

DCIM-GPOF Formulation for Electric Dipole Arrays in Multilayer Plasmonic Structures

Nadson Welkson, Thamyris Evangelista, Jean Silva, André Cruz and Karlo Costa

Abstract—This work presents a formulation for the analysis of electric dipole arrays in multilayered media, based on the combination of the Discrete Complex Image Method (DCIM) with the Generalized Pencil-of-Function (GPOF) method. The formulation adopts an automatic spectral path in the complex plane, reducing empirical tuning and facilitating the numerical evaluation of the Green's function. The approach captures the collective electromagnetic coupling, showing that the system response emerges from a globally coupled regime. The results reveal spatial patterns associated with the spectral structure of the medium and the array geometry, providing a systematic methodology for the analysis of stratified plasmonic structures.

Keywords—Coupled dipoles, Green's function, multilayer media, DCIM, GPOF.

I. INTRODUCTION

The electromagnetic interaction between electric dipoles and stratified media is a fundamental problem in applied electrodynamics, with relevance in areas such as plasmonics [1], nanoantennas [2], and photonic devices [3]. At optical frequencies, the presence of dielectric-metallic interfaces significantly alters the field distribution due to the excitation of surface plasmon modes and strong electromagnetic confinement [4].

In this regime, numerical analysis becomes particularly challenging, since nanometric wavelengths require extremely fine discretizations, substantially increasing the computational cost of conventional mesh-based methods such as FEM and FDTD [4].

In arrays of subwavelength nanoparticles, each element can be modeled as an induced electric dipole, whose moment depends on the local field, resulting from the superposition of the external field with the contributions of the other dipoles [5]. This mutual dependence leads to a coupled system, whose solution describes the collective behavior of the array.

The main challenge lies in evaluating the Green's function of the multilayer medium, which involves highly oscillatory and numerically costly Sommerfeld integrals [6]. Methods based on complex images allow replacing these integrals with discrete sums, significantly reducing computational cost [7].

A formulation is proposed for the analysis of finite arrays of vertical electric dipoles in multilayered media, based on the combination of the Discrete Complex Image Method (DCIM) [8] with the Generalized Pencil-of-Function (GPOF) method [9], [10]. The automatic spectral path previously presented in [11] is adopted here for the evaluation of the multilayer

Green's function. The contribution of this work lies in integrating this formulation with the coupled-dipole model to analyze collective and localized excitation regimes in finite arrays.

II. PROBLEM FORMULATION

A. Coupled Dipole Model

Consider N_{dip} electric dipoles at $\mathbf{r}_q$, with $q = 1, \dots, N_{\text{dip}}$, in layer 1 of an N -layer planar medium with interfaces $z = d_i$ ($i = 1, \dots, N - 1$) and semi-infinite outer layers. Each dipole responds linearly to the local field, with its moment given by

$$\mathbf{p}_q = \bar{\bar{\alpha}}_q \mathbf{E}(\mathbf{r}_q), \quad (1)$$

where $\bar{\bar{\alpha}}_q$ is the polarizability tensor and $\mathbf{E}(\mathbf{r}_q)$ is the local field that induces the moment of the q -th dipole [12].

The local field at each dipole results from the superposition of the incident field, the reflected self-interaction, and the fields generated by the remaining dipoles:

$$\mathbf{E}(\mathbf{r}_q) = \mathbf{E}^{\text{inc}}(\mathbf{r}_q) + \bar{\bar{G}}^{\text{ref}}(\mathbf{r}_q, \mathbf{r}_q) \cdot \mathbf{p}_q + \sum_{\substack{p=1 \\ p \neq q}}^{N_{\text{dip}}} \bar{\bar{G}}(\mathbf{r}_q, \mathbf{r}_p) \cdot \mathbf{p}_p. \quad (2)$$

where:

- $\mathbf{E}^{\text{inc}}(\mathbf{r}_q)$ is the incident field at $\mathbf{r}_q$;
- $\bar{\bar{G}}^{\text{ref}}(\mathbf{r}_q, \mathbf{r}_q)$ is the reflected self-interaction field produced by dipole q and evaluated at its own position;
- $\bar{\bar{G}}(\mathbf{r}_q, \mathbf{r}_p)$ is the complete dyadic Green's function of the multilayer medium, including direct and reflected contributions from a unit dipole at $\mathbf{r}_p$ to $\mathbf{r}_q$;
- the index p runs over the source dipoles, whereas q identifies the observation dipole;
- the condition $p \neq q$ excludes only the singular direct self-interaction from the summation, while the finite reflected self-interaction is retained explicitly.

