

Analysis of a Regular Tetrahedral Array for Underwater DOA Estimation

Rodrigo Passos Rocha[✉], Rigel P. Fernandes[✉], José A. Apolinário Jr.[✉], and Natanael Nunes de Moura Junior[✉]

Abstract—This paper presents a robust 3-D Direction of Arrival estimation framework using a compact regular tetrahedral array of four hydrophones, designed for shallow-water environments with adverse Signal-to-Noise Ratios (SNRs) down to -14 dB. The method combines GCC-PHAT for Time Difference of Arrival (TDOA) estimation with a closed-form Least Squares (LS) solver, enhanced by a recursive outlier-rejection mechanism to mitigate inconsistent TDOA pairs. Temporal coherence is exploited through multi-segment aggregation using mean and median operators. Simulations using synthetic signals generated by combining real recorded motorboat signatures with real underwater ambient noise demonstrate that discarding the two most inconsistent TDOA pairs reduces angular Root Mean Square Error (RMSE) by over 10° at -14 dB. Furthermore, median aggregation significantly outperforms mean aggregation; at a 5.12 s integration time and -14 dB SNR, the RMSE/Std improves from $13^\circ/5^\circ$ (LS-Mean, six pairs) to $2^\circ/1^\circ$ (LS-Median, four pairs). The results indicate that a 5.12 s integration window optimizes the trade-off between accuracy and update rate, proving suitable for real-time maritime situational awareness.

Keywords—Underwater acoustics, DOA estimation, GCC-PHAT, tetrahedral hydrophone array, outlier rejection.

I. INTRODUCTION

Underwater acoustic sensing plays a central role in maritime control and monitoring, allowing persistent surveillance, anomaly detection, and the protection of high-value assets. This need is amplified by the extensive exclusive economic zone (EEZ), characterized by rich natural resources, dense maritime traffic, and critical offshore infrastructure. Within this context, the Brazilian Navy's Blue Amazon Management System (SisGAAz) aims to provide continuous maritime situational awareness while mitigating the cost and scalability challenges associated with monitoring such a vast maritime territory [1]. Thus, suitable solutions require underwater sensing systems with a reduced number of sensors while preserving the intended surveillance and situational-awareness objectives.

Passive sonar generally has lower acquisition, installation, and maintenance costs than active systems [2]. Furthermore, the absence of acoustic emissions enables covert operation, reduces power consumption, and avoids environmental concerns

in ecologically sensitive regions [3], making passive sensing particularly attractive for persistent, large-area surveillance.

For a passive sonar deployed on the seabed and dedicated to maritime surveillance, an ideal configuration should provide 360° azimuthal coverage and a wide elevation aperture on the order of 90° above, while relying on as few sensors as possible. In this context, regular tetrahedral arrays (RTAs) have emerged as an attractive solution, since their compact three-dimensional geometry enables full 3-D Direction of Arrival (DOA) estimation with only four hydrophones [4], offering a favorable trade-off between directional coverage and sensor count.

Generalized Cross-Correlation with Phase Transform (GCC-PHAT) based on time-delay estimation is well established for extracting inter-sensor time-difference-of-arrival (TDOAs) in underwater acoustics [5]–[7], and Least Squares (LS) formulations are widely used in passive localization due to their simplicity and good performance [8]. Combining GCC-PHAT TDOAs with an LS formulation yields a natural DOA reconstruction problem, in which the source direction is estimated by fitting geometry-predicted to measured TDOAs [9], [10].

Beyond these classical building blocks, signal-enhancement and data-selection have been explored to improve TDOA-based DOA estimation. Freire [11] proposed a recursive iterative LS solution that progressively removes the sensor contributing most to the LS cost, thereby rejecting outliers. Multi-segment averaging [12] is also widely used to reinforce the signal-of-interest (SOI) and suppress noise.

Motivated by these developments, this work proposes an integrated processing pipeline and numerically evaluates its performance for robust 3-D DOA estimation with a compact four-hydrophone RTA. The pipeline combines GCC-PHAT-based TDOA estimation, a closed-form LS solution, an iterative outlier-rejection mechanism, and multi-segment DOA aggregation. Using semi-synthetic data built from motorboat recordings embedded in real shallow-water ambient noise, we quantify the impact of signal-to-noise ratio (SNR), segment configuration, and outlier removal, delineating practical operating regimes for reliable 3-D DOA estimation.

