

Miniaturized Dual-Band Fractal Microstrip Patch Antennas for TVWS Applications

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Abstract—This work presents a microstrip patch antenna (MPA) based on the Fractal outer model fractal for compact dual-band TV White Space (TVWS) applications. The proposed Fractal MPA with reflector plane operates at 560 MHz and 680 MHz on an FR-4 substrate. Our approach indicates a patch area reduction exceeding 70% at each operating frequency, confirming effective miniaturization. The antenna achieves realized gain of 6.3 dBi, radiation efficiency above 80%, and bandwidths of 40 MHz (lower band) and 60 MHz (upper band), respectively. Far-field analysis confirms linear polarization at 560 MHz and elliptical polarization at 680 MHz, demonstrating coupling robustness and suitability for compact dual-band TVWS systems.

Keywords—Fractal Fractal, Microstrip Patch Antennas, Dual-Band, UHF.

I. INTRODUCTION

Television white space (TVWS) systems have attracted significant interest for long-range and spectrum-efficient wireless communications by exploiting locally unused channels in the ultra-high frequency (UHF) television band [1], [2]. Owing to the favorable propagation characteristics of sub-GHz frequencies, TVWS is attractive for rural connectivity, cognitive-radio systems, sensing, and low-cost wireless links. In such applications, the antenna is a key component, since practical TVWS radios require radiators that combine adequate UHF performance with compact and easily integrated structures.

Microstrip patch antennas (MPAs) are attractive candidates for this purpose because of their low profile, low weight, low fabrication cost, and compatibility with planar circuits [3]. However, their dimensions are directly related to the operating wavelength. Consequently, in the UHF regime, conventional patch antennas become physically large, which hinders their integration into compact platforms [4], [5]. This limitation is even more critical in dual-band TVWS systems, where the antenna must operate at two distinct UHF frequencies while maintaining acceptable impedance and radiation performance.

Antenna miniaturization, however, is not a purely geometric task, since reducing the electrical size usually degrades bandwidth, impedance matching, gain, and radiation efficiency [6],

[7]. For this reason, compact UHF MPA designs remain a relevant challenge. Several miniaturization techniques have been reported for printed antennas, including slotting, reactive loading, shorting mechanisms, high-permittivity substrates, and contour perturbation [3]. Among these approaches, fractal geometries are particularly attractive because their self-similar and space-filling properties can increase the effective current path without a proportional increase in occupied area [8], [9]. Classical studies based on Koch and Peano geometries demonstrated the usefulness of non-Euclidean contours for reducing the resonant size of planar radiators [10]–[12], while more recent works have confirmed the relevance of fractal and fractal-inspired structures for compact UHF antennas and related electromagnetic devices [9], [13], [14]. Within this context, the Fractal has recently emerged as a promising geometry for microstrip antenna miniaturization. Although it was originally introduced in electroencephalogram signal analysis [15], subsequent studies showed that Fractal-based perturbations can reduce the resonant frequency of rectangular and circular patch antennas, yielding compact designs with useful radiation characteristics [16]–[18]. Nevertheless, most previous demonstrations have been concentrated at microwave frequencies, and its applicability to UHF patch antennas for TVWS scenarios remains underexplored.

Motivated by this gap, this work investigates a double Fractal MPA (FF-MPA) with a reflector plane for TVWS applications. The proposed geometry aims to reduce the antenna footprint while enabling dual-polarization characteristics and maintaining dual-band operation at 560 MHz and 680 MHz, compatible with TVWS radio architectures. The FF-MPA achieves a realized gain of approximately 6.3 dBi, along with bandwidths of 40 MHz and 60 MHz at 560 MHz and 680 MHz, respectively, exceeding the minimum requirement of 24 MHz per band. Further, analysis of the far-field electric field components confirms distinct polarization behaviors at each operating frequency, namely linear polarization at 560 MHz and elliptical polarization at 680 MHz. Moreover, an analytical cavity method indicates a patch area reduction exceeding 70%. These results demonstrate that the proposed perturbation enables meaningful size reduction while preserving electromagnetic performance suitable for dual-band TVWS operation.

