

Optimization-Based Sidelobe Control in Reduced Complexity Hardware-Combined Architectures

Aline G. T. Duarte, José A. Apolinário Jr., Cláudio A. B. Saunders F., and Antonio L. L. Ramos

Abstract—Phased arrays have become a fundamental technology in modern communication systems, including millimeter-wave 5G networks and Low Earth Orbit satellite links, where radiation pattern control is essential for spectral efficiency. In this context, an optimization-based framework under explicit angular constraints is proposed for beam synthesis. Based on the construction of the sidelobe steering matrix, formulations involving sidelobe energy minimization, the L_1 -norm, and the minimax criterion are analyzed within a unified framework. The approach is further extended to reduced-complexity subarray architectures, enabling a systematic investigation of performance–complexity trade-offs. Results show that the relative behavior of the optimization criteria is largely preserved under subarray constraints, achieving effective sidelobe suppression with reduced hardware complexity. Among the considered methods, the sidelobe energy formulation provides the best trade-off between performance and computational cost. Finally, a weight precomputation strategy is adopted to enable real-time operation by implementing efficient lookup tables (LUTs) in large-scale phased array systems.

Keywords—phased arrays, beam pattern synthesis, sidelobe control, subarray architectures, convex optimization, Low Earth Orbit satellites, millimeter-wave 5G

I. INTRODUCTION

Phased array antennas have become a fundamental technology in modern wireless communication systems, including millimeter-wave 5G networks and low Earth orbit (LEO) satellite communications, where adaptive beam steering and radiation pattern control are essential to ensure high spectral efficiency, coverage, and reliability [1], [2]. The recent deployment of LEO constellations has intensified the demand for high-gain, electronically steerable arrays operating under stringent requirements such as low latency, high throughput, and dynamic link adaptation [3]. In this context, precise control of the radiation pattern—particularly the trade-off between mainlobe (ML) shaping and sidelobe (SL) suppression—is critical for interference mitigation.

Beamforming techniques enable control of the array response through appropriate weight design. Classical approaches, such as minimum variance distortionless response (MVDR) and linearly constrained methods, provide effective solutions under stochastic signal models [2], but have limited ability to enforce spatial constraints over extended angular regions. Deterministic pattern synthesis methods address this limitation by imposing constraints at specific angular directions [1], [4]. Traditional sidelobe control relies on amplitude

tapering or windowing [5], while minimax designs provide peak SL control [1]. More recently, convex optimization-based methods have enabled flexible beam pattern synthesis by incorporating multiple criteria, including SL suppression, null placement, and beamwidth control [6]. Hybrid strategies combining different design approaches have also shown improved performance [7]. However, these approaches are often studied in isolation and primarily in full-array configurations.

Despite these advances, there is still a lack of a unified formulation that (i) enables structured SL control over angular regions, (ii) supports multiple optimization criteria within a common framework, and (iii) extends naturally to reduced-complexity (subarray-based) architectures. This limitation is particularly relevant in large-scale systems, where hardware constraints impose strict limits on the number of RF chains, making performance–complexity trade-offs central.

Motivated by these limitations, we propose an optimization-based framework for beam pattern synthesis under explicit angular constraints, leveraging a sidelobe steering matrix to represent the array response over undesired directions. This formulation enables a unified analysis of sidelobe energy minimization, the L_1 -norm, and minimax criteria, and extends naturally to subarray-based architectures via a linear mapping between the full and reduced domains.

A key contribution of this work is to show that the relative behavior of these optimization criteria is largely preserved under subarray constraints, enabling the design of reduced-complexity beamformers with predictable performance. In addition, the framework supports a practical implementation strategy based on offline weight computation and lookup tables (LUTs), eliminating the need for online optimization and enabling real-time operation in systems with stringent computational and hardware constraints [8], [9].

The remainder of this paper is organized as follows. Section II introduces the system model and array architectures. Section III presents the proposed formulation. Section IV describes the simulation setup and performance metrics. Section V discusses the results and performance–complexity trade-offs. Section VI concludes the paper.

