

Efficient DGTD Method with LTS and IWDL Formulation for Multi-Scale EM Scattering

Enzo Nagao, João Lopes, Dinael Guevara, Diego Tami, Cássio do Rego, Diego Parada and Marlon Lizarazo

Abstract—This work presents a third-order local time stepping (LTS) method along with an efficient imposition source strategy applied to the discontinuous Galerkin time domain (DGTD) method in order to improve execution time while preserving precision. This formulation was applied to solve a complex multi-scale electromagnetic (EM) scattering problem. Numerical results showed that the proposed method maintains the precision of the standard DGTD with global time stepping (GTS) and reduced the execution time by about 80%.

Keywords—DGTD, LTS, IWDL, multi-scale EM problems.

I. INTRODUCTION

The analysis of complex multi-scale electromagnetic (EM) scattering problems demands efficient numerical methods to solve Maxwell's equations. These problems involve regions requiring fine spatial resolution near the scatterer, making conventional approaches computationally expensive. The discontinuous Galerkin time-domain (DGTD) method offers high accuracy and flexibility for transient EM simulations [1], [2], [3], [4]. However, its standard global time-stepping (GTS) scheme restricts the time step to the smallest element in the mesh, reducing efficiency for multi-scale geometries. To overcome this limitation, local time-stepping (LTS) techniques [5] allow different regions to advance in time with variable step sizes while maintaining stability and accuracy. The DGTD method has become increasingly popular in computational electromagnetics because it combines important characteristics of finite-element and finite-volume methods while preserving explicit time integration strategies. Unlike classical finite-element time-domain approaches, DGTD does not require the assembly and inversion of large global matrices. Moreover, its discontinuous formulation naturally supports unstructured meshes and local adaptivity, making it highly suitable for complex EM scattering problems involving curved geometries, dielectric interfaces, and electrically large domains.

One of the major limitations of explicit time-domain methods is the stability condition imposed by the Courant–Friedrichs–Lewy (CFL) criterion. In multi-scale

problems, highly refined regions are often required near material discontinuities or around geometrically small features. Consequently, the smallest element in the mesh determines the global time step for the entire computational domain. This restriction leads to unnecessary computations in coarser regions and substantially increases the overall execution time. Local time stepping techniques were proposed to alleviate this issue by allowing different element groups to evolve using independent time-step values according to their local mesh size. Several LTS approaches have been proposed in the literature for DGTD methods. Early methods were mainly based on Leapfrog formulations and recursive strategies [6], [7], [8]. Although these techniques demonstrated considerable computational gains, many of them relied on low-order interpolation procedures between neighboring classes, which may introduce oscillations and amplitude errors. More recently, Runge–Kutta-based LTS approaches [9], [10], [11] have attracted attention because they provide improved numerical stability and higher-order accuracy. In particular, third-order Runge–Kutta (RK3) schemes provide an attractive compromise between computational efficiency, memory consumption, and stability properties. Furthermore, when combined with high-order interpolation schemes, RK3-LTS methods preserve the temporal convergence order of the original formulation while maintaining continuity between neighboring element classes.

Another key aspect in EM scattering simulations is the source excitation. The total-field/scattered-field (TFSF) formulation is commonly used to introduce plane-wave illumination and is often combined with the near-field/far-field (NFFF) technique for Radar Cross Section (RCS) evaluations [12]. In traditional time-domain methods such as finite difference time-domain (FDTD) and finite element time-domain (FETD), these formulations are applied on distinct surfaces to ensure field continuity between neighboring elements. In contrast, the DGTD method's discontinuous nature enables both TFSF and NFFF boundaries to coincide, reducing the computational domain and simulation time. This formulation is known as incident-wave directly loaded (IWDL) [13]. The IWDL formulation provides several important advantages over the traditional TFSF approach. First, the computational modeling becomes considerably simpler because the total-field region is eliminated. Second, the number of degrees of freedom (DoF) is reduced, decreasing memory consumption and execution time. Third, the elimination of the propagation region between the source excitation and the scatterer reduces numerical dispersion errors, improving the overall solution accuracy.

