

ROBUST ZERO-CROSSING PRECODING WITH MMSE CRITERION FOR CHANNELS WITH 1-BIT QUANTIZATION AND OVERSAMPLING

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ABSTRACT

The use of 1-bit analog-to-digital converters with temporal oversampling at the receiver has emerged as a solution for energy-constrained wireless systems with increased spectral efficiency. The corresponding receivers consume less energy and have a simpler hardware architectures compared to their high-resolution counterparts. Zero-crossing modulation implies that the information is conveyed in the time-instances of the zero-crossings. Different to existing methods the proposed zero-crossing precoding method takes into account imperfect channel state information. Its performance is evaluated numerically by using bit error rate and the average mean square error of the received signal. The results show that the proposed robust minimum mean squared error based space-time precoding approach yields noticeable performance gains.

Index Terms— Zero-crossing precoding, imperfect CSI, mean-square-error, oversampling, 1-bit quantization, faster-than-Nyquist signaling, MIMO Systems.

1. INTRODUCTION

The proliferation of wireless devices and IoT has increased the demand for receiver architectures that balance performance and energy efficiency [1]. In this context, systems with 1-bit ADCs and temporal oversampling represent interesting approaches [2, 3]. The existing methods present tailored waveforms that can exploit the higher resolution in time which then can partially compensate the information loss due to quantization, similar to the concept presented in [4]. However, these methods rely on perfect channel state information which is difficult to achieve in many practical systems. The present work proposes an improved waveform design which relies on an established zero-crossing precoding method [3]. The proposed waveform design exploits knowledge about the CSI mismatch in terms of the channel error covariance matrix.

Numerical simulations with Rayleigh fading evaluate the proposed scheme in terms of bit error rate and mean square error (MSE). The results show the advantages of the robust MMSE formulation across different system configurations and operating conditions. The remainder of the paper is organized as follows. Section II presents the system model and space-time signal representation. Section III describes the zero-crossing modulation scheme and the proposed robust MMSE precoder. Section IV presents numerical results, and Section V concludes the paper.

2. SYSTEM MODEL

A multiuser MIMO system with N_t transmit antennas serving N_u single-antenna users is considered. The receivers use 1-bit quantization with oversampling, as shown in Fig. 1. The input associated to the k th user is given by a block symbol sequence $\mathbf{x}_k$ with N complex symbols. Subsequently each sequence $\mathbf{x}_k$ will be mapped to an output pattern $\mathbf{c}_{\text{out},k}$. The system is defined by signaling rate $\frac{M_{\text{Tx}}}{T}$ and the sampling rate $\frac{M_{\text{Rx}}}{T} = \frac{M M_{\text{Tx}}}{T}$ and transmit filters and receive filters with impulse responses $g_{\text{Tx}}(t)$ and $g_{\text{Rx}}(t)$, respectively. The combined waveform is determined by the convolution of transmit and receive filter $v(t) = g_{\text{Tx}}(t) * g_{\text{Rx}}(t)$.

The space-time precoder generates the transmit vector $\mathbf{p}_x$, which incorporates spatial and temporal processing. Vector $\mathbf{p}_x$ is obtained by solving an optimization problem based on a chosen design criterion, such as the minimum mean square error (MMSE). The precoding vector is constructed by stacking the transmit sequences associated with each antenna, i.e.,

$$\mathbf{p}_x = [\mathbf{p}_{x1}^T \quad \mathbf{p}_{x2}^T \quad \cdots \quad \mathbf{p}_{xN_t}^T]^T. \quad (1)$$

After pulse shaping through the transmit filter $g_{\text{Tx}}(t)$, the signals are transmitted over a frequency-flat fading MIMO channel $\mathbf{H} = [\mathbf{h}_1^T, \mathbf{h}_2^T, \dots, \mathbf{h}_{N_u}^T]^T \in \mathbb{C}^{N_u \times N_t}$ which is the concatenated channel matrix for all users.