II. SYSTEM MODEL AND ARRAY ARCHITECTURES

A. Array Signal Model

The received signal at each antenna element is modeled as a time-delayed version of a plane wave impinging on the array, with the delay determined by the direction of arrival and the spatial position of the element. For an array with M elements, these direction-dependent delays capture the effect of wave propagation across the array aperture and are fully determined by the array geometry.

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Under the narrowband far-field assumption, the analytic signal at the m -th antenna element is modeled as

$$x_m(t) = s(t) e^{j\Omega_0 t} e^{-j\Omega_0 \Delta t_m} + n_m(t), \quad (1)$$

where $s(t)$ is the complex envelope of the incident signal, $\Omega_0 = 2\pi f_0$ is the carrier angular frequency, Δt_m is the propagation delay relative to a reference point, and $n_m(t)$ denotes additive noise.

For a plane wave arriving from direction (θ, ϕ) ,

$$\Delta t_m = \frac{1}{c} \mathbf{k}^T(\theta, \phi) \mathbf{p}_m, \quad (2)$$

where

$$\mathbf{k}(\theta, \phi) = -[\sin \theta \cos \phi \ \sin \theta \sin \phi \ \cos \theta]^T$$

is the unit propagation vector, $\mathbf{p}_m \in \mathbb{R}^3$ is the position of the m -th antenna element, and c is the propagation speed.

Stacking the received signals yields the classical narrowband array model

$$\mathbf{x}(t) = s(t) e^{j\Omega_0 t} \mathbf{a}(\theta, \phi) + \mathbf{n}(t), \quad (3)$$

where $\mathbf{a}(\theta, \phi) \in \mathbb{C}^M$ is the steering vector. Using $\Omega_0/c = 2\pi/\lambda$ and defining the array geometry matrix

$$\mathbf{P} = [\mathbf{p}_1 \ \cdots \ \mathbf{p}_M]^T,$$

the steering vector can be written compactly as

$$\mathbf{a}(\theta, \phi) = \exp\left(-j \frac{2\pi}{\lambda} \mathbf{P} \mathbf{k}(\theta, \phi)\right), \quad (4)$$

where $\exp(\cdot)$ denotes element-wise exponentiation.

B. Array Architecture

We adopt a uniform rectangular array (URA) architecture, in which the antenna elements are arranged on a regular two-dimensional grid with N_R rows and N_C columns. This configuration enables beam steering in both azimuth and elevation while preserving simplicity of implementation and scalability. The inter-element spacing is set to $d = \lambda/2$, which prevents spatial aliasing and the occurrence of grating lobes [1].

The antenna positions are defined on a rectangular lattice centered at the origin. The coordinates along each axis are given by

$$x_i = \left(i - \frac{N_C-1}{2}\right) d, \quad i = 0, \dots, N_C - 1, \quad (5)$$

$$y_j = \left(\frac{N_R-1}{2} - j\right) d, \quad j = 0, \dots, N_R - 1. \quad (6)$$

Each antenna element is indexed by the pair (i, j) , and its position vector is defined as

$$\mathbf{p}_m = [x_i \ y_j \ 0]^T, \quad (7)$$

where $m = 1, \dots, M$ is a linear index associated with (i, j) .

The array geometry matrix is formed by stacking the position vectors of all antenna elements as follows:

$$\mathbf{P} = [\mathbf{p}_1 \ \mathbf{p}_2 \ \cdots \ \mathbf{p}_M]^T \in \mathbb{R}^{M \times 3}. \quad (8)$$

This matrix fully characterizes the spatial distribution of the antenna elements and enables different array geometries

by adjusting their positions, assuming that all radiators are identical and ideal. Based on this geometric representation, array architectures can be defined based on the element connections to the RF chains. In this work, both full-array and reduced-complexity (subarray) architectures are considered.