II. DESIGN AND METHODS

Fig. 1(a) shows the geometry of the proposed dual-band FF-MPA. The radiating structure is composed of two FF-based patch elements printed on the front side of the FR-4 dielectric substrate, forming a 1×2 dual-polarized FF-MPA arrangement

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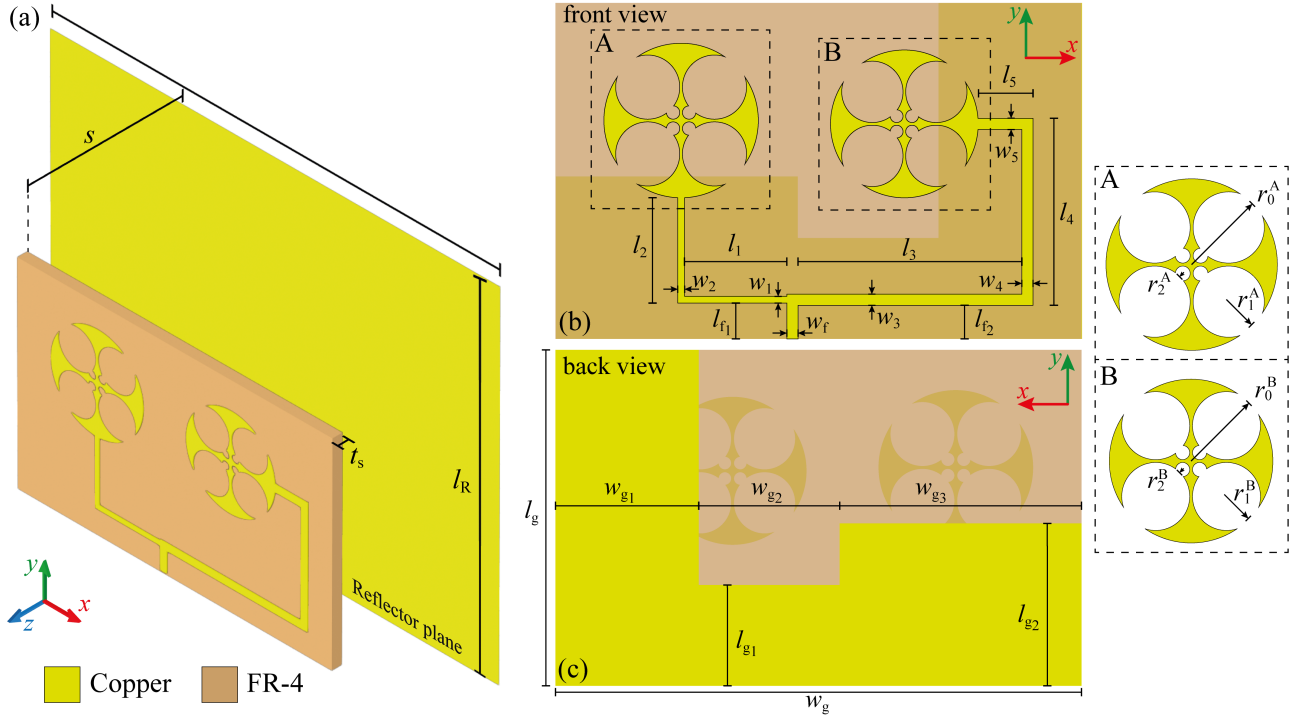


Fig. 1: (a) Schematic representation of the FF-MPA, (b) front and (c) back views with corresponding geometric parameters.

intended to support dual-band operation in the UHF TVWS range. One patch was designed for 560 MHz (with vertical feed) and another for 680 MHz (with horizontal feed). Finally, the FF-MPA is then backed by a metallic reflector plane. The reflector is positioned parallel to the antenna substrate in order to improve the forward radiation in $+z$ -axis and to reduce backward radiation. In this configuration, the reflector plane with length l_R and width w_R is spaced from s the FF-MPA with substrate thickness t_s .