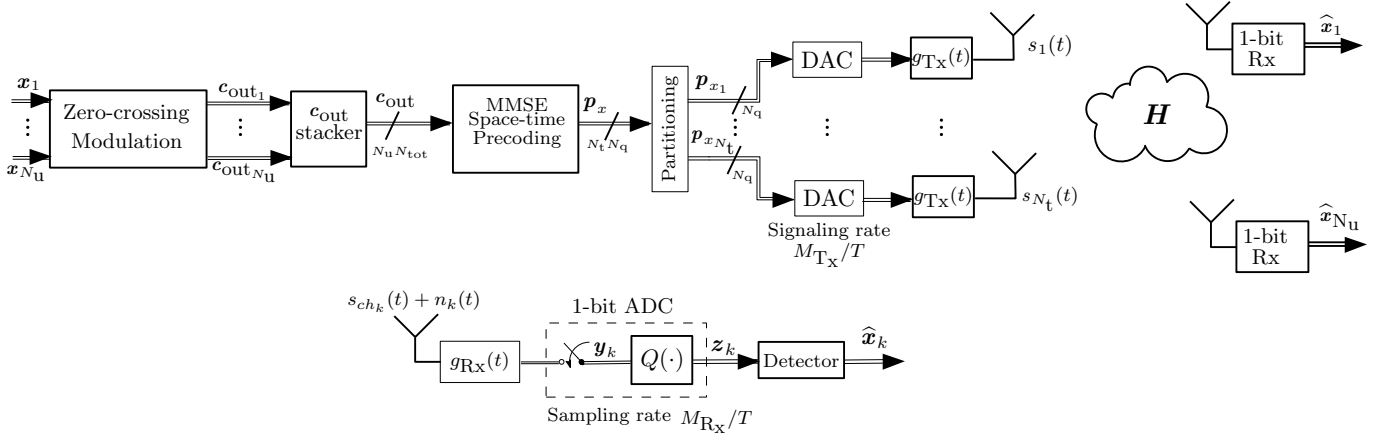


Fig. 1: Multiuser MIMO system with 1-bit ADCs and temporal oversampling

At the receivers, the signal is processed by the receive filter $g_{Rx}(t)$ and sampled with M_{Rx} -fold oversampling rate. The receive filter is represented by the Toeplitz matrix $\mathbf{G}_{Rx}$. The unquantized received signal at user k is given by

$$\mathbf{y}_k = (\mathbf{h}_k \otimes \mathbf{I}_{N_{tot}})(\mathbf{I}_{N_t} \otimes \mathbf{V}\mathbf{U})\mathbf{p}_x + \mathbf{G}_{Rx}\mathbf{n}_k, \quad (2)$$

where $\mathbf{V}$ represents the pulse shaping matrix resulting from the convolution of the transmit and receive filters, $\mathbf{U}$ is the up-sampling matrix, and $\mathbf{n}_k$ is the additive white Gaussian noise vector. The received signal at user k is quantized by a 1-bit analog-to-digital converter, resulting in

$$\mathbf{z}_k = Q_1(\mathbf{y}_k), \quad (3)$$

where $Q_1(\cdot)$ denotes the element-wise 1-bit quantization. For symbol detection a pattern matching method based on the Hamming distance is used. The stacked received signal can be expressed by a vector with length $N_u N_{tot}$ described by

$$\begin{aligned} \mathbf{z} &= Q_1(\mathbf{y}) \\ &= Q_1((\mathbf{H} \otimes \mathbf{I}_{N_{tot}})(\mathbf{I}_{N_t} \otimes \mathbf{V}\mathbf{U})\mathbf{p}_x + (\mathbf{I}_{N_u} \otimes \mathbf{G}_{Rx})\mathbf{n}) \\ &= Q_1(\mathbf{H}_{eff}\mathbf{p}_x + \mathbf{G}_{Rx,eff}\mathbf{n}), \end{aligned} \quad (4)$$

where $\mathbf{n}$ is the noise vector with covariance $\mathbf{C}_n = \mathbb{E}[\mathbf{n}\mathbf{n}^H]$. The effective channel matrix reads as

$$\mathbf{H}_{eff} = (\mathbf{H} \otimes \mathbf{I}_{N_{tot}})(\mathbf{I}_{N_t} \otimes \mathbf{V}\mathbf{U}). \quad (5)$$