1) *Full-Array Architecture*: In the full-array architecture, each antenna element is connected to an independent RF chain, enabling full control of both amplitude and phase. Let $\mathbf{w} \in \mathbb{C}^M$ denote the complex beamforming weight vector. This approach provides the maximum number of spatial degrees of freedom, allowing precise beam steering. However, it increases the cost, hardware complexity, and power consumption [10].

2) *Subarray Architectures*: To reduce hardware complexity, subarray-based architectures are described through a linear transformation between the full-array representation and a reduced-dimensional domain. Let $\mathbf{w}_s \in \mathbb{C}^{M_s}$ denote the subarray weight vector, where M_s is the number of subarray channels.

Fig. 1 illustrates the subarray architecture adopted herein, where the full array is partitioned into groups of elements connected to individual RF chains through analog combining. Signals from elements within each subarray are summed to produce a reduced set of outputs, which are then processed digitally, reducing hardware complexity.

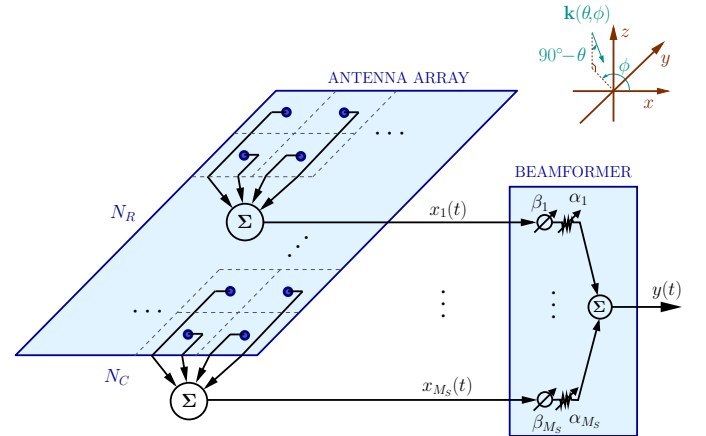


Fig. 1: Subarray architecture for a $N_R \times N_C$ URA.

For a URA $N_R \times N_C$, the total number of elements is $M = N_R N_C$. When a 2×2 subarray is employed as in Fig. 1, the array is divided into non-overlapping blocks of four adjacent elements, resulting in an equivalent grid with $\frac{N_R}{2}$ rows and $\frac{N_C}{2}$ columns. Consequently, assuming even dimensions, the number of subarrays is given as $M_s = N_R N_C / 4$.

This operation corresponds to a spatial array reduction, in which neighboring antenna elements are grouped into subarrays. The transformation between domains is defined as

$$\mathbf{w}_s = \mathbf{S} \mathbf{w}, \quad (9)$$

where $\mathbf{S} \in \{0, 1\}^{M_s \times M}$ is defined as

$$[\mathbf{S}]_{s,m} = \begin{cases} 1, & \text{if element } m \text{ belongs to subarray } s, \\ 0, & \text{otherwise,} \end{cases}$$

for $m = 1, \dots, M$ and $s = 1, \dots, M_s$, where m denotes the antenna element index and s denotes the subarray index.

For the adopted 2×2 partition, each row of $\mathbf{S}$ identifies the four antenna elements associated with a given subarray. Thus, the transformation $\mathbf{w}_s = \mathbf{S}\mathbf{w}$ maps the full-array weight vector into the reduced subarray domain.

To preserve the array-sampling properties, an equivalent subarray geometry is defined in which the spacing between adjacent subarray centers is $\lambda/2$ in both dimensions. This condition is achieved by setting the intra-element spacing to $d = 0.25\lambda$, so that the effective spacing between subarray centers remains $\lambda/2$, thereby preserving proper spatial sampling and preventing the occurrence of grating lobes. In contrast, alternative configurations such as 2×1 or 1×2 , with intra-element spacing of $d = 0.5\lambda$, violate this requirement in at least one dimension, possibly leading to spatial aliasing effects. Therefore, the 2×2 configuration is adopted as the reference architecture, as it provides an effective trade-off between hardware complexity reduction and beamforming performance.