The front view layout is detailed in Fig. 1(b). The two fractal patch radiators are denoted by A and B and are interconnected through a printed feeding network. Patch A is designed to operate at the lower target frequency of 560 MHz, whereas patch B is designed to operate at the upper target frequency of 680 MHz. The feeding structure is described by strip lengths dimensions l_1 , l_2 , l_3 , l_4 , and l_5 , and by the corresponding strip widths w_1 , w_2 , w_3 , w_4 , and w_5 . The input section is defined by the feed lengths l_{f1} (left-handed branch), l_{f2} (right-handed branch), and feed width w_f . These parameters control the electrical path length, the coupling between the two fractal patches, and the impedance transformation between the input port and the radiating elements.

The well-known problem of MPA's narrow bandwidth, particularly in the UHF band, was addressed through asymmetric truncation of the ground plane, ensuring that each patch achieves the minimum bandwidth required by the TVWS system. The back view in Fig. 1(c) depicts the non-uniform stepped ground plane, designed to tune the input impedance and enable dual-polarized and dual-band operation. The overall ground plane dimensions are represented by length l_g and width w_g . The horizontal segmentation of the ground is described by w_{g1} , w_{g2} , and w_{g3} , whereas the vertical extensions are defined

by l_{g1} and l_{g2} .

Finally, the dimensions of the surrounding dashed regions A and B indicate the local design domains associated with each FF-MPA element. The detailed construction of the FF outer model circular contours is shown in the insets on right side of Fig. 1. For radiator A, the fractal geometry is defined by the radius r_0^A , r_1^A , and r_2^A , while for radiator B the corresponding parameters are r_0^B , r_1^B , and r_2^B . These radius determine the circular arcs and internal voids that form the Firat-inspired perturbations.

The FF perturbations are introduced analytically based on the Firat outer model, originally proposed in [18] for microwave frequencies. Similarly, the fractal construction is initiated from the zeroth-order circle (C_0), corresponding to a circle centered at the origin. At each subsequent iteration, four circles are generated, each smaller than those of the previous iteration. These circles are arranged in a square configuration, symmetrically displaced from the origin to the coordinates $(\pm x_c, \pm y_c)$. The resulting set of circle elements can be systematically represented by

$$\begin{bmatrix} C_0 \\ C_k \end{bmatrix} = \begin{bmatrix} x^2 + y^2 = r_0^2 \\ [x \pm (r_k + x_c)]^2 + [y \pm (r_k + y_c)]^2 = r_k^2 \end{bmatrix}, \quad (1)$$

where C_k denotes the set of four circles at the k -th iteration and r_k is the corresponding radius. The parameters $x_c = y_c = 0.6$ mm define a displacement factor which, combined with r_k , determines the circle centers at $(\pm (r_k + x_c), \pm (r_k + y_c))$, resulting in four symmetric positions with respect to the origin. The r_k reduction is strictly related with the previous iteration as

$$r_k = \frac{0.415 r_{k-1}}{t}, \quad (2)$$

where 0.415 is the constant reduction factor related to the r_{k-1}^{th} -based circle and

$$t = \begin{cases} 1, & k = 1 \\ 2, & k = 2, 3, 4, \dots \end{cases} \quad (3)$$

is the the reduction factor for the r_k^{th} reference circle according to iteration k , established to keep the r_k^{th} -based circles within the conductive structure in order to maintain the current flow.

The outer model describes all Boolean difference among the zeroth-iteration with all subsequent circles. Thus, the final FF circular structure is described by

$$C_{\text{FF}} = C_0 - C_K, \quad (4)$$

where C_K represents the aggregate set of circles generated across all k iterations. In this work, the construction is limited to $k = 2$, resulting in a total of eight circles from the first and second iterations. The Boolean subtraction of C_K from C_0 yields the final FF outer model geometry.

