Performance Evaluation of High-Resolution Parameter Estimation in Low-Rank MIMO Channels with Interference

Jean C. S. Ferreira, Francisco Igor Pires and Fazal Asim

***Abstract*— This paper evaluates the performance of MUSIC, Root-MUSIC, and ESPRIT algorithms for angular parameter estimation in low-rank MIMO channels under co-channel interference. A MIMO channel model with angular parameters is formalized, taking into account interference from neighboring base stations. Performance is analyzed using the normalized mean square error (NMSE) for channel estimation and root mean square error (RMSE) for angle estimation. A Cramér–Rao lower bound is derived to assess the performance of the high-resolution algorithms. Simulation results show that the three algorithms achieve accurate estimation under interference, with Root-MUSIC offering similar performance at a lower computational cost. A two-stage estimation approach further improves accuracy by exploiting the channel's low-rank structure.**

***Keywords*— Parameter Estimation, Estimation Algorithms, MIMO System, Interference, Cramér–Rao lower bound.**

I. Introduction

The next generation of wireless communication systems promises to provide higher data rates by moving towards higher frequencies, making the channel sparser with fewer dominant paths. This can be dealt with as a geometrical channel model, where the problem of channel estimation boils down to angular parameter estimation [1]. Therefore, DoA (Direction of Arrival) estimation, which was generally studied as part of the broader field of array signal processing, now has substantial and growing research focused on radio direction finding—that is, estimating the direction of electromagnetic waves that impinge on array antennas.

Among the various DoA estimation techniques, subspace-based methods such as the ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques), MUSIC (Multiple Signal Classification) algorithm, and its more computationally efficient variant, Root-MUSIC, have gained prominence because of their high-resolution capabilities. These algorithms rely on singular value decomposition (SVD) on the received correlation matrix to distinguish multiple signal sources, even when they are very close together, proving effective in different propagation scenarios [2].

Although the performance of the ESPRIT, MUSIC and Root-MUSIC algorithms has been extensively studied in traditional array systems, their behavior in the presence of unwanted interference from other sources, especially in MIMO (Multiple Input Multiple Output) systems, where multiple transmitting and receiving antennas are present, still requires deeper analysis. In real wireless channels, signals often arrive via numerous paths with varying degrees of correlation, making the estimation process more complex. In addition, interference signals pose additional challenges in distinguishing between desired signals and noise or interference components. Therefore, this paper is based on the performance analysis of high-resolution algorithms in the presence of unwanted interference.

Jean C. S. Ferreira, Department of Teleinformatics Engineering, Federal University of Ceará, Fortaleza-CE, e-mail: jeancarlos8249@alu.ufc.br; Francisco Igor Pires, Department of Teleinformatics Engineering, Federal University of Ceará, Fortaleza-CE, e-mail: igorpires@alu.ufc.br; Fazal Asim, Department of Teleinformatics Engineering, Federal University of Ceará, Fortaleza-CE, e-mail: fazalasim@ufc.br. This work is partially supported by INCT-Signals under grant INCT-25255-82587.32.41/64 (FUNCAP) and 406517/2022-3 (CNPq), and FUNCAP/Universal under grant UNI-0210-00043.01.00/23, and CAPES (Finance code 001).

II. System and Channel Model

Consider a MIMO system where the base-station (BS) with M antennas is communicating with user-equipment (UE) with Q antennas. We also assume an interfering BS, which unintentionally sends signals to the intended UE as shown in Figure 1. Assuming the deployment of a uniform linear array (ULA) at both the BS and the UE, the channel between the BS and the UE is given as [3]:

$$\boldsymbol{H} = \sum_{r=1}^{R} \alpha_r \boldsymbol{b}(\phi_r) \boldsymbol{a}^{\mathrm{T}}(\theta_r) \in \mathbb{C}^{Q\times M}, \tag{1}$$

where α_r is the r-th complex path gain, $\boldsymbol{a}(\theta_r) \in \mathbb{C}^{M\times 1}$ is the r-th steering vector of the BS with θ_r as r-th angle of departure (AoD), and $\boldsymbol{b}(\phi_r) \in \mathbb{C}^{Q\times 1}$ is the r-th steering vector of the UE with ϕ_r as r-th angle of arrival (AoA), defined as:

$$\boldsymbol{a}(\mu_r) = [1, e^{-j\mu_r}, e^{-j2\mu_r}, \ldots, e^{-j(M-1)\mu_r}]^{\mathrm{T}}, \in \mathbb{C}^{M\times 1} \tag{2}$$

and

$$\boldsymbol{b}(\psi_r) = [1, e^{-j\psi_r}, e^{-j2\psi_r}, \ldots, e^{-j(Q-1)\psi_r}]^{\mathrm{T}}, \in \mathbb{C}^{Q\times 1} \tag{3}$$

where the spatial frequencies are defined in [3] as $\mu_r = \pi \sin\theta_r$ and $\psi_r = \pi \sin\phi_r$.