In contrast to most studies conducted in airborne acoustic scenarios and for impulsive signals, our main contribution is the numerical validation of a unified processing framework tailored to shallow-water passive sonar operating under continuous broadband propulsion noise. The main contributions of this work to the field of 3-D DOA estimation can be summarized as follows: (1) an integrated, closed-form 3-D DOA estimation pipeline for compact four-hydrophone regular tetrahedral ar-

Rodrigo Passos Rocha is with the Underwater Acoustics Program, Admiral Paulo Moreira Marine Research Institute (IEAPM), Arraial do Cabo-RJ, Brazil; E-mail: rprocha25@gmail.com.

Rigel P. Fernandes is with the Weapon Systems Directorate (DSAM) and the Program of Defense Engineering, Military Institute of Engineering (IME), Rio de Janeiro-RJ, Brazil; E-mail: rigelfernandes@gmail.com.

José A. Apolinário Jr. is with the Programs of Electrical and Defense Engineering, Military Institute of Engineering (IME), Rio de Janeiro-RJ, Brazil; E-mail: apolin@ime.eb.br.

Natanael Nunes de Moura Junior is with the Signal Processing Lab, COPPE/Poli, Federal University of Rio de Janeiro (UFRJ), Rio de Janeiro-RJ, Brazil; E-mail: natanael@lps.ufrj.br.

rays, specifically adapted to continuous propulsion noise; (2) a DOA enhancement strategy that combines GCC-PHAT-based TDOA estimation, multi-segment DOA aggregation, and a recursive iterative LS scheme for outlier rejection based on the removal of inconsistent TDOA pairs rather than entire sensors; (3) a systematic numerical assessment using data constructed by superimposing measured ambient noise and a modeled signal of interest at a hydrophone array, to quantify the impact of segment configuration and outlier removal; and (4) the identification of practical operating regimes that balance DOA accuracy and update rate, enabling reliable real-time 3-D DOA estimation in maritime scenarios.

II. PROBLEM STATEMENT AND ASSUMPTIONS

A set of operational assumptions is adopted to ensure the physical consistency and practical applicability of the proposed approach. In our simulation framework, TDOA values at the hydrophones are derived from a simple geometric model, assuming a single acoustic source, a constant sound speed, and a plane-wave incidence on the array. Under these conditions, more complex effects, such as refraction due to sound-speed gradients, internal waves, or acoustic ducting, are expected to have only a low impact on the relative delays across a compact array aperture at the considered ranges and frequencies, when compared with the modeled plane-wave propagation [13]. Their contribution is therefore neglected in the present analysis.

The sensor configuration considered is a four-hydrophone RTA, forming a compact three-dimensional geometry with the minimum number of hydrophones required for 3-D DOA estimation, as shown in Figure 1. This configuration provides directional resolution in both azimuth and elevation and avoids the angular ambiguities inherent to strictly collinear or coplanar arrays [4].

The acoustic source is modeled as a stationary point source within each analysis window. Acoustic emissions in the low-frequency range (up to approximately 1 kHz) are considered to be dominated by ship propulsion noise, primarily due to propeller cavitation [14], [15].

The SOI is a recorded motorboat signal, and ambient noise is modeled by real shallow-water noise recordings added as an additive background-noise process. The noise recordings are scaled to achieve prescribed SNR levels and used as a control parameter to assess the robustness of the proposed method.

III. METHOD AND SIGNAL GENERATION

The proposed method is a signal-processing pipeline for 3-D DOA estimation in shallow water using a compact RTA. Synthetic four-channel data are generated by delaying a recorded motorboat signal according to the RTA geometry and superimposing real shallow-water noise at controlled SNRs. The resulting array signals are processed by a GCC-PHAT-based TDOA estimator and an LS DOA estimator.

A. Hydrophone Array Geometry

An RTA with four hydrophones is adopted for far-field operation, following the configuration proposed by Luo et al. [4].