Building upon this formulation, two FF-MPAs are designed, as illustrated in Fig. 1. In these configurations, the zeroth-order radius is defined as $r_0 = r_0^A$ and $r_0 = r_0^B$ to target the operating frequencies of 560 MHz and 680 MHz, respectively. The remaining geometric parameters follow the construction previously described above. To validate the proposed approach and ensure methodological consistency, full-wave simulations are carried out using both ANSYS Electronics Desktop (HFSS Module) and CST Studio Suite, enabling cross-verification of the results. The agreement between HFSS and CST results should confirm the feasibility of the approach for dual-band UHF antenna applications. The antennas are analyzed over the frequency range from 500 MHz to 780 MHz. Both HFSS and CST simulations are performed using frequency-domain finite element method (FEM) solvers with adaptive meshing. In both tools, open (radiation) boundary conditions and wave ports are used to accurately model the antenna excitation and radiation behavior. The key performance metrics evaluated include the reflection coefficient S_{11} (dB), directivity D , gain G , realized gain G_r , and far-field radiation characteristics.

III. RESULTS AND DISCUSSION

The antenna dimensions were obtained through an analytical and systematic optimization process, while preserving the geometric characteristics imposed by the proposed FF-MPA outer model and ensuring operation at the target frequencies. The optimization procedure aimed to achieve proper impedance matching, stable radiation behavior, and effective dual-band performance. The optimized geometrical parameters are summarized in Table I.

Fig. 2(a) shows the simulated S_{11} (dB) obtained with HFSS (red) and CST (blue), confirming that the proposed FF-MPA exhibits two well-defined operating bands. The agreement between HFSS and CST is particularly relevant because the two tools are based on different numerical formulations, confirming that the result is robust and intrinsic to the proposed antenna. In the lower band, HFSS yields a -10 dB bandwidth (BW) between 545–585 MHz (BW = 40 MHz), while CST predicts a BW between 538–582 MHz (BW = 44 MHz), with

TABLE I: Optimized geometric parameters for FF-MPA.

Parameter	Dimension (mm)	Parameter	Dimension (mm)
l_1	45.75	w_1	3.00
l_2	47.04	w_2	3.50
l_3	100.0	w_3	5.00
l_4	83.50	w_4	5.00
l_5	24.59	w_5	5.00
l_g	150.0	w_g	235
l_{g1}	45.00	w_{g1}	64.0
l_{g2}	72.50	w_{g2}	63.0
l_{f1}	16.00	w_{g3}	108
l_{f2}	15.00	w_f	5.00
l_R	300.0	w_R	300
s	110.0	t_s	1.54
r_0^A	34.50	r_0^B	33.0
r_1^A	14.32	r_1^B	13.69
r_2^A	2.97	r_2^B	2.84

resonance dip near 560 MHz. In the upper band, HFSS and CST provide BWs of 660–721 MHz (BW = 61 MHz) and 665–722 MHz (BW = 57 MHz), respectively, with resonance dip around 692 MHz. Therefore, both solvers confirm a qualitative stable dual-band behavior, while all obtained BWs are considerably higher than the 24 MHz minimum specified for each TVWS band. An important aspect revealed by Fig. 2(a) is that the two resonances remain clearly separated. This indicates that the antenna does not behave as a single wideband radiator, but as a compact structure supporting two distinct resonant modes. It is important to highlight that analytical cavity method design of circular MPA for 560 MHz and 680 MHz would result in patch radii of 74 mm and 61 mm, respectively. Thus, the FF-MPA proposal reduces the patch area over 70% in