C. Beamforming Model

Given a complex weight vector $\mathbf{w} \in \mathbb{C}^M$, the beam pattern in a given direction (θ, ϕ) is defined as $BP(\theta, \phi) = \mathbf{w}^H \mathbf{a}(\theta, \phi)$ and its normalized version in dB is defined as

$$BP_{dB}(\theta, \phi) = 20 \log_{10} \left(\frac{|BP(\theta, \phi)|}{\max_{\theta, \phi} |BP(\theta, \phi)|} \right). \quad (10)$$

When subarray architectures are considered, the beamforming weights are defined in a reduced-dimensional domain, and the corresponding beam pattern is given by

$$BP(\theta, \phi) = \mathbf{w}_s^H \mathbf{S} \mathbf{a}(\theta, \phi), \quad (11)$$

which shows that the effective steering vector in the reduced domain is given by $\mathbf{S} \mathbf{a}(\theta, \phi)$ for the subarray cases.

III. OPTIMIZATION-BASED BEAM PATTERN SHAPING

This section presents the optimization framework adopted for beam pattern synthesis. The main steps of the proposed approach are detailed next.

A. Mainlobe and Sidelobe Mask Construction

The beam pattern synthesis problem is formulated by partitioning the angular domain into mainlobe (ML) and sidelobe (SL) regions over a discretized grid (θ, ϕ) . Fig. 2 illustrates this process.

Initially, a reference beam pattern is obtained using a conventional Delay-and-Sum (D&S) beamformer steered towards the desired direction (θ_0, ϕ_0) [2], as shown in Fig. 2(a). A -23 dB dashed contour, illustrated in Fig. 2(b), is used as a threshold to identify the ML region. Based on this reference level, the ML region is defined as the set of angular points whose magnitude exceeds this threshold. To improve robustness to modeling errors and ensure adequate beam coverage, this region is further expanded with safety margins in both the θ and ϕ dimensions. The resulting partition is depicted in Fig. 2(b), where the white region corresponds to the ML, the blue region defines the SL domain, and the red marker

indicates the desired steering direction. The black contour represents the final expanded boundary of the ML region.

Formally, the ML region is denoted by $\mathcal{M}_{ML} \subset \mathbb{R}^2$, while its complement $\mathcal{M}_{SL} = \mathcal{M}_{ML}^c$ defines the SL region. This partition enables explicit and flexible control of the mainlobe–sidelobe trade-off during the optimization process.

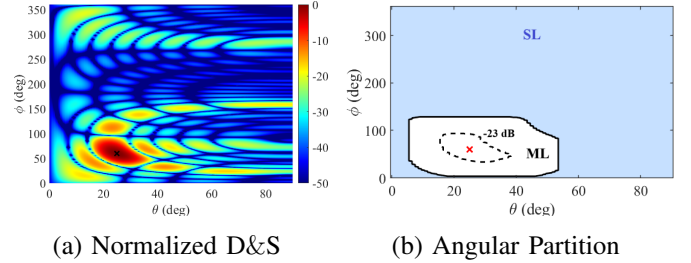


Fig. 2: (a) Normalized D&S beam pattern (in dB). (b) Angular partition used in the synthesis over the grid: white and blue regions correspond to $\mathcal{M}_{ML}$ and $\mathcal{M}_{SL}$, respectively.

B. Construction of the Sidelobe Steering Matrix

The SL steering matrix $\mathbf{A}_S$ is constructed by stacking steering vectors associated with angular directions $\{(\theta_k, \phi_k)\}_{k=1}^K \subset \mathcal{M}_{SL}$ in the SL region, i.e.,

$$\mathbf{A}_S = [\mathbf{a}(\theta_1, \phi_1) \quad \mathbf{a}(\theta_2, \phi_2) \quad \cdots \quad \mathbf{a}(\theta_K, \phi_K)], \quad (12)$$

where K denotes the number of angular grid points. This matrix provides a compact representation of the array response over undesired directions in the sidelobe region. Consequently, minimizing quantities involving $\mathbf{A}_S^H \mathbf{w}$ enables direct control of the sidelobe levels across the angular domain.