This compact 3-D aperture enables unambiguous estimation of both azimuth and elevation angles, avoiding the ambiguities inherent to planar arrays. The selected geometry generates distinct TDOA patterns across all sensor pairs, leveraging spatial complementarity for robust 3-D DOA estimation.

The hydrophones are placed with an inter-sensor spacing L , chosen according to the spatial Nyquist criterion for unaliased DOA estimation up to the maximum frequency of interest, and illustrated in Figure 1, which shows both a 3-D rendering and a realistic view of the array. For $f_{\max} = 1000$ Hz and $c = 1500$ m/s, this yields $L = 0.75$ m [16].

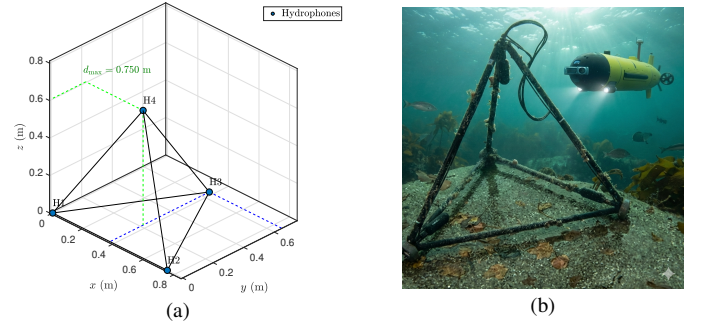


Fig. 1. RTA geometry. (a) MATLAB-based 3-D rendering of the array with all six inter-sensor distances equal to L . (b) AI-generated illustrative rendering of the same array in a realistic underwater setting.

B. Signal Synthesis and Noise Modeling

The processing chain operates on a four-channel array signal that is divided into short-time segments. Each segment has a fixed duration, and the DOA estimation is performed independently on each segment. The sampling rate for both source and ambient noise signals was set to $f_s = 44,100$ samples per second.

The source signal corresponds to a real motorboat recording extracted from the ShipsEar database, acquired by a single hydrophone and providing a representative underwater radiated-noise signature for a small surface vessel [17]. This single-channel signal is used to simulate a four-hydrophone RTA by generating four array channels with relative time delays consistent with the assumed DOA and the array geometry.

Since the motorboat recording is available as a discrete-time signal, these continuous-time delays are implemented by applying fractional-sample shifts using linear interpolation. This interpolation introduces a small discretization error in the effective delays, but, for the sampling rate and bandwidth considered, its impact on the relative TDOAs across the compact array aperture is expected to be minor compared with the underlying plane-wave model and is therefore neglected in this study.

A representative source direction is adopted for the simulations, with azimuth 35° and elevation 24° . This choice serves to illustrate the performance of the proposed pipeline for a generic 3-D direction that excites both azimuth and elevation components in the RTA, without being tied to any specific operational scenario.

Ambient noise is modeled using real shallow-water sea-noise recordings from Enseada dos Anjos, Arraial do Cabo

(RJ, Brazil), which is added as an additive background-noise process. For each channel and segment, a noise segment is selected and scaled to achieve a defined SNR at the array output, providing controlled yet close to realistic conditions for performance assessment.

The SNR is defined in terms of the variances σ_s^2 and σ_n^2 of the delayed source signal and the ambient-noise segment, respectively [10]. A challenging SNR of -14 dB is adopted to evaluate the robustness of the proposed DOA estimation method under adverse noise conditions.

Figure 2 illustrates the spectral and time–frequency characteristics of the source and noise signals. The motorboat signal in Figure 2(a-b) exhibits a broadband spectrum predominantly concentrated below 1 kHz and a time-varying spectrogram characteristic of a small maneuvering vessel over time. The ocean ambient noise recorded at Enseada dos Anjos in Figure 2(c-d) presents no persistent narrowband tonal components and displays marked temporal fluctuations, thereby providing a realistic and challenging interference background for TDOA/DOA estimation in shallow-water environments.