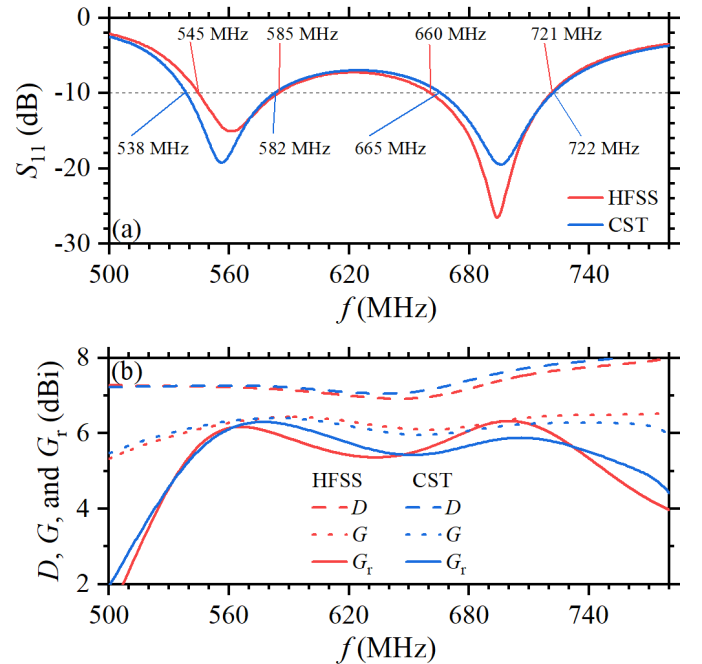


Fig. 2: (a) Return loss S_{11} (dB) and (b) D (dashed), G (dotted), and G_r (solid) for UHF FF-MPA using HFSS (red) and CST (blue).

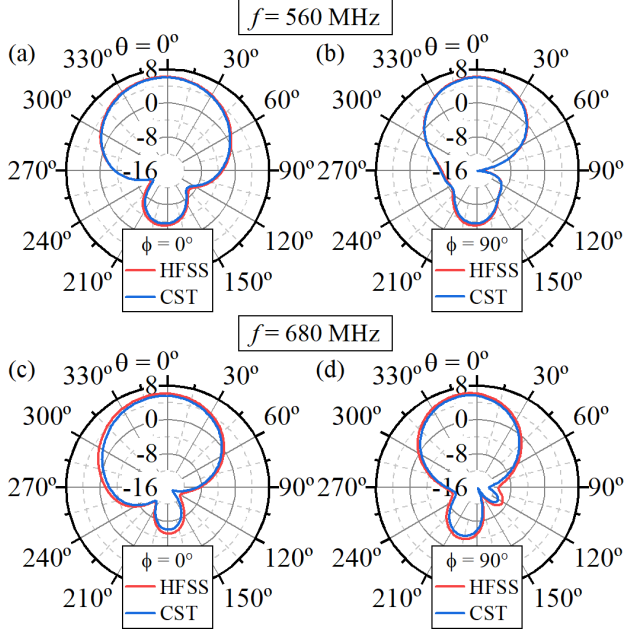


Fig. 3: Realized gain radiation pattern using HFSS (red) and CST (blue) for (a)–(b) 560 MHz and (c)–(d) 680 MHz in cut planes $\phi = 0^\circ$ (left) and $\phi = 90^\circ$ (right).

each frequency, confirming the effectiveness of the proposed miniaturization approach.

The radiation-related results shown in Fig. 2(b) further support the effectiveness of the proposed design. The directivity D remains relatively stable over the analyzed range, varying from 6.9 – 8 dBi in both solvers. The gain G is also stable, typically lying between 5.9 – 6.5 dBi, while the realized gain G_r exhibits two clear maxima coincident with the impedance-matched regions. G_r reaches its maximum approximately 6.3 dBi in both HFSS and CST. The difference between D and G suggests maximum radiation efficiency in the approximate value of 83%, and the small gap between G and G_r near resonance confirms effective impedance matching. These results strongly indicate that the combined use of Firat-based resonant elements, asymmetric ground truncation, and a reflector plane is an effective strategy for obtaining compact dual-band TVWS antennas with practical radiation performance.

Fig. 3 presents a direct comparison between HFSS (red) and CST (blue) results in terms of realized-gain radiation patterns of the proposed FF-MPA at 560 MHz and 680 MHz in the $\phi = 0^\circ$ and $\phi = 90^\circ$ cut-planes. For both operating bands, the antenna preserves a clear broadside radiation characteristic, with the main beam directed around $\theta = 0^\circ$. At 560 MHz (Fig. 3(a)–(b)), an excellent agreement between HFSS and CST is observed in both main planes, with nearly overlapping curves and consistent sidelobe behavior. The peak realized gain is approximately 6.3 dBi, consistent with the frequency-domain results. At 680 MHz (Fig. 3(c)–(d)), similar behavior is observed, with minor sidelobe deviations. The -3 dB beamwidths are about 100° ($\phi = 0^\circ$) and 71° ($\phi = 90^\circ$) at 560 MHz, and approximately 95° and 70° at 680 MHz. These results indicate that the miniaturized, reflector-backed design preserves a balance between directive broadside radiation and