The waveform impulse response matrix $\mathbf{V}$ with dimensions $N_{tot} \times N_{tot}$ with $N_{tot} = NM_{Rx} + 1$ and the receive filter matrix $\mathbf{G}_{Rx}$ are denoted as

$$\mathbf{V} = \begin{bmatrix} v(0) & v\left(\frac{T}{M_{Rx}}\right) & \cdots & v(TN) \\ v\left(-\frac{T}{M_{Rx}}\right) & v(0) & \cdots & v\left(T\left(N - \frac{1}{M_{Rx}}\right)\right) \\ \vdots & \vdots & \ddots & \vdots \\ v(-TN) & v\left(T\left(-N + \frac{1}{M_{Rx}}\right)\right) & \cdots & v(0) \end{bmatrix}. \quad (6)$$

and

$$\mathbf{G}_{Rx} = a_{Rx} \begin{bmatrix} [\mathbf{g}_{Rx}^T] & 0 \cdots 0 \\ 0 & [\mathbf{g}_{Rx}^T] & 0 \cdots 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 \cdots 0 & [\mathbf{g}_{Rx}^T] \end{bmatrix}_{N_{tot} \times 3N_{tot}}, \quad (7)$$

with $\mathbf{g}_{Rx} = [g_{Rx}(-T(N + \frac{1}{M_{Rx}})), g_{Rx}(-T(N + \frac{1}{M_{Rx}}) + \frac{T}{M_{Rx}}), \dots, g_{Rx}(T(N + \frac{1}{M_{Rx}}))]^T$ and $a_{Rx} = (T/M_{Rx})^{1/2}$. The M -fold upsampling matrix $\mathbf{U}$ with dimensions $N_{tot} \times N_q$ is defined by

$$U_{m,n} = \begin{cases} 1, & \text{for } m = M \cdot (n - 1) + 1 \\ 0, & \text{else.} \end{cases} \quad (8)$$

3. ZERO-CROSSING MODULATION AND PRECODING

The transmitted symbol sequence $\mathbf{x} = [x_1, \dots, x_{N_u}]$ is mapped to a desired binary output sequence $\mathbf{c}_{out}$, which represents the expected quantized signal at the receivers.

The desired output sequence for each user is constructed by concatenating the corresponding symbol segments. The overall sequence $\mathbf{c}_{out}$ is obtained by stacking the sequences of all users. The desired output pattern at the receivers sequence $\mathbf{c}_{out} = [\mathbf{c}_{out_1}^T, \dots, \mathbf{c}_{out_k}^T, \dots, \mathbf{c}_{out_{N_u}}^T]^T$. This implies that each symbol from the sequence $\mathbf{x}$ is represented at the receiver by M_{Rx} samples. Each symbol of the sequence $\mathbf{x}$ is mapped in a code segment which conveys the information in the time instance of the zero-crossing [5]. The process to extract out of $\mathbf{x}$ the sequence $\mathbf{c}_{out}$ with length N_{tot} with dimensions $N_{tot} = N \cdot M_{Rx} + 1$, where N is the number of symbols. The process is done for each user k and also independent for the real and imaginary parts of x_i . At the end $\mathbf{c}_{out}$ is obtained by stacking the modulated sequences for all the users.