While LTS and IWDL have been independently explored,

Enzo Nagao, ICTIN-UFLA, São Sebastião do Paraíso-MG, e-mail: enzo.nagao@estudante.ufla.br; João Lopes, ICTIN-UFLA, São Sebastião do Paraíso-MG, e-mail: joaorcardo.lopes@ufla.br; Dinael Guevara, GIDET-UFPS, Cúcuta-Colombia, e-mail: dinaelgi@ufps.edu.co; Diego Tami, ICT-UNIFEI, Itabira - MG, e-mail: diegotami@unifei.edu.br; Cássio do Rego, DELT-UFMG, Belo Horizonte-MG, e-mail: cassio@cpdee.ufmg.br; Diego Parada, ICTIN-UFLA, São Sebastião do Paraíso-MG, e-mail: diego.rozo@ufla.br; Marlon Lizarazo, ICTIN-UFLA, São Sebastião do Paraíso-MG, e-mail: marlonjesus@ufla.br; The authors would like to thank the Universidade Federal de Lavras for the financial support through the Programa de Inovação para Projetos em Ciência, Tecnologia e Ação Empreendedora, and the Brazilian agencies CAPES, FAPEMIG and CNPq (project 408384/2025-5).

their unified integration within a high-order DGTD framework represents a novel synergic approach. The primary contribution of this work lies in combining temporal adaptivity (RK3-LTS) with domain boundary reduction (IWDL) to eliminate total-field spatial overhead without degrading accuracy, achieving simultaneous temporal and spatial acceleration for multi-scale EM scattering.

II. THE DGTD METHOD FOR MAXWELL'S EQUATIONS

In this work, we focus on the discontinuous Galerkin discretization of Maxwell's curl equations defined as:

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\mu(\mathbf{r})\partial_t \mathbf{H}(\mathbf{r}, t) \quad (1)$$

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \varepsilon(\mathbf{r})\partial_t \mathbf{E}(\mathbf{r}, t) \quad (2)$$

where the electric, $\mathbf{E}$, and magnetic, $\mathbf{H}$, fields depend on the position vector, $\mathbf{r}$, and time t . In addition, ε and μ are known as electric permittivity and magnetic permeability, respectively.

These hyperbolic Eqs. 1 and 2 can be represented in the conservation form to apply the Galerkin discretization:

$$Q\partial_t \mathbf{q} + \nabla \cdot \mathbf{F}(\mathbf{q}) = 0 \quad (3)$$

where

$$Q = \begin{bmatrix} \mu(\mathbf{r}) & 0 \\ 0 & \varepsilon(\mathbf{r}) \end{bmatrix}, \quad \mathbf{q}(\mathbf{r}, t) = \begin{bmatrix} \mathbf{H}(\mathbf{r}, t) \\ \mathbf{E}(\mathbf{r}, t) \end{bmatrix},$$

$$\mathbf{F}(\mathbf{q}) = [\mathbf{F}_x, \mathbf{F}_y, \mathbf{F}_z]^T, \quad \mathbf{F}_i(\mathbf{q}) = \begin{bmatrix} -e_i \times \mathbf{E}(\mathbf{r}, t) \\ e_i \times \mathbf{H}(\mathbf{r}, t) \end{bmatrix},$$

with Q being the material matrix with the media information, $\mathbf{q}(\mathbf{r}, t)$ is the state vector, $\mathbf{F}(\mathbf{q})$ is known as the flux term and e_i represents the three Cartesian unit vectors, $i = x, y, z$. Once the discontinuous Galerkin spatial discretization has been applied, the semi-discrete form is rewritten as [1]:

$$\partial_t \mathbf{q} = \mathcal{L}(\mathbf{q}, t) \quad (4)$$

where $\mathcal{L}(\mathbf{q}, t)$ is an operator that combines the right-hand side (RHS) terms of Eq. 3.

The DGTD method discretizes the computational domain into non-overlapping elements. Inside each element, the electromagnetic fields are approximated by polynomial basis functions. One important characteristic of the DGTD formulation is that neighboring elements are coupled only through numerical fluxes imposed at their interfaces. This discontinuous nature provides high flexibility for parallel implementations and enables local refinement without introducing global continuity constraints. Another important advantage of the DGTD method is its compatibility with explicit time integration schemes. Since the resulting system of equations is solved locally within each element, the method avoids the solution of large sparse linear systems commonly found in implicit finite-element formulations. As a consequence, DGTD methods are particularly attractive for large-scale transient electromagnetic simulations.

In scattering problems, the numerical flux plays a crucial role in ensuring stability and accuracy. The flux terms weakly enforce the continuity conditions between adjacent elements and incorporate the boundary conditions into the formulation. Additionally, absorbing boundary conditions such as perfectly matched layers (PML) can be naturally integrated into the DGTD framework to truncate the computational domain.