The received data signal in the presence of unwanted interference can be written as:

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{S} + \sum_{i=1}^{I} \boldsymbol{H}_i \boldsymbol{S}_i + \boldsymbol{N} \in \mathbb{C}^{Q\times M}, \tag{4}$$

where $\boldsymbol{S} \in \mathbb{C}^{M\times M}$ is the orthogonal pilot signal matrix and $\boldsymbol{N} \sim \mathcal{CN}\left(\boldsymbol{0}_{Q\times M}, \sigma_n^2 \boldsymbol{I}_{Q\times M}\right)$ is the circularly symmetric additive white Gaussian noise matrix with variance σ_n^2. The matrices $\boldsymbol{H}_i \in \mathbb{C}^{Q\times M}$ and $\boldsymbol{S}_i \in \mathbb{C}^{M\times M}$ represent the i-th interfering channel and pilot signal matrices from the interfering BS, respectively.

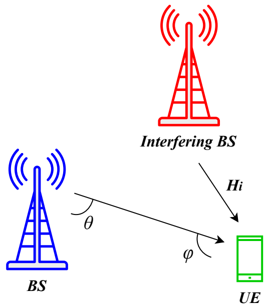

Fig. 1: System Model.

III. Channel Parameter Estimation

This section discusses two subspace high-resolution algorithms in the scenario where the desired UE also receives unwanted interference. SVD or eigenvalue decomposition (EVD) can be used to separate the signal subspace and the noise plus interference subspace to estimate the respective angular parameters. Finally, the intrinsic geometrical structure of the rank-one channel is exploited to solve the angle pairing issue between the AoD and AoA. We used two approaches to estimate the system parameters; the first approach, i.e., "approach 1", as given in subsection *A* and *B* when the SVD is applied directly on the received signal matrix $\boldsymbol{Y}$ at the UE. The second approach, i.e., "approach 2", is explained in section *D*.

A. MUSIC Algorithm

The MUSIC is a high-resolution parameter estimation algorithm based on the noise subspace of the received signal matrix. All impinging angles from every propagation path can be estimated by spectral scanning the angular domain and identifying the peaks of the MUSIC spectrum [4]. The noise subspace is calculated as

$$\hat{\boldsymbol{R}}_{yy} = \frac{1}{N}\sum^{N} \boldsymbol{Y}\boldsymbol{Y}^{\mathrm{H}}, \tag{5}$$

where N is the number of snapshots received. Applying the EVD on the covariance matrix $\hat{\boldsymbol{R}}_{yy}$ or the SVD on the received signal $\boldsymbol{Y}$. Assuming SVD on $\boldsymbol{Y}$ leads to:

$$\mathrm{svd}(\boldsymbol{Y}) = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^{H} = \begin{bmatrix}\boldsymbol{U}_s & \boldsymbol{U}_{o+i}\end{bmatrix}\begin{bmatrix}\boldsymbol{\Sigma}_s & 0 \\ 0 & \boldsymbol{\Sigma}_{o+i}\end{bmatrix}\begin{bmatrix}\boldsymbol{V}_s^{H} \\ \boldsymbol{V}_{o+i}^{H}\end{bmatrix}, \tag{6}$$

where the $\boldsymbol{U}_s$ and $\boldsymbol{U}_{o+i}$ are the left singular vectors related to signal and noise plus interference subspace. $\boldsymbol{V}_s$ and $\boldsymbol{V}_{o+i}$ are right singular vectors related to signal and noise plus interference subspaces. Finally, $\boldsymbol{\Sigma}_s$ and $\boldsymbol{\Sigma}_{o+i}$ are singular values related to impinging signals and interference plus noise floor. The MUSIC spectrum is defined as:

$$S_{MUSIC}(\nu) = \frac{\boldsymbol{a}^{H}(\nu)\boldsymbol{a}(\nu)}{\boldsymbol{a}^{H}(\nu)\boldsymbol{U}_{o+i}\boldsymbol{U}_{o+i}^{H}\boldsymbol{a}(\nu)} = \frac{1}{\boldsymbol{a}^{H}(\nu)\boldsymbol{C}_{o+i}\boldsymbol{a}(\nu)} \tag{7}$$

where $\nu \in \{\mu, \psi\}$, are respective spatial frequencies and $\boldsymbol{C}_{o+i} = \boldsymbol{U}_{o+i}\boldsymbol{U}_{o+i}^{\mathrm{H}}$ is the projection onto the noise plus interference subspace.