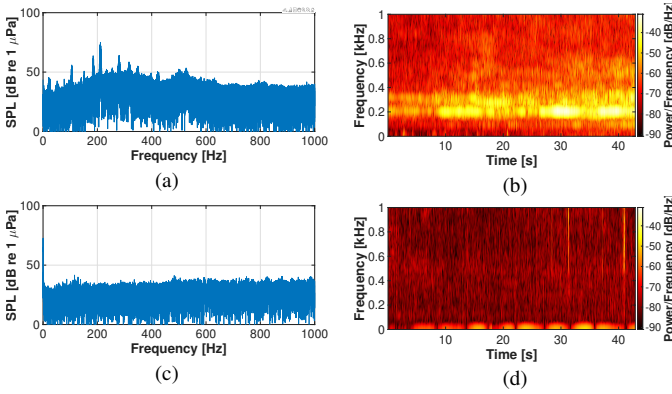


Fig. 2. Spectral and time–frequency analysis (25 ms Hamming window, 75% overlap). (a) Power spectrum of the motorboat signal. (b) Spectrogram of the motorboat signal. (c) Power spectrum of the ambient noise. (d) Spectrogram of the ambient noise.

C. DOA Estimation via GCC-PHAT and Closed-Form LS

The TDOAs between hydrophone signals are estimated using the GCC-PHAT framework, originally proposed by Knapp and Carter [5], and widely adopted in microphone array processing [10], [18]. Let $x_i[n]$ denote the discrete-time signal received at hydrophone i , comprising the delayed motorboat signal plus ambient noise.

For each short-time segment and each hydrophone pair (i, j) , the GCC-PHAT cross-correlation between $x_i[n]$ and $x_j[n]$ is computed in the frequency domain and transformed back to the time domain. The TDOA estimate $\hat{\tau}_{ij}$ is obtained as the lag corresponding to the maximum magnitude of the resulting cross-correlation $\hat{r}_{ij}(\tau)$, within a physically admissible lag interval $\mathcal{L}$ defined from the maximum inter-sensor distance and the sound speed, so that non-candidate delays are discarded [10]:

$$\hat{\tau}_{ij} = \arg \max_{\tau \in \mathcal{L}} |\hat{r}_{ij}(\tau)|. \quad (1)$$

Based on the estimated delay, and the delay based on the unknown vector $\mathbf{a}_{\text{DOA}}$, the least squares cost function, is defined as [11], [19], [20]:

$$\xi = \sum_{i,j} (\hat{\tau}_{i,j} - \Delta \mathbf{p}_{i,j}^T \mathbf{a}_{\text{DOA}})^2, \quad (2)$$

for all possible pairs, $N = M(M-1)/2$ for the case of M microphones.

Minimizing the cost function with respect to $\mathbf{a}_{\text{DOA}}$, the following is true:

$$\mathbf{a}_{\text{DOA}} = \mathbf{R}^{-1} \mathbf{d}, \quad (3)$$

where $\mathbf{d} = \Delta \bar{\mathbf{p}}^T \boldsymbol{\tau}$, $\boldsymbol{\tau} = [\tau_{1,2} \ \tau_{1,3} \ \dots \ \tau_{1,M} \ \tau_{2,3} \ \dots \ \tau_{M-1,M}]^T$ and $\mathbf{R} = \Delta \bar{\mathbf{p}}^T \Delta \bar{\mathbf{p}}$, $\Delta \bar{\mathbf{p}}$ are assembled as follows:

$$\Delta \bar{\mathbf{p}} = [\Delta \bar{\mathbf{p}}_{1,2} \ \Delta \bar{\mathbf{p}}_{1,3} \ \dots \ \Delta \bar{\mathbf{p}}_{M-1,M}]^T. \quad (4)$$

The solution provided by Equation 3 may not have a unit norm, which must be ensured through normalization. Only after normalization can the azimuth and zenith angles be accurately computed using trigonometric functions.

IV. DOA ESTIMATION ENHANCEMENT: STRATEGIES AND PERFORMANCE ANALYSIS

This section investigates a set of strategies to improve DOA estimation performance under varying conditions. Data-selection techniques are explored, such as the choice of segment length and the elimination of problematic cross-correlations, in order to mitigate the impact of low-quality measurements on the LS solution. Complementarily, signal-enhancement techniques, specifically the use of multiple short-time segments, are considered to obtain more reliable TDOA estimates.