A. Third-order LTS

The main motivation for using the LTS approach is to avoid forcing large elements to evolve using unnecessarily small time steps. In conventional GTS implementations, the time step is limited by the smallest element in the entire mesh due to the CFL stability condition. For multi-scale meshes, this leads to a considerable waste of computational resources because most elements are capable of advancing using larger stable time steps.

In this work, the temporal integration is performed using an explicit LTS scheme based on the third-order Runge–Kutta (RK3) method [5]. This LTS approach ensures continuity between different element classes through third-order interpolations. By employing interpolations with the same order of accuracy as the RK3 scheme, the overall precision of the solution is preserved. The temporal integration process employed in this work is illustrated in Fig. 1 considering the simplest LTS configuration composed of two refinement classes, namely Class 0 and Class 1. In this case, the largest elements belonging to Class 0 evolve using the global time step Δt_0 , while the refined elements of Class 1 advance using a smaller local time step equal to $\Delta t_0/2$.

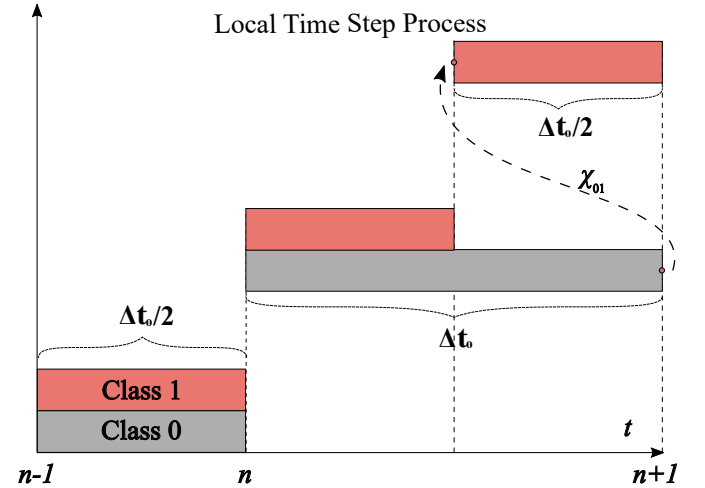


Fig. 1: Marching time process with two element classes.

The LTS procedure begins by advancing all classes from time level t_{n-1} to t_n while storing the required field information at the interfaces between neighboring classes. After this synchronization stage, the elements belonging to each class evolve independently according to their own locally stable time step. Thus, during a complete time interval of size Δt_0 , the elements of Class 0 are updated only once, whereas the elements of Class 1 are updated twice using $\Delta t_0/2$. Since neighboring classes evolve at different temporal resolutions,