B. Root-MUSIC Algorithm

The Root-MUSIC algorithm comes into play due to the high cost of spectral search. Unlike traditional MUSIC, which requires an exhaustive search over the angles of interest, root-MUSIC estimates the angles by finding the roots of a polynomial constructed from the noise subspace. The algorithm applies to arrays with a uniform structure (such as linear or planar arrays with regular spacing) [4].

C. ESPRIT Algorithm

The ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) is also a high-resolution subspace-based algorithm, but in contrast to MUSIC and Root-MUSIC, it works on the signal subspace instead of the noise subspace. ESPRIT's fundamental principle lies in exploiting the geometric shift invariance of the antenna array. It works by dividing the array into two identical subarrays, where we can map one onto the other by a simple translational shift by Δ (separation distance between the two subarrays). The angles of arrival and/or departure are then directly calculated from the eigenvalues of the resulting rotation operator, eliminating the need for any search function. This method also requires the array to have a specific structure that allows for this subarray division, such as a Uniform Linear Array (ULA) [5].

D. Two-stage Parameter Estimation

In this section, we explain another strategy by estimating the MIMO channel first and exploiting its geometrical structure to estimate the respective parameters in the second stage using the MUSIC and root-MUSIC algorithms, respectively. As explained previously, we have to send the orthogonal pilots rather than data signals. The method works very well for rank-one channels but could be a reasonable approximation for higher rank channels. The channel can be estimated *via* matched filtering as $\hat{\boldsymbol{H}} = \boldsymbol{Y}\boldsymbol{S}^{\mathrm{H}}$.

Where the following approximation can be assumed for the dominant line-of-sight (LOS) channel as $\hat{\boldsymbol{H}} \approx \hat{\boldsymbol{b}}(\psi) \otimes \hat{\boldsymbol{a}}^{\mathrm{T}}(\mu)$ [3]. By exploiting the structure, the following rank-one optimization function can be minimized:

$$\left\{\hat{\boldsymbol{a}}(\mu), \hat{\boldsymbol{b}}(\psi)\right\} = \arg\min \left\| \hat{\boldsymbol{H}} - \boldsymbol{b}(\psi)\boldsymbol{a}^{\mathrm{T}}(\mu) \right\|_{\mathrm{F}}^{2}. \tag{8}$$

The problem could be solved *via* the Least Squares Kronecker Factorization [3]. Finally, we can estimate the angles by applying ESPRIT, Root-MUSIC, or MUSIC to the estimated AoD and AoA.

IV. Simulation Results

This section presents simulation results to analyze the performance of the ESPRIT, MUSIC, and Root-MUSIC algorithms to estimate the AoD and AoA in the presence of interference. We consider a single propagation path ($R = 1$), a unit complex gain ($\alpha_1 = 1$), and one interfering BS ($I = 1$) for ease of simplicity. The interference variance is fixed as unitary ($\sigma_i^2 = 1$), and the noise variance σ_n^2 is varied according to different SINR values. The angular parameters, i.e., $\theta = 10°$ (AoD), $\phi = 20°$ (AoA), $M = 12$ transmit antennas at the

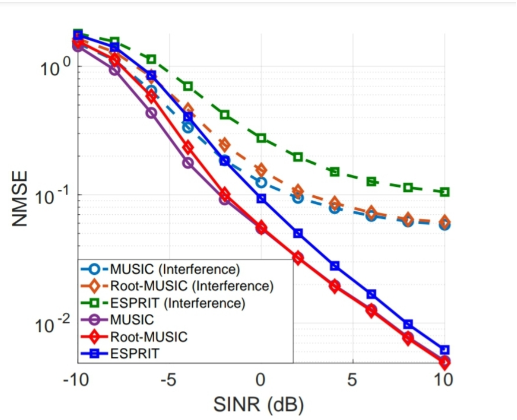

Fig. 2: NMSE of the estimated channel $\hat{\boldsymbol{H}}$ with and without interference.

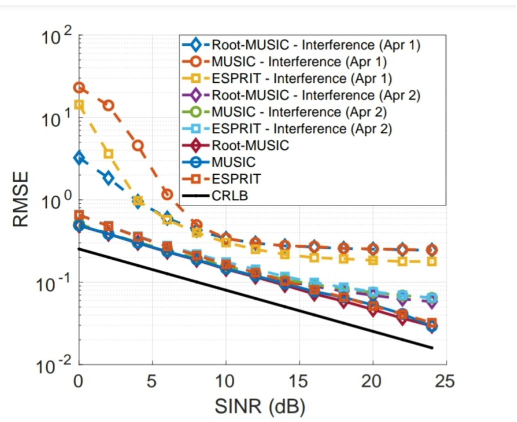

Fig. 3: RMSE of the AoA (ϕ) with and without interference.