In this way, the essence of mutual coupling is revealed: each dipole is simultaneously a source and a receiver. The moment $\mathbf{p}_q$ that dipole q acquires depends on the field it senses at $\mathbf{r}_q$; this field, in turn, is partially produced by the moments $\mathbf{p}_p$ of all other dipoles $p \neq q$. The self-consistent solution of the system provides the dipole moments and, consequently, the total scattered field at any point in space.

Eq. (2) highlights a fundamental aspect of the system: even if only one dipole is directly excited, all others exhibit induced moments due to the electromagnetic interaction mediated by the Green's function. Consequently, the system response is not local, but governed by mutual interactions.

Nadson Welkson, Thamyris Evangelista and Jean Silva; Department of Electrical Engineering, UNIFESSPA, Marabá-PA, e-mail: nadsonwelkson@unifesspa.edu.br, thamyris.evangelista@unifesspa.edu.br and jean.carneiro@unifesspa.edu.br. André Cruz and Karlo Costa; Department of Electrical Engineering, UFPA, Belém-PA, e-mail: andre@ufpa.edu.br and karlo@ufpa.edu.br.

In the specific case treated in this work, two distinct excitation regimes are considered, both consistent with the coupled system formulation:

- Coherent collective excitation (uniform mode): all dipoles are excited simultaneously and in phase with unit amplitude, producing a global response governed by constructive interference and mutual coupling.
- Localized excitation (single mode): only one dipole is directly excited with unit amplitude, while the others respond exclusively through electromagnetic coupling mediated by the Green's function of the medium.

In both cases, the field cannot be interpreted as a superposition of independent contributions, but as the solution of a coupled system dominated by the interaction between the dipoles and the multilayer medium. Thus, the problem is governed by the spectral structure of the Green's function.

For z -oriented dipoles, $\mathbf{p}_q = p_{z,q} \mathbf{a}_z$, and only G_{zz} contributes. Eqs. (1) and (2) then yield

$$p_{z,q} = \alpha_{zz}^{(q)} \left[E_z^{\text{inc}}(\mathbf{r}_q) + G_{zz}^{\text{ref}}(0, z_q, z_q) p_{z,q} + \sum_{\substack{p=1 \\ p \neq q}}^{N_{\text{dip}}} G_{zz}(\rho_{qp}, z_q, z_p) p_{z,p} \right]. \quad (3)$$

where $\rho_{qp} = |\boldsymbol{\rho}_q - \boldsymbol{\rho}_p|$, $\alpha_{zz}^{(q)}$ is the vertical polarizability, E_z^{inc} is the incident field, taken as unity for each directly excited dipole and zero otherwise, and $G_{zz}^{\text{ref}}(0, z_q, z_q)$ is the reflected self-interaction of dipole q , with $\rho_{qq} = 0$ because the source and observation points correspond to the same dipole.

For a point electric dipole source in layer 1 and an observation point in layer i , the Green's function admits the Sommerfeld spectral representation [13]:

$$G_{zz}(\rho, z, z') = \frac{1}{4\pi\omega\epsilon_i} \int_0^{+\infty} dk_\rho \frac{k_\rho^3}{k_{1z}} J_0(k_\rho \rho) A_i^{TM}(k_\rho) \times e^{-jk_{1z}z'} \left[e^{jk_{iz}z} + \tilde{R}_{i,i+1}^{TM}(k_\rho) e^{-jk_{iz}(z+2d_i)} \right], \quad (4)$$

Here, ϵ_i and μ_i are the layer parameters, $k_i = \omega\sqrt{\mu_i\epsilon_i}$, $k_{iz} = \sqrt{k_i^2 - k_\rho^2}$, and J_0 is the zeroth-order Bessel function. For dipole coupling, $\rho = \rho_{qp}$, $z = z_q$, and $z' = z_p$.