This construction can be interpreted as defining a virtual interference field over the sidelobe region, allowing the optimization problem to directly shape the array response across undesired directions.

C. Beamforming Formulations

Using $\mathbf{A}_S$, the beamforming design is expressed as a constrained optimization problem that minimizes the sidelobe response while enforcing a distortionless constraint in the desired direction. Different criteria lead to the formulations described below, where $\mathbf{a}_0 = \mathbf{a}(\theta_0, \phi_0)$ denotes the steering vector associated with the signal of interest.

1) **Sidelobe energy formulation** [2]: The sidelobe (SL) energy formulation minimizes the total energy radiated in the sidelobe region, leading to

$$\min_{\mathbf{w}} \|\mathbf{A}_S^H \mathbf{w}\|_2^2 \quad \text{s.t.} \quad \mathbf{w}^H \mathbf{a}_0 = 1. \quad (13)$$

This formulation can be equivalently written as

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{Q}_{SL} \mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^H \mathbf{a}_0 = 1, \quad (14)$$

where $\mathbf{Q}_{SL} = \mathbf{A}_S \mathbf{A}_S^H$.

This problem corresponds to a quadratic minimization with a linear equality constraint, for which a closed-form solution

exists. By applying the method of Lagrange multipliers, the optimal weight vector is obtained as

$$\mathbf{w}_{\text{OPT}} = \frac{\mathbf{Q}_{SL}^{-1} \mathbf{a}_0}{\mathbf{a}_0^H \mathbf{Q}_{SL}^{-1} \mathbf{a}_0}. \quad (15)$$

This solution has a structure analogous to the well-known MVDR [2], in which $\mathbf{Q}_{SL}$ plays the role of the spatial covariance matrix associated with the sidelobe region. As a result, the sidelobe energy formulation can be interpreted as minimizing the power radiated toward undesired directions while preserving unit gain in the signal-of-interest direction.

2) **L_1 -norm Formulation [6]:** To promote sparsity in the sidelobe response and enhance peak suppression, the following L_1 -norm formulation is considered:

$$\min_{\mathbf{w}} \|\mathbf{A}_S^H \mathbf{w}\|_1 \quad \text{s.t.} \quad \mathbf{w}^H \mathbf{a}_0 = 1. \quad (16)$$

This problem does not admit a closed-form solution and is solved using CVX [11].

3) **Minimax Formulation [12]:** The minimax formulation directly minimizes the maximum sidelobe level across the angular domain:

$$\min_{\mathbf{w}} \|\mathbf{A}_S^H \mathbf{w}\|_{\infty} \quad \text{s.t.} \quad \mathbf{w}^H \mathbf{a}_0 = 1. \quad (17)$$

This formulation is directly associated with the L_{∞} norm. For any vector $\mathbf{z}$, the infinity norm is defined as

$$\|\mathbf{z}\|_{\infty} = \max_i |z_i|. \quad (18)$$

Minimizing $\|\mathbf{A}_S^H \mathbf{w}\|_{\infty}$ corresponds to a minimax formulation, where the worst-case sidelobe level is minimized. This problem does not admit a closed-form solution and is solved using CVX. It provides explicit control over the peak sidelobe level [12], [11].

IV. SIMULATION SETUP AND PERFORMANCE METRICS

Simulations use a 16×8 URA at $f_0 = 10.5$ GHz, steered towards $(\theta_0, \phi_0) = (25^\circ, 60^\circ)$. This size balances directivity and hardware complexity, even with subarrays. The full and reduced 2×2 architectures ($N_R N_C / 4$ channels) are evaluated using a 1° resolution over the area shown in Fig. 2(b). All configurations share the same grid and steering direction to ensure a fair comparison.