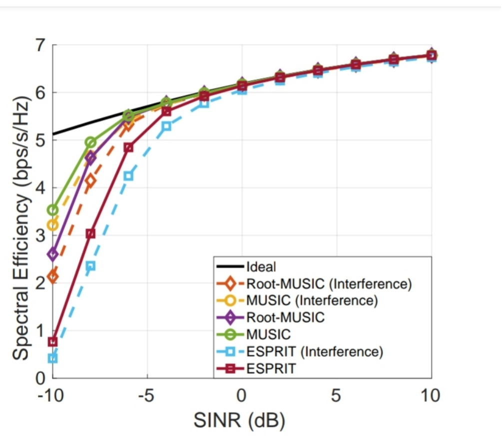

Fig. 4: Spectral Efficiency vs SINR values with and without interference.

BS, $Q = 12$ UE antennas, and transmit power $p_T = 1$ Watt. For the MUSIC algorithm, a spectral search is conducted in the range of $-30°$ to $30°$ with a step size of $0.1°$. All simulations were performed with 10,000 Monte Carlo runs. The signal-to-interference plus noise ratio (SINR) is defined as SINR $= \frac{P_t}{\sigma_n^2+\sigma_i^2}$.

The normalized mean square error (NMSE) metric is used to evaluate the accuracy of the estimated channel matrix $\hat{\boldsymbol{H}}$ across multiple Monte Carlo runs for the ESPRIT, MUSIC, and Root-MUSIC algorithms, allowing assessment of the impact of interference on channel estimation. Additionally, the root mean square error (RMSE) is employed to quantify the precision of the estimated angular parameters, such as AoA and AoD. Spectral efficiency (SE) is also analyzed to capture the system-level impact of the estimation accuracy. In this context, the precoder and combiner are designed based on the reconstructed channel obtained from the estimated angular parameters, using the SVD [3].

Figure 2 shows that interference degrades the performance of all estimation methods, increasing the NMSE. However, MUSIC and Root-MUSIC maintain nearly identical and robust performance, while ESPRIT is more affected by interference, indicating higher sensitivity to signal corruption. In Figure 4, the spectral efficiency (SE) is more impacted at low SINR, but the difference diminishes at higher SINR values, where performance approaches the ideal case. Figure 3 compares the RMSE of 'approach-1' (based on SVD) and 'approach-2' (a two-stage method combining matched filtering, rank-one approximation, and high-resolution algorithms). The second approach significantly improves RMSE due to noise rejection and correct angle pairing. Although none of the methods reach the Cramér-Rao Lower Bound (CRLB) [1], the two-stage method reduces the gap to this theoretical bound.

V. Conclusions

This paper assesses the performance of MUSIC, root-MUSIC, and ESPRIT algorithms for high-resolution parameter estimation in low-rank MIMO channels with interference. All methods showed robust performance in the presence of interference, while "approach-2" shows improved performance compared to "approach-1" due to the additional noise rejection in the first stage.

References

[1] Fazal-E-Asim, F. Antreich, C. C. Cavalcante, A. L. F. de Almeida, and J. A. Nossek, *"Two-dimensional channel parameter estimation for millimeter-wave systems using Butler matrices,"* IEEE Trans. Wirel. Commun, vol. 20, no. 4, pp. 2670–2684, 2021.

[2] E. Tuncer and B. Friedlander, *"Classical and Modern Direction-of-Arrival Estimation,"*. Ed. Elsevier, USA, 2009, ch. 1.

[3] Fazal-E-Asim, A. L. F. De Almeida, B. Sokal, B. Makki and G. Fodor, *"Two-Dimensional Channel Parameter Estimation for IRS-Assisted Networks,"* in IEEE Transactions on Communications, doi: 10.1109/TCOMM.2024.3522047.

[4] A. Vesa, *"Direction of Arrival Estimation using MUSIC and Root–MUSIC Algorithm,"* in Proc. 18th Telecommunications Forum (TELFOR), Belgrade, Serbia, Nov. 2010.

[5] N. P. Waweru, D. B. O. Konditi, e P. K. Langat, *"Performance Analysis of MUSIC, Root-MUSIC and ESPRIT DOA Estimation Algorithm,"* International Journal of Electronics and Communication Engineering, vol. 8, no. 1, pp. 209–216, Jan. 2014.