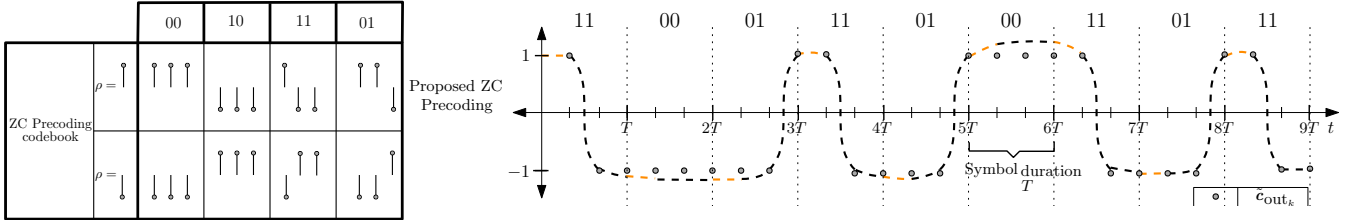


Fig. 2: Mapping process for construction of $\mathbf{c}_{\text{out}}$ for ZC precoding, $M_{\text{Rx}} = 3$

3.1. Zero-Crossing Modulation

Each user's input sequence x_k ($k = 1, \dots, N_u$) is processed through the Time-Instance Zero-Crossing (TI ZX) Modulator, which maps the information into zero-crossing time instances, generating output sequences $\mathbf{c}_{\text{out},k}$. Using $M_{\text{Tx}} > 1$ corresponds to faster-than-Nyquist signalling, where more than one symbol is transmitted per Nyquist interval. This is related to the oversampling factor M_{Rx} by

$$M \cdot M_{\text{Tx}} = M_{\text{Rx}}, \quad (9)$$

where M is the factor that relates the transmitter and receiver sampling rates. Since the pattern of each segment depends on the zero-crossing time instance the codeword $\mathbf{c}_{s,i}$ depends also on the last sample of the previous sequence segment which we term $\rho_{i-1} \in \{1, -1\}$. Hence, each input symbol is associated with two possible codewords.

Finally, the pattern sequence $\tilde{\mathbf{c}}_{\text{out},k} = [\mathbf{c}_{s,0}^T, \dots, \mathbf{c}_{s,N-1}^T]^T$ with total length $N M_{\text{Rx}} + 1$ for each user is obtained by concatenating the $\mathbf{c}_s$ segments of each symbol. Given the dependency of the construction of $\mathbf{c}_s$ on ρ_{i-1} of the previous code segment $\mathbf{c}_{s,i-1}$, one pilot sample $pb \in \{1, -1\}$ is included as the first sample of the sequence $\tilde{\mathbf{c}}_{\text{out},k}$ to map and later detect the first transmit symbol. Fig. 2 illustrates an example for the construction of $\tilde{\mathbf{c}}_{\text{out},k}$ with $M_{\text{Tx}} = 3$.

3.2. Imperfect CSI Model and Error Covariance Matrix

It is assumed that the transmitter has access only to the known channel component $\hat{\mathbf{H}}$ and that the coefficients $c_1 = \sqrt{\sigma_e^2}$ and $c_2 = \sqrt{1 - \sigma_e^2}$ can be estimated. The channel error term is unknown and defined as $c_1 \Delta \mathbf{H}$. With this the actual channel matrix can be written as

$$\mathbf{H} = c_2 \hat{\mathbf{H}} + c_1 \Delta \mathbf{H}. \quad (10)$$

For the case $\sigma_e^2 = 0$ the channel is perfectly known and when $\sigma_e^2 = 1$ it is completely unknown. In the following it is assumed that statistical information about $\Delta \mathbf{H}$ is available. After applying the pulse shaping filter $\mathbf{B} = \mathbf{V}\mathbf{U}$, the components of the effective channel are

$$\begin{aligned} \hat{\mathbf{H}}_{\text{eff}} &= (\hat{\mathbf{H}} \otimes \mathbf{I}_{N_{\text{tot}}}) (\mathbf{I}_{N_t} \otimes \mathbf{B}), \\ \Delta \mathbf{H}_{\text{eff}} &= (\Delta \mathbf{H} \otimes \mathbf{I}_{N_{\text{tot}}}) (\mathbf{I}_{N_t} \otimes \mathbf{B}). \end{aligned} \quad (11)$$