The Fresnel coefficients are calculated for TM polarization [13], and the generalized reflections are obtained by recursion from bottom to top, with propagation factor $\Phi_{i+1} = \exp[j2k_{(i+1)z}(d_{i+1} - d_i)]$:

$$\tilde{R}_{i,i+1}^{TM} = \frac{r_{i,i+1}^{TM} + \tilde{R}_{i+1,i+2}^{TM} \Phi_{i+1}}{1 + r_{i,i+1}^{TM} \tilde{R}_{i+1,i+2}^{TM} \Phi_{i+1}}, \quad \tilde{R}_{N,N+1}^{TM} = 0. \quad (5)$$

The progressive amplitudes are calculated from $A_1^{TM} = 1$:

$$A_i^{TM} = A_{i-1}^{TM} \frac{e^{jk_{(i-1)z}d_{i-1}} S_{i-1,i}^{TM}}{e^{jk_{iz}d_{i-1}}}, \quad (6)$$

with

$$S_{i-1,i}^{TM} = \frac{t_{i-1,i}^{TM}}{1 + r_{i-1,i}^{TM} \tilde{R}_{i,i+1}^{TM} \exp[j2k_{iz}(d_i - d_{i-1})]}. \quad (7)$$

After solving the coupled linear system in Eq. (3) for all dipole moments, the field generated by the array at an observation point $\mathbf{r}$ is

$$E_z(\mathbf{r}) = \sum_{p=1}^{N_{\text{dip}}} G_{zz}(\rho_p, z, z_p) p_{z,p}, \quad (8)$$

where p indexes the source dipoles whose contributions are summed at the observation point $\mathbf{r}$, $\rho_p = |\boldsymbol{\rho} - \boldsymbol{\rho}_p|$ is the horizontal distance to dipole p , and z_p is its height. The incident excitation is accounted for through the calculated moments $p_{z,p}$. The self-singularity is excluded from the matrix and avoided numerically in the maps.

III. KERNEL DEFINITION

For the application of the DCIM-GPOF method, it is necessary to explicitly define which spectral kernels of the multilayer Green's function are approximated by the GPOF.

A. Source Layer ($i = 1$)

In the source layer, G_{zz} decomposes into two contributions:

$$G_{zz}^{(1)}(\rho, z, z') = G_{zz}^{\text{dir}}(\rho, z, z') + G_{zz}^{\text{ref}}(\rho, z, z'). \quad (9)$$

The direct term corresponds to the homogeneous-medium Green's function for a vertical dipole, which admits a closed-form spatial expression [11], [12]:

$$G_{zz}(\rho, R, k) = \frac{e^{-jkR}}{R^3} \left[\left(\frac{2R^2 - 3\rho^2}{R^2} \right) (1 - jkR) - \frac{\rho^2}{R^2} (kR)^2 \right], \quad (10)$$

where $R = \sqrt{\rho^2 + z_d^2}$, $z_d = |z - z'|$ and $G_{zz}^{\text{dir}}(\rho, z, z') = G_{zz}(\rho, R, k_1)/(4\pi j\omega\epsilon_1)$.

The reflected term incorporates the interface effects and admits the Sommerfeld spectral representation:

$$G_{zz}^{\text{ref}} = \frac{1}{4\pi\omega\epsilon_1} \int_0^\infty dk_\rho \frac{k_\rho^3}{k_{1z}} J_0(k_\rho \rho) \tilde{R}_{1,2}^{TM}(k_\rho) e^{-jk_{1z}(z+z')}. \quad (11)$$