A critical aspect of this evaluation is selecting SNRs that significantly challenge DOA estimation, so that the impact of the enhancement techniques can be clearly observed and their resilience to adverse noise levels demonstrated.

A. Outlier Removal via Recursive Iterative Least Squares

To enhance the robustness of the LS-based DOA estimator, an iterative outlier rejection scheme, inspired by recursive iterative LS strategies for sensor selection [11], is employed. This method is designed to mitigate the impact of erroneous TDOA measurements by progressively identifying and removing the most problematic hydrophone pairs. In this setup, outlier TDOA estimates arise naturally due to the low-SNR conditions, without injected artificial errors into the data.

For each segment and each of the six sensor pairs, its TDOA measurements are temporarily omitted, and the DOA is re-estimated using only the remaining five pairs. Subsequently, the predicted TDOAs $\tilde{\tau}_{s,m}$ for the excluded pair are computed from this re-estimated DOA. The squared error between the measured TDOAs $\tau_{s,m}$ and the predicted TDOAs $\tilde{\tau}_{s,m}$ defines a residual $e_{s,m}$ for each pair s and segment m , as given by:

$$e_{s,m} = (\tau_{s,m} - \tilde{\tau}_{s,m})^2. \quad (5)$$

The sensor pair exhibiting the largest residual is identified as an outlier, as it contributes most significantly to the LS cost. This identified outlier pair is then removed from the LS estimation, and the procedure can be repeated for subsequent iterations.

B. Multi-Segment TDOA Processing: Aggregation and Segment Length Optimization

The use of multiple short-time segments is a fundamental strategy to improve TDOA-based DOA estimation in noisy environments [12] by performing DOA estimation over several short, partially independent realizations of the same source signal and aggregating the resulting estimates.

For each segment, the GCC-PHAT algorithm is applied independently to every hydrophone pair, yielding a set of segment-level TDOA estimates. These TDOAs are then used to compute a segment-level azimuth and elevation estimate $\hat{\mathbf{D}}_m = (\hat{\phi}_m, \hat{\theta}_m)$ via the LS-based estimator.

An aggregated DOA is then obtained by applying two natural aggregation strategies, the sample mean $\bar{\mathbf{D}}_{\text{mean}}$ and the sample median $\bar{\mathbf{D}}_{\text{med}}$, yielding the final DOA estimates:

$$\bar{\mathbf{D}}_{\text{mean}} = \frac{1}{N_{\text{seg}}} \sum_{m=1}^{N_{\text{seg}}} \hat{\mathbf{D}}_m, \quad \bar{\mathbf{D}}_{\text{med}} = \text{median}(\{\hat{\mathbf{D}}^{(1)}, \dots, \hat{\mathbf{D}}^{(N_{\text{seg}})}\}). \quad (6)$$

This multi-segment DOA aggregation can be interpreted as a form of statistical averaging in the DOA domain, where multiple noisy realizations of the same source direction are combined to improve estimation robustness.

The choice of segment length is also critical, as it directly governs the trade-off between estimation accuracy and the temporal resolution of DOA updates. The effective update rate is defined as the total integration time used to form a refined DOA estimate, given by the product of the number of segments and the segment duration.

The results in Table I summarize the angular error performance of the multi-segment GCC-PHAT combined with the recursive iterative LS estimator under an SNR of -14 dB and 200 independent noise realizations. All combinations of number of segments and segment durations were evaluated, and the best-performing configurations were grouped by effective update rate, which is limited to a maximum of 10.24 s to ensure practically useful refresh times, corresponding to a maximum spatial displacement of approximately 53 m at 10 knots. For each method and configuration, the table reports the root mean square error (RMSE) and standard deviation (Std) of the angular error, enabling a direct comparison between LS-Mean and LS-Median aggregation across different update rates and pair configurations.

Quantitatively, the results in Table I confirm that combining outlier removal with multi-segment median aggregation substantially improves DOA estimation accuracy. For a fixed aggregation strategy, discarding the most inconsistent TDOA pairs systematically reduces the angular error by eliminating measurements that are incompatible with the assumed geometric model. For example, with LS-Median at an update of 0.64 s, the RMSE decreases from 21.43° for six pairs to 12.75° for four pairs, and the Std from 11.75° to 8.35° .