The uncertainty covariance matrix $\mathbf{Q}_{\Delta}$ is defined as

$$\mathbf{Q}_{\Delta} = \mathbb{E} [\Delta \mathbf{H}_{\text{eff}}^H \Delta \mathbf{H}_{\text{eff}}]. \quad (12)$$

3.3. Kronecker Product Identities

For the solution, two key identities were implemented, which will be instrumental in deriving the robust MMSE precoder.

Identity 1 (Mixed-Product Property). For any matrices $\Delta \mathbf{H} \in \mathbb{C}^{N_u \times N_t}$ and $\mathbf{B} \in \mathbb{C}^{N_{\text{tot}} \times N_q}$, the following identity holds:

$$(\Delta \mathbf{H} \otimes \mathbf{I}_{N_{\text{tot}}}) (\mathbf{I}_{N_t} \otimes \mathbf{B}) = \Delta \mathbf{H} \otimes \mathbf{B}. \quad (13)$$

Using Identity 1 the effective channel error reads as

$$\Delta \mathbf{H}_{\text{eff}} = \Delta \mathbf{H} \otimes \mathbf{B}. \quad (14)$$

With this the error covariance has the following structure

$$\mathbb{E} [\Delta \mathbf{H}_{\text{eff}} \Delta \mathbf{H}_{\text{eff}}^H] = \mathbb{E} [\Delta \mathbf{H} \Delta \mathbf{H}^H] \otimes (\mathbf{B} \mathbf{B}^H). \quad (15)$$

3.4. Proposed Robust Space-Time MMSE Precoding

This section devises the MMSE design of the transmit vector $\mathbf{p}_x$ by taking into account imperfect CSI at the transmitter and a total power constraint.

3.4.1. Problem Formulation

The goal of the proposed MMSE-based precoding is to design a precoding vector $\mathbf{p}_x$ and a scaling factor $f > 0$ that minimize the MSE between the received signal and the desired output pattern, while satisfying a transmit power constraint. The optimization problem can be cast as

$$\begin{aligned} \min_{f > 0, \mathbf{p}_x} \quad & \mathbb{E} \left\{ \|f (\mathbf{H}_{\text{eff}} \mathbf{p}_x + \mathbf{G}_{\text{Rx,eff}} \mathbf{n}) - \mathbf{c}_{\text{out}}\|_2^2 \right\} \\ \text{subject to:} \quad & \mathbf{p}_x^H \mathbf{A}^H \mathbf{A} \mathbf{p}_x \leq E_0, \end{aligned} \quad (16)$$

where E_0 is the maximum transmit energy, $\mathbf{A} = \mathbf{I}_{N_t} \otimes \mathbf{G}_{\text{Tx}}^T \mathbf{U}$, represents upsampling and the transmit filter [3]. Due to the fact that $\mathbf{H}_{\text{eff}}$ is only partially known at the transmitter the problem needs to be reformulated.

3.4.2. Objective Function Derivation

Using $\mathbf{H}_{\text{eff}} = c_2 \hat{\mathbf{H}}_{\text{eff}} + c_1 \Delta \mathbf{H}_{\text{eff}}$, expanding the objective in (16) and taking expectations yields

$$\begin{aligned} J = & f^2 \mathbf{p}_x^H \left(c_2^2 \hat{\mathbf{H}}_{\text{eff}}^H \hat{\mathbf{H}}_{\text{eff}} + c_1^2 \mathbf{Q}_\Delta \right) \mathbf{p}_x \\ & - 2f \operatorname{Re} \left\{ c_2 \mathbf{p}_x^H \hat{\mathbf{H}}_{\text{eff}}^H \mathbf{c}_{\text{out}} \right\} \\ & + f^2 \operatorname{tr} \left\{ \mathbf{G}_{\text{Rx,eff}} \mathbf{C}_n \mathbf{G}_{\text{Rx,eff}}^H \right\} + \|\mathbf{c}_{\text{out}}\|_2^2. \end{aligned} \quad (17)$$

Cross-terms vanish by the zero-mean and independence assumptions.