The reduced reflected-field spectral kernel is defined as

$$\tilde{G}_{zz}^{(1)}(k_\rho) = \tilde{R}_{1,2}^{TM}(k_\rho), \quad (12)$$

upon which the GPOF is applied, resulting in an equivalent exponential representation. Each term of the expansion is interpreted as an image source at a complex position, allowing one to write

$$G_{zz}^{\text{ref}}(\rho, z, z') \approx \frac{1}{(4\pi j\omega\epsilon_1)} \sum_{m=1}^{M_1} A_m^{(1)} G_{zz}(\rho, R_m^{(1)}, k_1), \quad (13)$$

$$R_m^{(1)} = \sqrt{\rho^2 + (z + z' + jB_m^{(1)})^2}, \quad (14)$$

where:

- M_1 is the order of the exponential approximation;
- $A_m^{(1)}$ are the complex coefficients (amplitudes) determined by least squares;
- $B_m^{(1)}$ are the exponential constants extracted from the GPOF eigenvalues.

B. Intermediate Layers ($2 \leq i \leq N-1$)

In the intermediate layers, the spectral kernel decomposes into two physically distinct contributions:

$$\tilde{G}_{zz,\downarrow}^{(i)}(k_\rho) = A_i^{TM}(k_\rho), \quad \tilde{G}_{zz,\uparrow}^{(i)}(k_\rho) = A_i^{TM}(k_\rho) \tilde{R}_{i,i+1}^{TM}(k_\rho), \quad (15)$$

representing, respectively, the downward component (direct propagation from the source) and the upward component (internal reflections). Each is treated independently by the GPOF, generating the sets $\{A_{m,\downarrow}^{(i)}, B_{m,\downarrow}^{(i)}\}$ and $\{A_{m,\uparrow}^{(i)}, B_{m,\uparrow}^{(i)}\}$.

The spatial reconstruction is then given by the superposition of the two families of images:

$$G_{zz}^{(i)}(\rho, z, z') \approx \frac{1}{4\pi j\omega\epsilon_i} \left[\sum_m A_{m,\downarrow}^{(i)} G_{zz}(\rho, R_{m,\downarrow}^{(i)}, k_i) + \sum_m A_{m,\uparrow}^{(i)} G_{zz}(\rho, R_{m,\uparrow}^{(i)}, k_i) \right], \quad (16)$$

$$\begin{aligned} R_{m,\downarrow}^{(i)} &= \sqrt{\rho^2 + (z' - z + jB_{m,\downarrow}^{(i)})^2}, \\ R_{m,\uparrow}^{(i)} &= \sqrt{\rho^2 + (z + z' - 2d_i + jB_{m,\uparrow}^{(i)})^2}. \end{aligned} \quad (17)$$

C. Last Layer ($i = N$)

In the last semi-infinite layer, the field is composed exclusively of the transmitted wave. The corresponding transmitted-field spectral kernel reduces to the progressive amplitude:

$$\tilde{G}_{zz}^{(N)}(k_\rho) = A_N^{TM}(k_\rho), \quad (18)$$

whose approximation via GPOF provides the parameters $(A_m^{(N)}, B_m^{(N)})$. The spatial reconstruction is written as

$$G_{zz}^{(N)}(\rho, z, z') \approx \frac{1}{4\pi j\omega\epsilon_N} \sum_{m=1}^{M_N} A_m^{(N)} G_{zz}(\rho, R_m^{(N)}, k_N), \quad (19)$$

$$R_m^{(N)} = \sqrt{\rho^2 + (|z| + z' + jB_m^{(N)})^2}. \quad (20)$$

In this formulation, propagation effects, evanescent modes, plasmonic poles, and *branch point* contributions are captured by the image sources in complex coordinates, eliminating the need for direct numerical integration in all layers.

IV. AUTOMATIC SPECTRAL PATH

The selected spectral kernels are sampled along an automatic complex path adapted to the physical and spatial parameters of each configuration. The automatic spectral path previously presented in [11] is parameterized in the complex k_z plane as

$$k_z(t_n) = k_1 [1 - j(N_1 t_n + W_1)], \quad t_n \in [0, 1], \quad (21)$$

$$t_n = \frac{n-1}{PS-1}, \quad n = 1, \dots, PS. \quad (22)$$

Here, N_1 controls the path depth, W_1 its initial offset from the real axis, and PS the number of spectral samples. The radial wavenumber follows from $k_\rho^2 = k_1^2 - k_z^2$. For each configuration, these parameters are recalculated by the automatic procedure in [11], considering the operating frequency, material properties, source position, and observation domain. Thus, they are configuration-dependent rather than fixed inputs. The transformation maps the original spectral integration path onto discrete points in the complex plane, avoiding problematic regions near the real axis during the GPOF approximation.