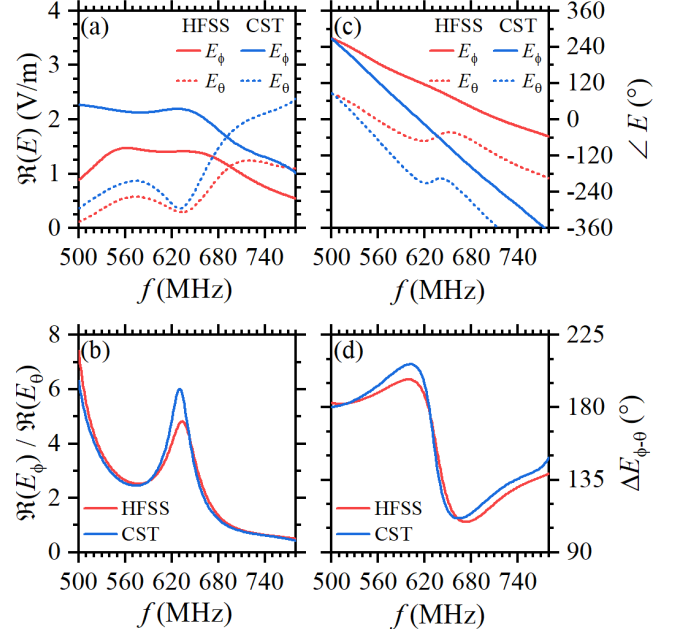


Fig. 4: Polarization characteristics of the proposed FF-MPA from HFSS (red) and CST (blue): (a) $\Re(E_\phi)$ and $\Re(E_\theta)$; (b) component ratio $\Re(E_\phi)/\Re(E_\theta)$; (c) $\angle E_\phi$ and $\angle E_\theta$; and (d) phase difference $\Delta E_{\phi-\theta}$.

wide angular coverage, suitable for practical UHF links.

An important implication is the pattern stability across frequency and simulation platform. Similar beam profiles at both frequencies confirm predictable dual-band spatial coverage. This is notable since miniaturization often degrades radiation patterns and gain in UHF MPAs. In the proposed antenna, the FF perturbation enables compact dual-band operation without destroying the fundamental broadside radiation mechanism of the patch. At the same time, the reflector effectively suppresses backward radiation and sustains a realized gain level above ~ 6 dBi in both operating bands. From an application viewpoint, this is a strong advantage for UHF TVWS links, because it combines compactness, dual-band functionality, and directive radiation in a single printed structure.

From a physical standpoint, this behavior is closely related to the resonant modes supported by the antenna. Specifically, Patch A predominantly resonates at 560 MHz through a fundamental linear mode, characterized by a current distribution aligned along a x -axis of the structure. In contrast, Patch B, operating at 680 MHz, exhibits an elliptical (near-circular) mode due to orthogonal field excitation. The polarization characteristics of the proposed FF-MPA are evaluated in Fig. 4, which reports the real parts and phases of the far-field electric field components, as well as the corresponding ratio and inter-component phase difference, as obtained from both HFSS (red curves) and CST (blue curves). In Fig. 4(a), in particular, both solvers indicate that the E_ϕ (solid) component dominates E_θ (dotted) over most of the analyzed frequency range. At 560 MHz, $\Re(E_\phi)$ is significantly larger than $\Re(E_\theta)$, confirming that the radiated field is primarily vertically polarized. Specifically, HFSS predicts $\Re(E_\phi) = 1.47$ V/m and $\Re(E_\theta) = 0.55$ V/m, while CST yields $\Re(E_\phi) = 2.14$ V/m

and $\Re(E_\theta) = 0.84$ V/m. Despite magnitude differences, both solvers indicate strong imbalance, which is characteristic of linear polarization associated with the resonant mode of patch A. At 680 MHz, however, the orthogonal components become comparable. HFSS predicts $\Re(E_\phi) = 1.26$ V/m and $\Re(E_\theta) = 0.92$ V/m, while CST gives $\Re(E_\phi) = 1.81$ V/m and $\Re(E_\theta) = 1.47$ V/m. This improved balance indicates excitation of orthogonal modes of Patch B, with comparable strengths, enabling a circular or elliptical polarization at the upper band.