3.4.3. Lagrangian Derivation

Based on the expanded MSE cost function (16) the following notations are useful

$$\begin{aligned} \mathbf{R} &= c_2^2 \hat{\mathbf{H}}_{\text{eff}}^H \hat{\mathbf{H}}_{\text{eff}} + c_1^2 \mathbf{Q}_\Delta, \\ \sigma_n^2 &= \operatorname{tr} \left\{ \mathbf{G}_{\text{Rx,eff}} \mathbf{C}_n \mathbf{G}_{\text{Rx,eff}}^H \right\} = (N_u N_{\text{tot}}) N_0, \\ \mathbf{v} &= c_2 \hat{\mathbf{H}}_{\text{eff}}^H \mathbf{c}_{\text{out}}. \end{aligned} \quad (18)$$

With this, the Lagrangian is given by

$$\begin{aligned} \mathcal{L}(\mathbf{p}_x, f, \lambda) = & f^2 \mathbf{p}_x^H \mathbf{R} \mathbf{p}_x - 2f \operatorname{Re} \left\{ \mathbf{p}_x^H \mathbf{v} \right\} + f^2 \sigma_n^2 + \lambda (\mathbf{p}_x^H \mathbf{A}^H \mathbf{A} \mathbf{p}_x - E_0). \end{aligned} \quad (19)$$

Setting $\partial \mathcal{L} / \partial \mathbf{p}_x^* = \mathbf{0}$ yields

$$(f^2 \mathbf{R} + \lambda \mathbf{A}^H \mathbf{A}) \mathbf{p}_x = f \mathbf{v}. \quad (20)$$

Setting $\partial \mathcal{L} / \partial f = 0$ yields

$$f (\mathbf{p}_x^H \mathbf{R} \mathbf{p}_x + \sigma_n^2) = \operatorname{Re} \left\{ \mathbf{p}_x^H \mathbf{v} \right\}. \quad (21)$$

From the KKT conditions and considering that the power constraint is active with $\mathbf{p}_x^H \mathbf{A}^H \mathbf{A} \mathbf{p}_x = E_0$, it follows that

$$\lambda f^{-2} = \sigma_n^2 E_0^{-1}. \quad (22)$$

Inserting (22) into (20), the optimal precoder takes the form

$$\mathbf{p}_{x,\text{opt}} = f^{-1} \left(\mathbf{R} + \frac{\sigma_n^2}{E_0} \mathbf{A}^H \mathbf{A} \right)^{-1} \mathbf{v}. \quad (23)$$

with $\mathbf{w}$

$$\mathbf{w} = \left(\mathbf{R} + \frac{\sigma_n^2}{E_0} \mathbf{A}^H \mathbf{A} \right)^{-1} \mathbf{v}, \quad (24)$$

Enforcing the active power constraint on (23) yields the optimal scaling factor

$$f = \sqrt{E_0^{-1} \mathbf{w}^H \mathbf{A}^H \mathbf{A} \mathbf{w}}. \quad (25)$$

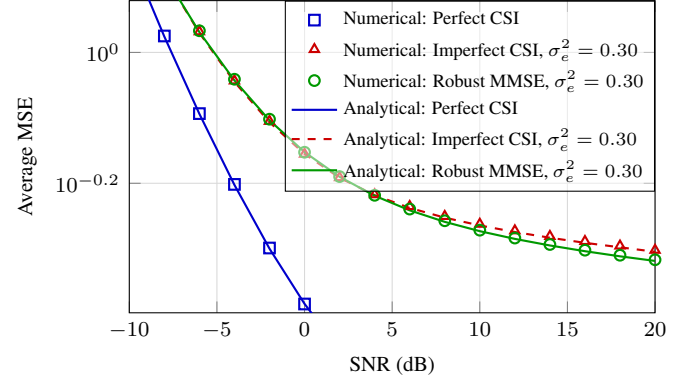


Fig. 3: Average MSE versus SNR for $\sigma_e^2 = 0.30$.