V. RESULTS

A. Test Structure

A finite 7×7 square array of vertical electric dipoles is considered in layer 1, with $d_1 = 0$ and all elements positioned at $z' = 20$ nm. The dipole spacing is 125 nm, and the operating wavelength is $\lambda = 633$ nm. For all dipoles, $\alpha_{zz}^{(a)} = 1$ is adopted, and logarithmic field values are referenced to unity in the numerical units.

For the analysis, a three-layer nonmagnetic structure ($\mu_r = 1$), with the following complex relative permittivities, is adopted [14]:

- Layer 1 (air): $\epsilon_r = 1.0 - j0.01$, semi-infinite;
- Layer 2 (gold): $\epsilon_r = -11.53 - j1.20$, thickness $h_2 = 50$ nm;
- Layer 3 (prism): $\epsilon_r = 2.3013 - j0.01$, semi-infinite.

The small loss terms assigned to air and the prism were retained from the reference configuration adopted in [11].

The spatial domain is $x, y, z \in [-2, 2] \mu\text{m}$, with the electric field evaluated through vertical (x, z) and horizontal (x, y) two-dimensional cuts rather than full volumetric discretization.

Spatial discretization is applied only within each analyzed plane, with $\Delta x = \Delta z = 1$ nm for the (x, z) plane and $\Delta x = \Delta y = 1$ nm for the (x, y) plane.

The dipoles are positioned in layer 1 at a subwavelength distance of 20 nm from the metallic interface, favoring coupling to TM-polarized surface modes.

B. Coherent Collective Excitation (Uniform Mode)

The result presented in Fig. 1 indicates the excitation of plasmonic modes at the gold-air interface, located at $z \approx 0$. In the logarithmic field distribution shown in Fig. 1(a), a strong field concentration is observed near the interface, followed by decay in the normal direction, a behavior consistent with surface plasmon polaritons (SPPs).

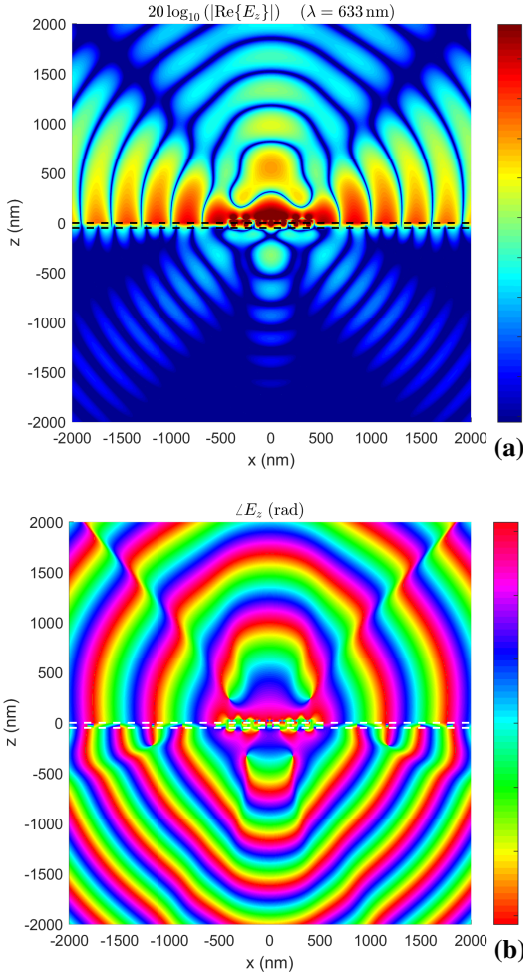


Fig. 1. E_z in the (x, z) plane: (a) $20 \log_{10}(|\text{Re}\{E_z\}|)$ [dB] and (b) $\arg(E_z)$ [rad].