The performance is evaluated using standard metrics from array processing and antenna theory to characterize sidelobe behavior and ML resolution. These metrics are presented next.

1) **Sidelobe Level:** The Sidelobe Level (SLL) is defined as the maximum radiation level in the sidelobe region [1], [4], i.e.,

$$\text{SLL} = \max_{(\theta, \phi) \in \mathcal{M}_{\text{SL}}} BP_{dB}(\theta, \phi). \quad (19)$$

2) **Integrated Sidelobe Level:** The Integrated Sidelobe Level (ISL) quantifies the total sidelobe energy relative to the ML [2], [13], defined as

$$\text{ISL} = 10 \log_{10} \left(\frac{\sum_{(\theta, \phi) \in \mathcal{M}_{\text{SL}}} |BP(\theta, \phi)|^2}{\sum_{(\theta, \phi) \in \mathcal{M}_{\text{ML}}} |BP(\theta, \phi)|^2} \right). \quad (20)$$

3) **Half-Power Beamwidth (HPBW):** The half-power beamwidth (HPBW) is defined as the angular width of the ML at the -3 dB level, evaluated along a given angular cut (e.g., θ or ϕ) [12], [5],

$$\text{HPBW}_{\theta} = \theta_{\text{right}} - \theta_{\text{left}}, \quad \text{HPBW}_{\phi} = \phi_{\text{right}} - \phi_{\text{left}}. \quad (21)$$

V. RESULTS AND DISCUSSION

A. Beam Pattern Analysis: Full vs. Subarray Architectures

Fig. 3 shows the beam patterns obtained with the sidelobe energy formulation for both configurations. The full-array configuration yields a narrower, more directive ML and a more uniform sidelobe distribution, owing to its higher degrees of freedom. In contrast, the 2×2 subarray produces a broader ML due to the reduced number of degrees of freedom, while still achieving effective sidelobe suppression.

Fig. 4 shows the beam patterns for the reduced array under different optimization criteria. The L_1 -norm formulation promotes sparsity, leading to localized sidelobe suppression, whereas the minimax formulation directly controls the peak sidelobe level, yielding a more uniform sidelobe envelope.

These results show that, despite dimensionality reduction, the relative behavior of the optimization criteria is preserved, leading to consistent trade-offs between sidelobe suppression and beamwidth.

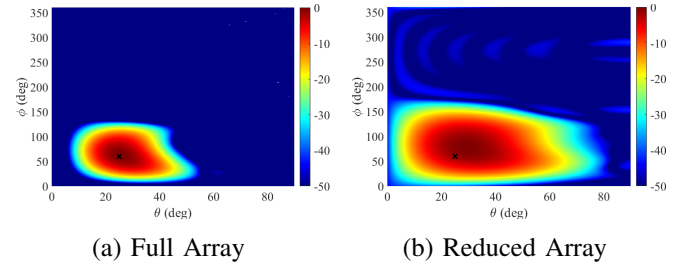


Fig. 3: Normalized beam patterns for a 16×8 (full and reduced) URA using the sidelobe energy formulation.

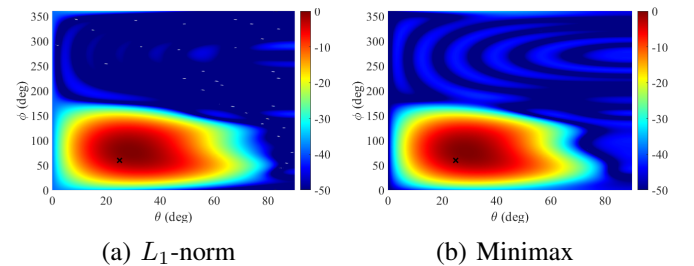


Fig. 4: Normalized beam patterns of the reduced array.