4. NUMERICAL RESULTS

Monte Carlo simulations use $N_t = 15$, $N_u = 2$, $N_r = 1$, $M_{\text{Tx}} = M_{\text{Rx}} = 2$, $N = 10$, $\epsilon_{\text{Tx}} = \epsilon_{\text{Rx}} = 0.22$, and 1000 channel realizations. The channel coefficients follow Rayleigh fading and the additive noise is complex Gaussian with power spectral density N_0 . The SNR is defined as

$$\text{SNR} = \frac{E_0 / (N_q T)}{N_0 (1 + \epsilon_{\text{Rx}}) / T} = \frac{E_0}{N_q N_0 (1 + \epsilon_{\text{Rx}})}. \quad (26)$$

The imperfect CSI model follows (10), where the uncertainty level 2ϵ determines the power of the unknown channel component. The following precoding strategies are compared: 1) MMSE with perfect CSI, for which the precoder is designed using the actual channel matrix $\mathbf{H}$; 2) MMSE with CSI mismatch, for which the precoder is designed using $\mathbf{bH}$ but evaluated over the actual channel $\mathbf{H}$; and 3) robust MMSE, for which the precoder is designed using $\mathbf{bH}$ together with the channel uncertainty covariance matrix $\mathbf{Q}$, as in (23), and evaluated over $\mathbf{H}$.

The analytical MSE is evaluated as

$$J_{\text{an}} = \frac{1}{L} \left(\left\| f c_2 \hat{\mathbf{H}}_{\text{eff}} \mathbf{p}_x - \mathbf{c}_{\text{out}} \right\|_2^2 + f^2 c_1^2 \mathbf{p}_x^H \mathbf{Q}_\Delta \mathbf{p}_x + f^2 \sigma_n^2 \right), \quad (27)$$

where $L = N_u N_{\text{tot}}$. The numerical MSE is obtained directly from the channel and noise realizations as

$$J_{\text{num}} = \frac{1}{L} \left\| f (\mathbf{H}_{\text{eff}} \mathbf{p}_x + \mathbf{G}_{\text{Rx,eff}} \mathbf{n}) - \mathbf{c}_{\text{out}} \right\|_2^2. \quad (28)$$

Fig. 3 confirms the agreement between the analytical and numerical MSE. At low SNR, noise dominates the performance, whereas at higher SNR the effect of CSI mismatch becomes more significant. By incorporating $\mathbf{Q}_\Delta$ into the optimization, the robust precoder reduces the residual MSE compared with the mismatched-CSI scheme.

Fig. 4 shows a similar trend in terms of BER. The mismatched-CSI precoder is affected by the discrepancy

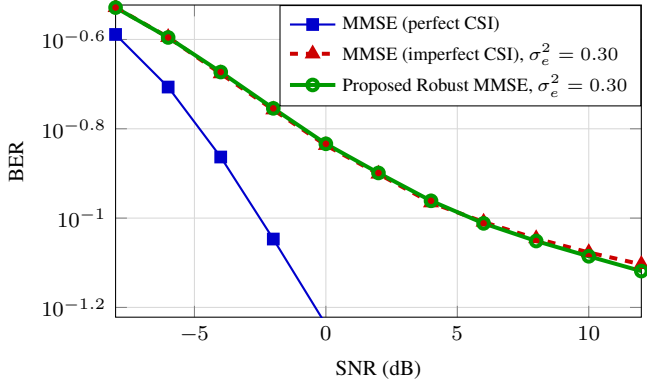


Fig. 4: BER versus SNR for the considered MMSE precoders.