This transition in magnitude balance is directly reflected in component ratio $\Re(E_\phi)/\Re(E_\theta)$ in Fig. 4(b). Around 560 MHz, the component ratio remains high, reaching 2.67 in HFSS and 2.54 in CST, indicating strong imbalance and vertical linear polarization. Near 680 MHz, in contrast, the ratio decreases to 1.37 (HFSS) and 1.23 (CST), indicating improved balance. This reduction leads to a pronounced minimum in the curve, which, although not reaching the ideal condition, clearly indicates a transition from circularly to an elliptically polarized regime.

The polarization behavior is further clarified by the phase responses in Figs. 4(c)-(d). At 560 MHz, the phase difference $\Delta E_{\phi-\theta} = \angle E_\phi - \angle E_\theta$ is close to 180° (approximately 189.21° in HFSS and 193.66° in CST). Combined with the strong amplitude imbalance, this phase opposition reinforces that the resulting field is predominantly linear. At 680 MHz, the phase departs from 180° , enabling elliptical polarization with improved amplitude balance. The phase difference at this frequency is 109.53° (HFSS) and 114.31° (CST), approaching the 90° quadrature condition.

The handedness of the polarization is further supported by the polarization ratio. Calculations of left-handed circular polarization (LHCP) and right-handed circular polarization (RHCP) at 560 MHz yielded a LHCP-RHCP ratio of approximately -1.84 dB, indicating that neither circular component is dominant, which is consistent with linear polarization. In contrast, at 680 MHz, the ratio increases to 25.47 dB, indicating LHCP dominance consistent with elliptical polarization of the upper-band.

It is also worth noting that the HFSS and CST results show strong qualitative agreement across all figures. The close overlap indicates low sensitivity to numerical implementation and supports fabrication reproducibility. Moreover, minor discrepancies are attributed to differences in numerical methods, meshing, and post-processing. Nevertheless, both solvers consistently predict the same polarization behavior, confirming the robustness of the proposed FF-MPA and that the observed characteristics are not solver-dependent artifacts.

IV. CONCLUSIONS

This paper presented a miniaturized MPA modified by Firat outer model fractal for TVWS applications. Two circular patches were designed and optimized for 560 MHz and 680 MHz using two different numerical tools. The improvement of bandwidth was achieved with the truncation of the ground plane and, to maintain the broadside behavior and gain, a reflector structure was applied. The reflection coefficient and radiation results showed good agreement between the two numerical approaches and proposed technological application in TVWS UHF. Overall, the proposed dual-band FF-MPA

supports two distinct polarization regimes within the UHF dual-band TVWS, combining compactness and stable radiation characteristics. The lower resonance at 560 MHz exhibits a vertical linearly polarized broadside mode, while the upper resonance at 680 MHz provides a quasi-left-handed circularly polarized response due to the combined effect of improved amplitude balance and near-quadrature phase difference between orthogonal field components. This dual-polarization capability enhances polarization diversity, improves link robustness, and mitigates polarization mismatch in multipath environments.