Additionally, a laterally extended field distribution is observed along the gold–air interface, consistent with the collective response of the dipole array and the excitation of a surface mode. Owing to the finite thickness of the gold film, part of this surface-bound field couples through the metal layer to propagating waves in the higher-index prism.

In the prism region, below the gold layer ($z < -h_2$), a directional wave pattern is observed in both the logarithmic field distribution (Fig. 1(a)) and the phase map (Fig. 1(b)). This behavior is consistent with Surface Plasmon Coupled Emission (SPCE) [7], in which energy coupled to surface-plasmon modes radiates into the higher-index dielectric medium with a well-defined angular distribution.

Fig. 2 presents $20 \log_{10}(|\text{Re}\{E_z\}|)$ in horizontal planes near the dipole array. At $z = 20$ nm, the region occupied by the array is clearly identified by the local field maxima occurring in the vicinity of each dipole.

Moreover, the field distribution departs from radial symmetry and reflects the square geometry of the array. This behavior is evidenced by the deformation of the field contours and by the reduced field levels near the corners, showing that the spatial response is strongly influenced by the dipole arrangement.

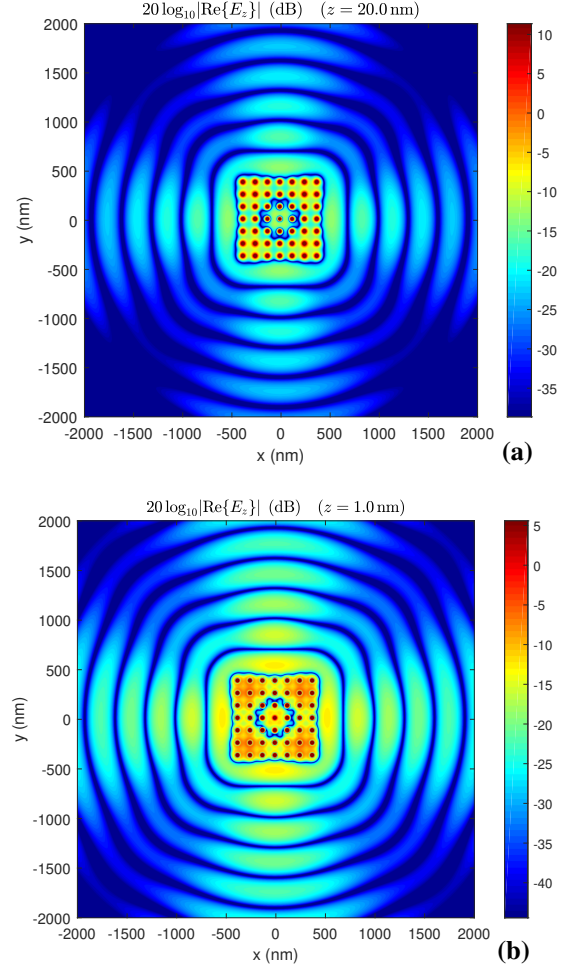


Fig. 2. $20 \log_{10}(|\text{Re}\{E_z\}|)$ [dB] in the (x, y) plane at (a) $z = 20$ nm and (b) $z = 1$ nm.

The horizontal maps are also consistent with the surface-confined field observed in the (x, z) plane. In Fig. 2(a), at $z = 20$ nm, strong local fields are observed around the dipoles. In Fig. 2(b), at $z = 1$ nm, a considerable field level remains near the gold–air interface, supporting the confinement and enhancement associated with the plasmonic surface mode.

C. Localized Excitation (Single Mode)

Fig. 3 compares the response of an isolated dipole with that of the finite array when only the central dipole is directly excited. For the isolated dipole in Fig. 3(a), the field exhibits an approximately circular pattern, consistent with the radial symmetry of a z -oriented dipole in a laterally isotropic multilayered medium.