B. Performance Evaluation

Table I summarizes the performance of the evaluated methods in terms of SLL, ISL, and HPBW. For the full-array configuration (128 antennas), the sidelobe energy formulation achieves the lowest ISL (-72.98 dB), whereas the minimax formulation achieves the lowest SLL (-78.58 dB), confirming

its effectiveness in controlling the worst-case SL level. The L_1 -norm formulation exhibits intermediate behavior, providing a compromise between overall SL energy reduction and peak SL suppression.

TABLE I: Performance comparison of the evaluated methods

Arch.	Method	SLL (dB)	ISL (dB)	HPBW _{θ}	HPBW _{ϕ}
FULL	SL Energy	-69.15	-72.98	16	43
FULL	L_1 -norm	-59.13	-69.15	16	44
FULL	Minimax	-78.58	-66.72	15	43
SUB 2×2	SL Energy	-37.70	-40.89	25	64
SUB 2×2	L_1 -norm	-35.00	-40.11	25	64
SUB 2×2	Minimax	-43.08	-36.81	24	64

For the reduced 2×2 subarray architecture, 128 antennas are combined into 32 channels, and performance degradation occurs due to the fewer degrees of freedom, mainly reflected in the increased half-power beamwidth (HPBW). Nevertheless, all methods maintain effective SL suppression. In this configuration, the minimax formulation continues to achieve the lowest SLL (−43.08 dB), whereas the sidelobe energy formulation yields the lowest overall sidelobe energy.

Results indicate that the reduced architecture preserves the relative behavior of the methods observed in the full-array case, with consistent trade-offs between SL suppression and beamwidth.

C. Complexity Trade-off

Table II reports the execution time required by each method to compute the beamforming weight vector for both full-array and reduced subarray configurations. All simulations were performed with an 11th Gen Intel(R) Core(TM) i5-1135G7 processor and 32 GB of RAM. As expected, the subarray structure significantly reduces the computational cost.

TABLE II: Beamforming weight computation time

Method	FULL (s)	SUB 2×2 (s)	Speedup
SL energy	2170 μ	120 μ	17.93
L_1 -norm	17.15	6.41	2.68
Minimax	32.26	7.17	4.50

The sidelobe energy formulation yields the lowest execution time because it admits a closed-form solution, resulting in a speedup of approximately $18\times$. In contrast, the L_1 -norm and minimax formulations, solved via convex optimization, require higher computational effort and exhibit less pronounced speedups ($2.7\times$ and $4.5\times$, respectively).

These results show that the reduced architecture not only decreases hardware complexity but also significantly improves computational efficiency, making optimization-based beamforming more practical for real-time implementation.

D. Lookup Table Perspective

The reduced-dimensional formulation enables a practical lookup table (LUT)-based implementation of beamforming weights. By solving the optimization problem offline for a predefined set of steering directions, the corresponding subarray weights $\mathbf{w}_s$ can be stored and retrieved in real time, thereby avoiding online optimization.

This approach is particularly beneficial for computationally intensive formulations such as L_1 -norm and minimax. Combined with subarray mapping, the LUT strategy is well-suited to satellite and 5G communication scenarios, where real-time processing is critical, and system cost is a key constraint.

VI. CONCLUSION AND FUTURE WORK

This work presented an optimization-based framework for full- and reduced-complexity beamformer synthesis using the sidelobe steering matrix $\mathbf{A}_S$. The proposed approach enables structured control of the radiation pattern and supports a unified treatment of different design criteria, including sidelobe energy, L_1 -norm, and minimax formulations. Results demonstrate that the reduced 2×2 subarray architecture significantly reduces hardware and computational complexity while preserving the relative behavior of the optimization criteria, maintaining effective sidelobe suppression and mainlobe characteristics at the cost of a moderate increase in beamwidth.

Future work will assess the potential reduction in ML and SL performance caused by practical antenna characteristics and mutual coupling, allowing for a more accurate evaluation under real-world conditions.

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