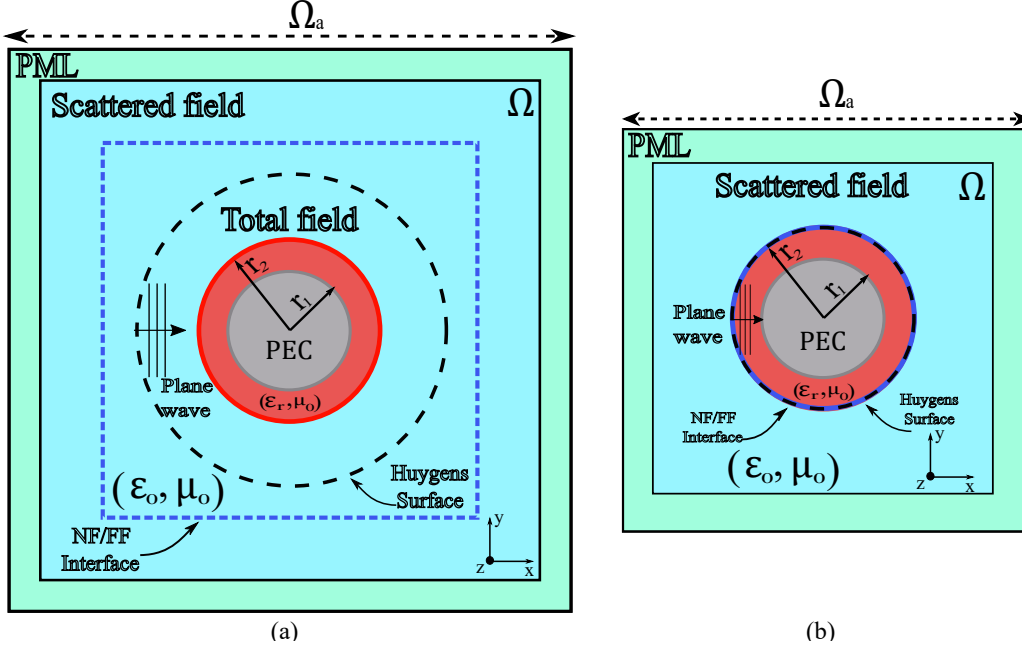


Fig. 2: Computational domain for a PEC-coated cylinder: TFSF (a) and IWDL (b) formulations.

interpolation procedures are required to preserve the continuity of the numerical solution across the interfaces. In the proposed formulation, third-order interpolation polynomials, χ_{01} , are employed to exchange temporal information between adjacent classes while maintaining the same order of accuracy of the RK3 integration scheme. After the smaller time-step updates are completed, both classes become synchronized again at time level t_{n+1} , and the process is repeated for the subsequent time interval. This strategy considerably reduces the total number of unnecessary element-time updates in coarse regions while preserving the stability and accuracy of the standard global time-stepping approach.

B. IWDL Formulation

The IWDL formulation is introduced as an acceleration strategy for solving scattering problems. Such problems typically consist of four regions: the scatterer interior, total and scattered fields, and the perfectly matched layer (PML) (see Fig. 2(a)). The IWDL approach simplifies the domain by merging the first two regions and applying the TFSF and NFFF formulations on the same surface over the scatterer (see Fig. 2(b)). This leads to a significant reduction in the number of elements and computational time while improving accuracy by eliminating dispersion errors [13]. In traditional TFSF formulations, the incident wave is introduced through an excitation surface surrounding the scatterer, while a second surface is required for the near-field/far-field transformation. Consequently, the computational domain contains an intermediate total-field region between the excitation surface and the scatterer.

The DGTD method presents a particular advantage because its discontinuous nature allows independent unknowns to be associated with the internal and external faces of each element. Taking advantage of this characteristic, the IWDL

formulation applies the incident field directly on the scatterer boundary while simultaneously using the same surface for field extrapolation. As a result, the total-field region disappears and the computational domain is reduced from four regions to only three regions. This simplification directly decreases the total number of elements and the degrees of freedom required in the simulation. Moreover, fewer surfaces must be treated during the modeling process, simplifying the implementation of electromagnetic scattering problems. Another important benefit of the IWDL approach concerns numerical accuracy. In conventional TFSF formulations, the incident wave propagates from the excitation surface to the scatterer and the scattered field propagates from the scatterer to the near-field/far-field surface. These propagation paths introduce numerical dispersion errors. Since the IWDL formulation removes these intermediate regions, the associated dispersion errors are significantly reduced [13]. Therefore, the IWDL formulation not only accelerates the simulation but also improves the quality of the computed electromagnetic fields and RCS responses.

III. RESULTS AND CONCLUSIONS

All simulations were executed on a workstation equipped with an AMD Ryzen 7 5800H CPU, 16 GB of RAM, and an NVIDIA GTX 1650 GPU (4 GB VRAM). The proposed method was tested on a PEC-coated dielectric cylinder with relative permittivity $\epsilon_r = 3$, non-magnetic, immersed in vacuum. The cylinder has inner and outer radii of $r_1 = 0.5\lambda$ and $r_2 = 0.7\lambda$, respectively, and is illuminated by a 300 MHz plane wave with PML absorbing boundaries, as illustrated in Fig. 2. The TFSF formulation used 32,748 elements, while the IWDL formulation reduced the mesh to 15,930 elements, cutting the domain size by more than 50%. Time steps for TFSF were $\Delta t_0 = 5.44$ ps, $\Delta t_1 = 2.72$ ps, and $\Delta t_2 = 1.36$ ps, while the IWDL formulation used Δt_1 and Δt_2 . Finally, the