When the finite array is considered in Fig. 3(b), the response changes significantly. Although only the central dipole is directly excited, the remaining elements acquire induced dipole moments. This behavior follows from the Green's matrix $\mathbf{G}_{zz}$, which describes the electromagnetic interaction among all dipoles in the array.

The field generated by the directly excited dipole contributes to the local field at the other elements, inducing moments that reradiate and modify the local fields throughout the system.

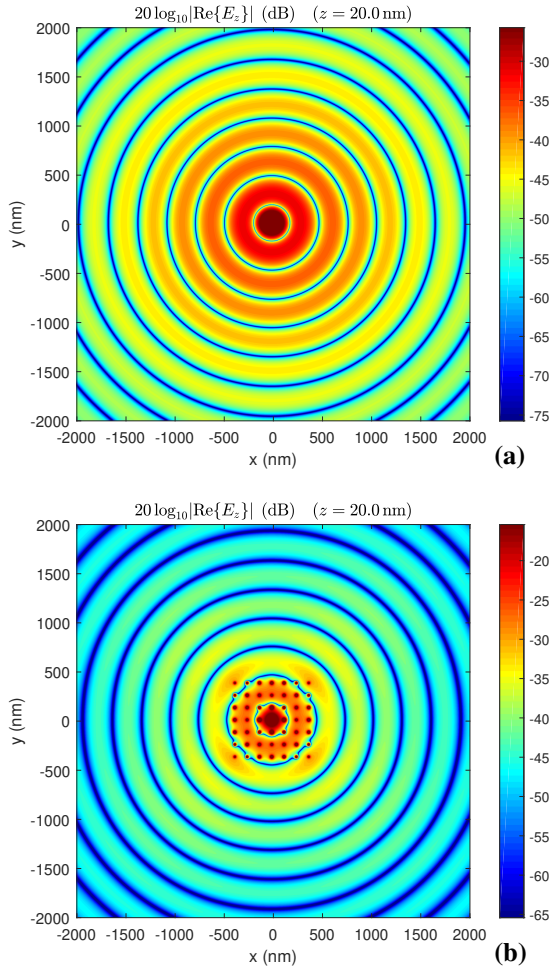


Fig. 3. $20 \log_{10}(|\text{Re}\{E_z\}|)$ [dB] in the (x, y) plane at $z = 20$ nm: (a) isolated dipole and (b) central dipole excited within the array.

Consequently, the response is determined collectively rather than by the central dipole alone.

This effect is evident in Fig. 3(b), where the field distribution is modified by the square geometry of the array and departs from the radial symmetry observed for the isolated dipole. A central region of high field levels is formed, while the outer field contours retain an approximately radial character.

The spatial modification across the array indicates that the interaction is not restricted to the directly excited element and its nearest neighbors. Instead, the response involves the coupled moments distributed throughout the array.

Therefore, Fig. 3 shows that the response cannot be interpreted as the superposition of independently responding dipoles, but as the field generated by dipole moments obtained from a globally coupled system mediated by the multilayer Green's function.

VI. CONCLUSION

This work demonstrated that the combination of DCIM with GPOF, associated with the previously developed automatic spectral path, enables a systematic evaluation of the Green's

function in multilayered media. The formulation replaces Sommerfeld integrals with discrete complex-image representations that retain the relevant spectral contributions. Furthermore, the results show that dipole arrays exhibit a coupled response, even when one element is directly excited.

The formulation enables effects of array geometry, source position, layer thickness, and material properties on field confinement, collective coupling, and directional radiation to be investigated without volumetric discretization. Thus, the proposed approach provides a consistent framework for the analysis and design of plasmonic sensors, emitters, and stratified electromagnetic systems.

ACKNOWLEDGMENTS

UNIFESSPA and UFPA are acknowledged.