TABLE I. Angular error statistics for multi-segment GCC-PHAT/LS with mean and median aggregation (200 noise realizations, SNR = -14 dB).

Method	Upd. 0.64 s 8×80 ms		Upd. 1.28 s 8×160 ms		Upd. 2.56 s 32×80 ms		Upd. 5.12 s 32×160 ms		Upd. 10.24 s 64×160 ms	
	RMSE	Std	RMSE	Std	RMSE	Std	RMSE	Std	RMSE	Std
Mean:										
LS (6)	26.12°	12.32°	18.03°	9.36°	19.72°	7.23°	13.10°	5.29°	11.74°	4.00°
LS (5)	26.03°	12.25°	17.72°	10.34°	23.15°	8.22°	13.63°	6.08°	12.99°	4.65°
LS (4)	25.50°	11.79°	18.29°	10.07°	24.55°	8.95°	14.44°	6.79°	13.49°	5.22°
Median:										
LS (6)	21.43°	11.75°	14.02°	8.59°	14.96°	7.91°	7.22°	3.90°	6.09°	3.11°
LS (5)	15.19°	9.49°	8.14°	6.32°	11.43°	6.46°	3.21°	1.92°	2.41°	1.49°
LS (4)	12.75°	8.35°	6.04°	4.60°	9.34°	5.89°	2.22°	1.18°	1.65°	0.65°

Aggregating multiple short-time segments using the median, together with an appropriate choice of segment length, further enhances robustness by reinforcing consistent DOA estimates and attenuating the impact of occasional large errors across segments. Intuitively, longer effective integration times provide more statistical evidence about the source direction, so that consistent segment-level estimates are reinforced while inconsistent ones are downweighted by the median. This effect is clearly observed in Table I: at an update of 5.12 s LS-Median(4) achieves $2.22^\circ/1.18^\circ$, corresponding to a reduction of more than 10° in RMSE and a much tighter concentration around the true DOA.

The segment length and resulting update rate therefore play a key role in balancing accuracy and responsiveness. Shorter updates (e.g., 0.64 s) offer faster DOA refresh but relatively higher error, whereas very long updates (e.g., 10.24 s) yield excellent accuracy at the cost of slower tracking. A joint analysis of Table I indicates that an intermediate update of 5.12 s, corresponding to a spatial displacement of only about 26 m at 10 knots, offers a favorable compromise: with LS-Median(4), it already provides RMSE/Std of $2.22^\circ/1.18^\circ$, close to the best performance at 10.24 s, while maintaining a significantly faster update rate that is more suitable for real-time or slowly time-varying scenarios.

The 2D visualization in Figure 3 is consistent with the quantitative trends reported in Table I. As the update time increases, the LS-Median(4) estimates form progressively tighter clusters around the true target, and the angular RMSE reported in the legend decreases from approximately 11.67° (e.g., 8×80 ms) to 1.59° for the longest effective integrations.

The configuration with update 5.12 s, previously identified as a good compromise between accuracy and responsiveness, already yields a compact cloud of estimates very close to the true target, visually confirming that it achieves near-optimal localization performance while maintaining a practical update rate.

All MATLAB scripts and configuration files used in this work are available in a public GitHub repository [21].

V. CONCLUSION

This work demonstrated that the GCC-PHAT framework, when combined with an RTA, may be an interesting option for 3-D DOA estimation, offering a low-cost scalable solution.

Regarding the proposed enhancement methods, the recursive iterative LS scheme significantly improved estimation

