz
Subcarrier spacing Δf	120 KHz
Wavelength	1.07×10^{-2} m
$i_{\max}$	500
δ	10^{-6}
Symbols M	4
Subcarriers Q	4
Time-slots T	256

Their right-invertibility requires $LT \geq N$ and $LMQT \geq N^2$. For (7), $\mathbf{P}_3 = ([\mathcal{X}]_{(2)}\mathbf{D}(\mathbf{c}(\tau)) \otimes \mathcal{G})^T \diamond \mathbf{I}_M$ and $\mathbf{P}_4 = ([\mathcal{X}]_{(3)}\mathbf{D}(\mathbf{d}(\nu)) \otimes \mathcal{G})^T \diamond \mathbf{I}_Q$ require $NQ \geq 1$, $NM \geq 1$, and $MQ \geq K$ (with $Q, M \geq 1$). Since the known core prevents arbitrary non-singular rotations [23], these conditions ensure uniqueness up to trivial scaling ambiguities.

V. SIMULATION RESULTS

We evaluate the proposed NTFE algorithm in terms of NMSE and RMSE for a monostatic OFDM sensing scenario assisted by a group-connected BD-RIS. While the proposed NTFE algorithm is generalized for a group-connected architecture with K groups, the following simulations consider a baseline scenario consisting of a single target and a single active BD-RIS group operating in the far-field. The reported processing considers one group with $N = N_y \times N_z$ reflecting elements, uniform planar arrays at the ST, far-field propagation, random unitary intra-group phase-shift matrices fixed over blocks of MQ OFDM symbols, Hadamard pilots, and uniformly distributed target angles in $(0^\circ, 90^\circ)$. Channel and parameter errors are measured by $\text{NMSE}(\mathbf{X}) = \mathbb{E}\{\|\mathbf{X} - \hat{\mathbf{X}}\|_F^2 / \|\mathbf{X}\|_F^2\}$ and $\text{RMSE}(\mathbf{x}) = \sqrt{\mathbb{E}\{\|\mathbf{x} - \hat{\mathbf{x}}\|_2^2\}}$, respectively. The SNR is defined as $\text{SNR} = \|\mathbf{Y}\|_F^2 / \sigma_Z^2 \|\mathbf{Z}\|_F^2$, and the delay and Doppler estimates are normalized by T_s . Unless otherwise stated, the parameters in Table II are used, and all curves are averaged over 5000 Monte Carlo realizations. We compare NTFE with three benchmarks: the sequential 4D maximum likelihood (ML) method of [19], a direct LS estimate of (1), acting as a performance upper-bound when the spatial and temporal structure is not exploited, and the Kronecker factorization (KF)-based method of [25] that factorizes the Kronecker product in (1). Fig. 4 shows the NMSE for estimating $\mathbf{H}_{\text{eff}} = \{\mathbf{p} \otimes \mathbf{F}_{\tau\nu}^T \otimes \mathcal{G}\}^T$, reconstructed from the parameters estimated by NTFE. The direct LS estimate, $\hat{\mathbf{H}}_{\text{eff}} = \mathbf{S}^\dagger \mathbf{Y}$, requires $T \geq N^4$ and performs about 10 dB worse than KF, which benefits from the Kronecker structure. In turn, KF remains roughly 25 dB worse than NTFE, since it does not exploit the full geometric structure of the sensing model. Fig. 5 reports the parameter RMSE. By decoupling the delay-Doppler and angle domains, NTFE accurately estimates the temporal parameters and sep-

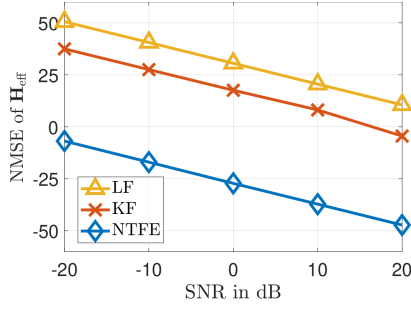
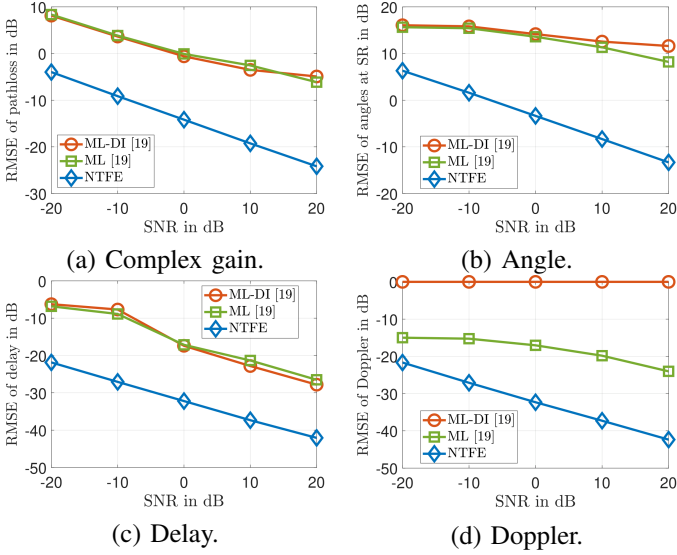

 Fig. 4: NMSE of H_{eff} estimation for NTFE, LS, and KF [25].


Fig. 5: Parameter RMSE of the proposed NTFE algorithm.

arates the azimuth and elevation components induced by the BD-RIS. The complex-gain accuracy follows the quality of the preceding spatial and temporal estimates. NTFE achieves a performance improvement of 10 dB compared with the standard 4D ML benchmark for all estimated parameters. This gain is mainly due to reduced dependence on delay-Doppler initialization and mitigated error propagation from sequential ML searches. We also show the performance of the Doppler ignorant (DI) ML variant from [19]; although it does not estimate Doppler, it performs similarly to the standard ML method on the remaining parameters in the considered scenario. The computational complexity of Alg. 1 scales as $\mathcal{O}(I_{\text{ALS}}N_k(QM^5 + MQ^5 + N_k^7 LMQT))$, where I_{ALS} denotes the number of alternating iterations required for convergence. In our numerical evaluations, the proposed NTFE algorithm exhibits fast convergence behavior, typically satisfying an error tolerance of $\epsilon = 10^{-6}$ within 15 iterations across both stages.

VI. CONCLUSION

We proposed the NTFE algorithm for single-target sensing in a group-connected BD-RIS-assisted monostatic sensing scenario. Using a 3rd order nested Tucker model, the method decouples the temporal and spatial parameters, sup-

ports identifiable estimation up to trivial ambiguities, and outperforms state-of-the-art benchmarks in terms of effective-channel NMSE and parameter RMSE.

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