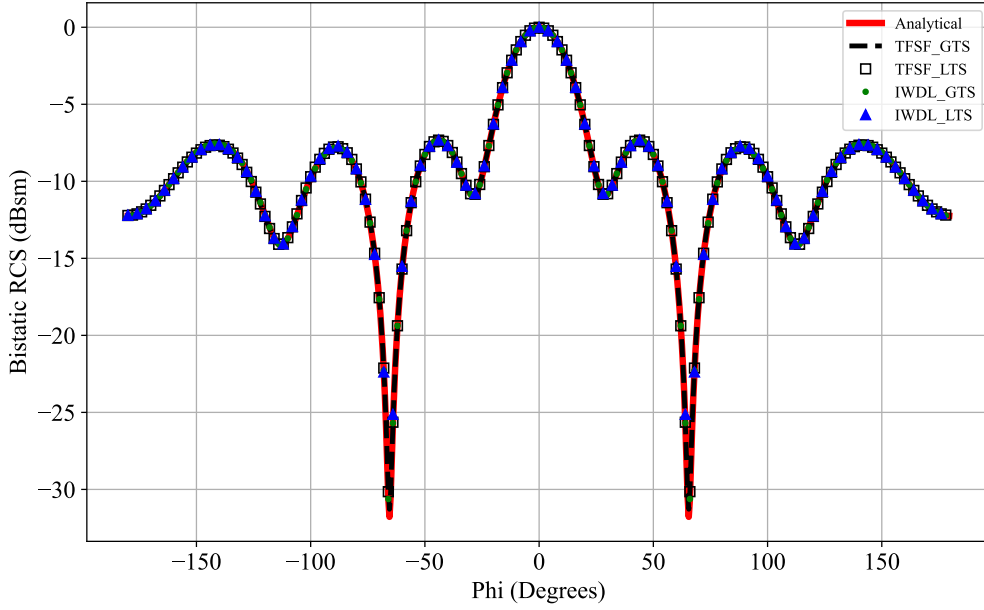


Fig. 3: RCS of the PEC-coated cylinder for TMz polarization.

simulation was carried out for 66.66 ns. The proposed test problem represents a challenging multi-scale electromagnetic scattering scenario because the coated structure requires local mesh refinement near the dielectric interfaces while larger elements can still be employed in other regions of the domain. Such configurations are particularly suitable for evaluating the efficiency of LTS schemes since they naturally generate different stable time-step requirements throughout the mesh.

Table I summarizes the execution time, speedup, and bistatic RCS error for all approaches, demonstrating that errors remain below 0.07% for all cases. The bistatic RCS error reported in Table I was calculated using the normalized mean absolute error defined as $RCS_{error} = \frac{1}{N} \sum_{i=1}^N |RCS_{analytical}^{(i)} - RCS_{numerical}^{(i)}|$, where N is the total number of observation points. In terms of performance, LTS reduced execution time by roughly 50%, and IWDL alone provided a speedup of about 2.5x. The combination of LTS and IWDL achieved a total speedup of 4.56x, corresponding to an 80% reduction in runtime, demonstrating the efficiency and accuracy of the proposed approach for multi-scale EM scattering problems.

TABLE I: Numerical results for the PEC coated problem

Method	Execution Time(s)	Speedup	RCS_error
GTS-TFSF	9134.06	–	0.0651
LTS-TFSF	4401.77	2.07	0.06512
GTS-IWDL	3694.8	2.47	0.06339
LTS-IWDL	2001.32	4.56	0.0634

The results indicate that the LTS strategy successfully reduces the number of element-time updates without compromising the numerical stability of the DGTD method. The relatively small difference between the RCS errors of the GTS

and LTS approaches confirms that the third-order interpolation employed in the proposed implementation effectively preserves the temporal accuracy of the RK3 scheme. The IWDL formulation also demonstrated considerable computational advantages. By removing the total-field region, the number of elements was reduced by more than half, leading to significant reductions in both memory consumption and execution time. Furthermore, the elimination of the propagation region contributed to a slight improvement in the RCS error when compared with the traditional TFSF formulation.

Fig. 3 presents the bistatic RCS for TMz polarization. As observed, all numerical solutions are in good agreement with the analytical reference solution. The curves obtained using the proposed LTS-IWDL formulation accurately reproduce the scattering behavior over the entire angular range. The numerical results also confirm that the combination of both acceleration techniques provides complementary benefits. While the LTS approach reduces the temporal integration cost, the IWDL formulation decreases the spatial computational burden. Consequently, the combined formulation achieves significantly larger performance gains than those obtained by each strategy independently. Overall, the proposed DGTD-LTS-IWDL formulation proved to be an efficient and accurate alternative for solving complex multi-scale electromagnetic scattering problems.