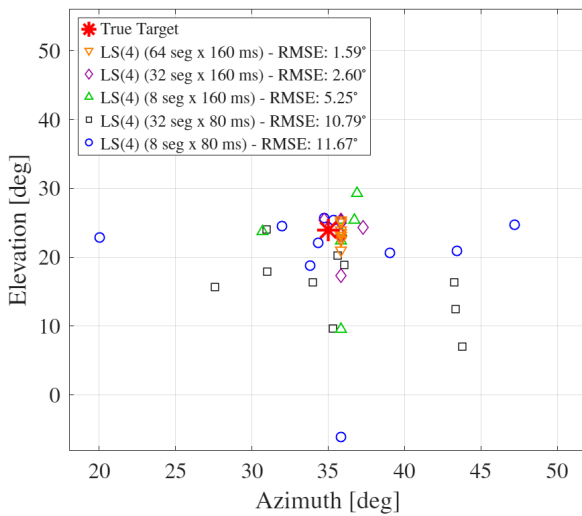


Fig. 3. Spatial distribution of LS(4) DOA estimates over 10 noise realizations. Each color and marker type corresponds to one multi-segment configuration, together with the angular RMSE between the corresponding set of estimated DOA vectors and the true target direction (red star).

accuracy and reduced the standard deviation by identifying and removing outlier hydrophone pairs. Under the challenging SNR of -14 dB, the best overall performance was achieved when the two most unreliable pairs were discarded, and the LS solution was computed over the remaining four pairs.

Furthermore, using the median across multiple short-time segments proved effective at reinforcing the repetition of consistent directions in low-SNR environments. It was notably more robust than the mean, suppressing disparate values caused by transient interference and occasional large errors. The results also indicate that increasing the number of segments improves accuracy for the configurations tested, but with diminishing returns as the effective update time becomes very long. A joint analysis of segment parameters and update rate suggests that an intermediate integration time around 5.12 s (e.g., 32 segments of 160 ms with LS-Median and outlier removal down to 4 pairs) offers a favorable compromise, delivering angular errors close to those obtained at 10.24 s while maintaining a significantly faster DOA refresh that is more suitable for real-time applications, corresponding to a spatial displacement of approximately 26.3 meters at 10 knots.

Future research will focus on validating this processing pipeline with experimental data, which will be crucial for transitioning this methodology from simulation to practical applications.