REFERENCES

- [1] C. K. Vithanawasam, Y. L. Then, and H. T. Su, "A review of microstrip antenna designs for TV white space applications," *Indones. J. Electr. Eng. Comput. Sci.*, vol. 19, no. 2, pp. 855–863, 2020.
- [2] M. Rahman and A. Saifullah, "A comprehensive survey on networking over TV white spaces," *Pervasive Mob. Comput.*, vol. 59, p. 101072, 2019.
- [3] M. U. Khan, M. S. Sharawi, and R. Mittra, "Microstrip patch antenna miniaturization techniques: A review," *IET Microw. Antennas Propag.*, vol. 9, no. 9, pp. 913–922, 2015.
- [4] K. Sarabandi, A. M. Buerkle, and H. Mosallaei, "Compact wideband UHF patch antenna on a reactive impedance substrate," *IEEE Antennas Wirel. Propag. Lett.*, vol. 5, pp. 503–506, 2006.
- [5] T. Alam, M. T. Islam, and M. Cho, "Near-zero metamaterial inspired UHF antenna for nanosatellite communication system," *Sci. Rep.*, vol. 9, p. 3441, 2019.
- [6] M. B. Fallahpour and R. Zoughi, "Antenna miniaturization techniques: A review of topology- and material-based methods," *IEEE Antennas Propag. Mag.*, vol. 60, no. 1, pp. 38–50, 2018.
- [7] M. R. Ghaderi and N. Amiri, "CubeSat antenna designs in the last 2 decades (2002–2023): A survey," *Int. J. Aeronaut. Space Sci.*, vol. 26, no. 1, pp. 327–375, 2025.
- [8] D. H. Werner and S. Ganguly, "An overview of fractal antenna engineering research," *IEEE Antennas Propag. Mag.*, vol. 45, no. 1, pp. 38–57, 2003.
- [9] F. Venneri, S. Costanzo, and A. Borgia, "Fractal metasurfaces and antennas: An overview for advanced applications in wireless communications," *Appl. Sci.*, vol. 14, no. 7, p. 2843, 2024.
- [10] C. P. Baliarda, J. Romeu, and A. Cardama, "The Koch monopole: A small fractal antenna," *IEEE Trans. Antennas Propag.*, vol. 48, no. 11, pp. 1773–1781, 2000.
- [11] H. Oraizi and S. Hedayati, "Miniaturization of microstrip antennas by the novel application of the Giuseppe Peano fractal geometries," *IEEE Trans. Antennas Propag.*, vol. 60, no. 8, pp. 3559–3567, 2012.
- [12] C. Borja and J. Romeu, "On the behavior of Koch island fractal boundary microstrip patch antenna," *IEEE Trans. Antennas Propag.*, vol. 51, no. 6, pp. 1281–1291, 2003.
- [13] F. Venneri, S. Costanzo, A. Borgia, and G. Buonanno, "Fractal metamaterial surfaces for UHF sensing applications," *IEEE Access*, vol. 12, pp. 65 810–65 824, 2024.
- [14] H. Zhang, D. Han, M. Ke, L. Luo, Z. Huang, and G. Zhang, "PD Koch complementary fractal UHF antenna based on AMC metasurface," *Sensors*, vol. 26, no. 4, p. 1398, 2026.
- [15] T. Tuncer, S. Dogan, and A. Subasi, "A new fractal pattern feature generation function based emotion recognition method using EEG," *Chaos Solitons Fract.*, vol. 144, p. 110671, 2021.
- [16] A. L. M. de Araújo, M. H. A. M. Melo, I. K. C. de Melo, H. D. de Andrade, R. V. A. Lira, and I. B. T. da Silva, "Firat fractal applied as miniaturization technique on rectangular microstrip patch antenna," in *2023 SBMO/IEEE MTT-S International Microwave and Optoelectronics Conference (IMOC)*, 2023, pp. 85–87.
- [17] M. S. da Silva, A. L. M. de Araújo, L. O. de Sousa, L. A. Sarmento, J. A. I. de Araujo, S. M. de Holanda, G. G. Machado, and I. B. T. da Silva, "Miniaturization behavior of microstrip patch antenna modified by Firat-based circular fractal," in *2025 SBMO/IEEE MTT-S International Microwave and Optoelectronics Conference (IMOC)*, 2025.
- [18] M. S. da Silva, S. M. de Holanda, L. O. de Sousa, I. B. T. da Silva, and F. de A. Brito Filho, "Firat outer model fractal evaluation on microstrip patch antennas," in *2026 IEEE 2nd Latin American Conference on Antennas and Propagation (LACAP)*, 2026.