While the present work validates the proposed LTS-IWDL framework on a benchmark multi-scale geometry, future research will focus on extending this formulation to complex 3D real-world applications, such as antenna modeling, dispersive metamaterials, and electromagnetic biosensors, as well as incorporating multi-GPU parallel acceleration.

ACKNOWLEDGMENT

The authors would like to thank the Universidade Federal de Lavras for the financial support through the Programa de Inovação para Projetos em Ciência, Tecnologia e Ação Empreendedora, and the Brazilian agencies CAPES, FAPEMIG and CNPq (project 408384/2025-5).

REFERENCES

- [1] J. S. Hesthaven, and T. Warburton, *Nodal discontinuous Galerkin methods: algorithms, analysis, and applications*, Springer Science & Business Media, 2007.
- [2] Y. S. Tian, H. Cao, L. Wang, Q. Sun, Q. H. Liu and W. Chen, “Hybrid Element-Level and Subdomain-Level DGTD Method for Multiscale Electromagnetic Modeling,” *IEEE Antennas and Wireless Propagation Letters*, vol. 25, no. 03, pp. 1214-1218, 2026.
- [3] H. Chen, Q. Yang, L. Li, B. Wei, J. Dong and Z. Li, “A new DGTD algorithm for calculating radiation from rotationally symmetric antennas,” *Computer Physics Communications*, vol. 327, p. 110279, 2026.
- [4] L. Li, X. He, Q. Yang, B. Wei and H. Zheng, “Simulation and Analysis of Electromagnetic Properties of Magnetized Ferrite by SO-DGTD Algorithm,” *IEEE Transactions on Plasma Science*, vol. 54, no. 6, pp. 3307-3314, 2026.
- [5] M. J. Lizarazo, and E. J. Silva, “A Parallel-GPU DGTD Algorithm with a Third-Order LTS Scheme for Solving Multi-Scale Electromagnetic Problems,” *Mathematics*, vol. 12, no. 23, p. 3663, 2024.
- [6] M. Li, Q. Wu, Z. Lin, Y. Zhang and X. Zhao, “Efficient Improved Local Time Stepping with the Leapfrog Scheme for Transient 3-D Electromagnetic Analyses,” *ACES Journal*, vol. 38, no. 08, pp. 587-593, 2023.
- [7] J. Alvarez, L. D. Angulo, A. R. Bretones, C. M. D. J. van Coevorden and S. G. Garcia, “Efficient antenna modeling by DGTD: Leapfrog discontinuous Galerkin timedomain method,” *IEEE Antennas and Propagation Magazine*, vol. 57, no. 03, pp. 95-106, 2015.
- [8] G. Xie, C. Wang, D. Han, Z. Huang, S. Sun and R. Zhao, “Matrix Exponential-Based Symplectic DGTD Algorithm for Simulating Drude-Type Dispersive Media,” *IEEE Transactions on Antennas and Propagation*, vol. 74, no. 6, pp. 6000-6005, 2026.
- [9] D. Feng, S. Liu, X. Wang, and G. Li, “High-order GPU-DGTD method based on unstructured grids for GPR simulation,” *Journal of Applied Geophysics*, vol. 202, p. 104666, 2022.
- [10] S. Xue, C. Yin, J. Li, J. Zhu, and W. Lui, “A High Efficiency Discontinuous Galerkin Method for 3D Ground-Penetrating Radar Simulation,” *Remote Sensing*, vol. 17, no. 2, p. 228, 2025.
- [11] R. Xiang, M. Khamitova, S. C. Sierra, X. Ma, J. Wang and H. Bagci, “Analysis of Generalized Nonlocal Optical Response in Plasmonic Nanostructures using DGTD,” *2026 International Applied Computational Electromagnetics Society Symposium (ACES)*, pp. 1-2, 2026.
- [12] M. J. Lizarazo, D. P. Roza, D. T. Lopez, D. Guevara, C. do Rego and E. J. da Silva, “A GPU-DGTD Algorithm with Drude Dispersive Modeling for Photonic Applications,” *2026 IEEE 2nd Latin American Conference on Antennas and Propagation (LACAP)*, pp. 1-2, 2026.
- [13] D. Han, Y. Zhao, R. Zhao, L. Wu, and J. Hu, “An improved incident-wave formulation for broadband electromagnetic scattering by discontinuous Galerkin time-domain method,” *International Journal of RF and Microwave Computer-Aided Engineering*, vol. 32, no. 9, p. e23281, 2022.