REFERENCES

- [1] M. S. Rodrigues, "Plano Estratégico da Marinha PEM 2040," *Revista da EGN*, vol. 27, no. 1, pp. 28–29, 2021. [Online]. Available: <https://doi.org/10.21544/1809-3191.v27n1.p.13-30>
- [2] A. Sutin, B. Bunin, A. Sedunov, N. Sedunov, L. Fillinger, M. Tsionskiy, and M. Bruno, "Stevens passive acoustic system for underwater surveillance," in *2010 International Waterside Security Conference (WSS)*. IEEE, 2010, pp. 1–6. [Online]. Available: <https://doi.org/10.1109/WSSC.2010.5730286>
- [3] T. M. Cox, T. J. Ragen, A. J. Read, E. Vos, R. W. Baird, K. Balcomb, J. Barlow, J. Caldwell, T. Cranford, and L. Crum, "Understanding the impacts of anthropogenic sound on beaked whales," *Journal of Cetacean Research and Management*, 2006. [Online]. Available: <https://doi.org/10.47536/jcrm.v7i3.729>
- [4] J. Luo, J. Zhang, H. Yang, and Y. Guan, "All-directional DOA estimation for ultra-wideband regular tetrahedral array using wrapped PDoA," *Sensors*, vol. 22, no. 4, p. 1532, 2022. [Online]. Available: <https://doi.org/10.3390/s22041532>
- [5] C. Knapp and G. Carter, "The generalized correlation method for estimation of time delay," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 24, no. 4, pp. 320–327, 1973. [Online]. Available: <https://doi.org/10.1109/TASSP.1976.1162830>
- [6] R. P. Fernandes, J. A. Apolinário Jr, and J. M. de Seixas, "Enhancing TDE-based drone DoA estimation with genetic algorithms and zero cyclic sum," 2023. [Online]. Available: <https://doi.org/10.21528/CBIC2023-115>
- [7] G. Giunta and L. Pallotta, "Improving delay estimation in underwater acoustic applications by the additional use of cross-cross-correlation," pp. 1–5, 2023. [Online]. Available: <https://doi.org/10.1109/VTC2023-Spring57618.2023.10199279>
- [8] R. P. Fernandes, J. A. Apolinário Jr, and J. M. de Seixas, "A reduced complexity acoustic-based 3D DoA estimation with zero cyclic sum," *Sensors*, vol. 24, no. 7, p. 2344, 2024. [Online]. Available: <https://doi.org/10.3390/s24072344>
- [9] K. Doğançay, "Bearings-only target localization using total least squares," *Signal Processing*, vol. 85, no. 9, pp. 1695–1710, 2005. [Online]. Available: <https://doi.org/10.1016/j.sigpro.2005.03.007>
- [10] R. P. Fernandes, A. Borzino, A. L. Ramos, and J. A. Apolinário Jr, "Investigating the potential of UAV for gunshot DoA estimation and shooter localization," *Proceedings of the Simpósio Brasileiro de Telecomunicações e Processamento de Sinais, Belém, Brazil*, vol. 30, 2016. [Online]. Available: <https://doi.org/10.14209/sbirt.2016.52>
- [11] I. L. Freire, P. C. Prandel, and J. A. Apolinário Jr, "Sobre a escolha de sinais em arranjos de microfones estimando DoA com GCC-PHAT," *XXX Simpósio Brasileiro de Telecomunicações*, 2012. [Online]. Available: <https://doi.org/10.14209/sbirt.2012.76>
- [12] B. Baumann, C. W. Merkle, R. A. Leitgeb, M. Augustin, A. Wartak, M. Pircher, and C. K. Hitzengerger, "Signal averaging improves signal-to-noise in OCT images: But which approach works best, and when?" *Biomedical Optics Express*, vol. 10, no. 11, pp. 5755–5775, 2019. [Online]. Available: <https://doi.org/10.1364/BOE.10.005755>
- [13] D. Abraham, *Underwater Acoustic Signal Processing: Modeling, Detection, and Estimation*, ser. Modern Acoustics and Signal Processing. Springer International Publishing, 2019. [Online]. Available: <https://books.google.com.br/books?id=azuIDwAAQBAJ>
- [14] M. Chen, J. Liu, Q. Si, Y. Liang, Z. Jin, and J. Yuan, "Investigation into the hydrodynamic noise characteristics of electric ducted propeller," *Journal of Marine Science and Engineering*, vol. 10, no. 3, p. 378, 2022. [Online]. Available: <https://doi.org/10.3390/jmse10030378>
- [15] ITTC Specialist Committee on Hydrodynamic Noise, "The specialist committee on hydrodynamic noise," 2017. [Online]. Available: <https://www.ittc.info/media/7837/17-sc-hydrodynamic-noise-compressed.pdf>
- [16] D. H. Johnson and D. E. Dudgeon, *Array Signal Processing: Concepts and Techniques*. Prentice Hall, 1993.
- [17] D. Santos-Domínguez, S. Torres-Guijarro, A. Cardenal-López, and A. Pena-Gimenez, "ShipsEar: An underwater vessel noise database," *Applied Acoustics*, vol. 113, pp. 64–69, 2016. [Online]. Available: <https://doi.org/10.1016/j.apacoust.2016.06.008>
- [18] A. M. C. R. Borzino, J. A. Jr., and M. L. R. de Campos, "Estimating direction of arrival of long range gunshot signals," in *2014 International Telecommunications Symposium (ITS)*, São Paulo, Brazil, Aug. 2014, pp. 1–5. [Online]. Available: <https://doi.org/10.1109/ITS.2014.6947995>
- [19] A. M. Borzino, J. A. Apolinário, and M. L. de Campos, "Robust doa estimation of heavily noisy gunshot signals," in *2015 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. IEEE, 2015, pp. 449–453. [Online]. Available: <https://doi.org/10.1109/ICASSP.2015.7178009>
- [20] R. P. Fernandes, "Acoustic-based drone localization and DOA estimation for highly noisy environments," Ph.D. dissertation, Instituto Militar de Engenharia, Rio de Janeiro, RJ, Brazil, July 2025. [Online]. Available: <https://repositorio.marinha.mil.br/handle/ripcmb/848001>
- [21] R. Rocha, "MATLAB scripts and data for TDOA/DOA estimation with a tetrahedral hydrophone array," <https://github.com/rrocha25/RodrigoRocha/tree/main>, 2026, accessed: 12 Feb. 2026.