REFERENCES

- [1] A. F. S. da Cruz, K. Q. da Costa, V. Dmitriev, N. W. P. de Souza, Q. Zaman, and T. Del Rosso, "Electromagnetic model of a nanodipole array above a double-layer graphene by periodic Green's function," *IET Microw. Antennas Propag.*, vol. 14, no. 15, pp. 2088–2096, Dec. 2020.
- [2] K. Q. da Costa, V. Dmitriev, T. L. T. Santos, N. W. P. de Souza, J. L. Souza, and G. L. Silvano, "Simple and efficient computational method to analyze cylindrical plasmonic nanoantennas," in *Proc. MOMAG 2012 - 15th SBMO Brazilian Symposium on Microwaves and Optoelectronics and 10th CBMag Brazilian Congress on Electromagnetism*, João Pessoa, PB, Brazil, 2012, pp. 1–5.
- [3] Q. Zaman, J. S. Costa, A. R. J. Barreto, J. F. D. F. Araujo, L. D. Carlos, A. N. Carneiro Neto, M. Cremona, Z. Ahmed, A. F. S. da Cruz, N. W. P. de Souza, et al., "Dielectric-loaded waveguides as advanced platforms for diagnostics and application of transparent thin films," *Langmuir*, vol. 37, no. 11, pp. 3248–3260, Mar. 2021.
- [4] A. F. S. da Cruz, N. W. P. de Souza, and K. Q. da Costa, "Electromagnetic model of a planar array of nanodipoles on a gold plane via 3D Green's function," in *Técnicas de Processamento de Sinais e Telecomunicações*, H. A. Holzmann, Ed. Ponta Grossa, PR: Atena Editora, 2019, ch. 16, pp. 168–181.
- [5] M. M. Tajdini and H. Mosallaei, "Characterization of large array of plasmonic nanoparticles on layered substrate: dipole mode analysis integrated with complex image method," *Opt. Express*, vol. 19, no. S2, pp. A175–A193, 2011.
- [6] K. A. Michalski and D. R. Wilton, "On the computation of plane-wave impressed fields and far-zone fields of arbitrary current sources in planar multilayer dielectric and plasmonic media," *J. Electromagn. Waves Appl.*, vol. 31, no. 11–12, pp. 1177–1193, Jul. 2017.
- [7] N. W. P. de Souza, "Analysis of plasmonic sensors based on stratified nanostructures using the discrete complex image method," Ph.D. dissertation, Graduate Program in Electrical Engineering, Federal University of Pará, Belém, PA, Brazil, 2018.
- [8] Y. L. Chow, J. J. Yang, D. G. Fang, and G. E. Howard, "A closed-form spatial Green's function for the thick microstrip substrate," *IEEE Trans. Microw. Theory Techn.*, vol. 39, no. 3, pp. 588–592, Mar. 1991.
- [9] Y. Hua and T. K. Sarkar, "Generalized pencil-of-function method for extracting poles of an EM system from its transient response," *IEEE Trans. Antennas Propag.*, vol. 37, no. 2, pp. 229–234, Feb. 1989.
- [10] T. K. Sarkar and O. Pereira, "Using the matrix pencil method to estimate the parameters of a sum of complex exponentials," *IEEE Antennas Propag. Mag.*, vol. 37, no. 1, pp. 48–55, Feb. 1995.
- [11] N. W. P. de Souza, T. S. Evangelista, A. F. S. da Cruz, and K. Q. da Costa, "DCIM-GPOF Formulation with Automatic Spectral Path for Multilayer Plasmonic Structures," paper accepted to the XXII Brazilian Symposium on Microwaves and Optoelectronics (SBMO 2026), 2026.
- [12] M. A. Yurkin and A. G. Hoekstra, "The discrete dipole approximation: An overview and recent developments," *J. Quant. Spectrosc. Radiat. Transfer*, vol. 106, no. 1–3, pp. 558–589, 2007.
- [13] W. C. Chew, *Waves and Fields in Inhomogeneous Media*. New York: IEEE Press, 1995.
- [14] N. W. P. de Souza, J. S. Costa, R. C. dos Santos, A. F. S. da Cruz, T. Del Rosso, and K. Q. da Costa, "Modal analysis of surface plasmon resonance sensor coupled to periodic array of core-shell metallic nanoparticles," in *Resonance*, J. Awrejcewicz, Ed. Rijeka, Croatia: InTech, 2017, vol. 1, ch. 9, pp. 169–188.