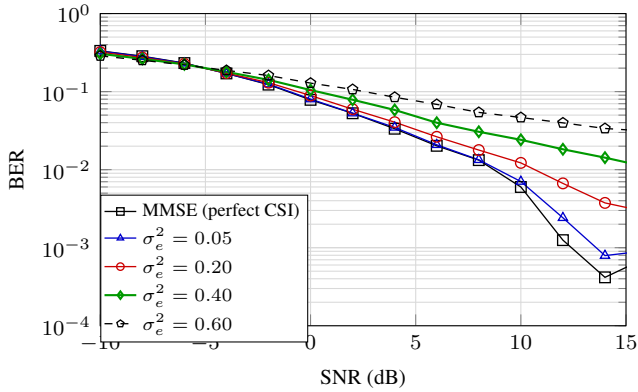


Fig. 5: BER versus SNR for different uncertainty levels σ_e^2 .

between $\hat{\mathbf{H}}$ and $\mathbf{H}$, while the proposed robust design mitigates this degradation that becomes more evident as the noise contribution decreases.

Fig. 5 illustrates the effect of channel uncertainty. For small σ_e^2 , the robust and conventional designs exhibit similar performance because the uncertainty penalty is weak. In the limiting case $\sigma_e^2 \rightarrow 0$, the channel uncertainty vanishes and

$$c_1^2 \mathbf{Q}_\Delta \rightarrow \mathbf{0}. \quad (29)$$

Consequently, the robust formulation approaches the conventional MMSE formulation associated with vanishing CSI uncertainty. As σ_e^2 increases, the term $\mathbf{p}_x^H \mathbf{Q}_\Delta \mathbf{p}_x$ becomes more relevant and the robust design increasingly penalizes directions that are sensitive to channel perturbations, resulting in a larger BER improvement.

5. CONCLUSIONS

A robust space-time MMSE precoder is proposed for zero-crossing modulation. The optimal precoding vector is derived in closed form via a Lagrangian formulation of the constrained MSE minimization problem, yielding a tractable solution that explicitly incorporates the error covariance matrix

of the effective channel. The numerical results confirm the considered MSE expression and that incorporating the error covariance matrix into the precoder design provides measurable improvements in both MSE and BER performance. The proposed robust precoder achieves a lower error floor than the conventional zero-crossing precoder.

Acknowledgements

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6. REFERENCES

- [1] J. Chen, L. Zhang, Y.-C. Liang, X. Kang, and R. Zhang, "Resource allocation for wireless-powered IoT networks with short packet communication," *IEEE Trans. Wireless Commun.*, vol. 18, no. 2, pp. 1447–1461, Feb. 2019.
- [2] A. Gokceoglu, E. Björnson, E. G. Larsson, and M. Valkama, "Spatio-temporal waveform design for multiuser massive MIMO downlink with 1-bit receivers," *IEEE J. Sel. Topics Signal Process.*, vol. 11, no. 2, pp. 347–362, March 2017.
- [3] D. M. V. Melo, L. T. N. Landau, R. C. de Lamare, P. F. Neuhaus, and G. P. Fettweis, "Zero-crossing precoding techniques for channels with 1-bit temporal oversampling ADCs," *IEEE Trans. Wireless Commun.*, vol. 22, no. 8, pp. 5321–5336, 2023.
- [4] S. Shamai (Shitz), "Information rates by oversampling the sign of a bandlimited process," *IEEE Trans. Inf. Theory*, vol. 40, no. 4, pp. 1230–1236, July 1994.
- [5] D. M. V. Melo, L. T. N. Landau, and R. C. de Lamare, "Zero-crossing precoding with MMSE criterion for channels with 1-bit quantization and oversampling," in *WSA 2020; 24th International ITG Workshop on Smart Antennas*, 2020, pp. 1–6.