On the blind source separation of nonlinear mixtures

Alexandre Miccheleti Lucena, Kenji Nose-Filho, Ricardo Suyama

***Abstract*— In this paper we analyze the method proposed by Ehsandoust et. al for the blind source separation of nonlinear mixtures. Interestingly, the initially proposed method based on deriving the observed signals, performing an adaptive linear blind source separation, smooth the coefficients of the separation matrices and integrate the solution can be reduced, in some cases, by simply performing an adaptive linear blind source separation and smooth the coefficients of the separation matrices. Also, we extend the results for different sets of signals such as autoregressive signals and propose an alternative method, based on a General Regression Neural Network, for the smoothing of the Jacobian matrix.**

***Keywords*— Blind source separation, nonlinear mixtures, independent component analysis, nonlinear regression, autoregressive model.**

I. Introduction

Blind source separation can be considered one of the main problems within the realm of signal processing theory and has vast applications in various fields, such as audio processing, biomedical signals, and telecommunications, among others [1]–[3]. Essentially, the goal is to estimate signals of interest (the sources) from observations corresponding to an unknown mixture of the original signals, and different techniques have been developed to tackle this problem. They explore different characteristics of the signals and the mixing process, leading to well-established paradigms such as Independent Component Analysis (ICA) [4], Sparse Component Analysis [5], and Nonnegative Matrix Factorization [6], among others.

It is interesting to note, however, that most of the developed tools apply to linear mixtures, and the development of solutions for scenarios where the mixing process is nonlinear has been less intense and targeted at specific models. Among the commonly addressed models are the post-nonlinear model, the linear-quadratic model, and the exponential model [7].

One of the reasons why attention has been focused on the aforementioned nonlinear models is due to the fact that the statistical independence of the sources remains a guiding criterion for seeking solutions, thus allowing the framework developed for the linear context to be extensively reused.

In this sense, the work proposed by [8] introduced a new perspective for the separation of nonlinear mixtures by still exploring the concept of ICA. The idea, of course, does not apply to all types of nonlinearity, but rather to models where the distortion can be considered "smooth," and the sources also do not exhibit abrupt variations.

In this scenario, the mixing process can be locally approximated as a linear mixture. Thus, considering that the sources are independent, ICA can be applied to the different local mixtures so that, globally, the source signals would be recovered. The method presents interesting results, but the original work did not address some aspects related to its effectiveness for other types of nonlinearity, or even other types of sources.

Therefore, in the present work, a new study of this approach is conducted, aiming to verify under what conditions the method is effective and whether there are any simplifications that can be explored to improve the quality of the source estimates, thereby enabling its application in practical scenarios. We propose a modification of the BATIN algorithm [8] by including a General Regression Neural Network (GRNN) as an alternate to the nonlinear regression step and evaluate its performance.

With this objective in mind, the article is structured as follows. In Section II, we discuss the fundamentals of the nonlinear separation problem and detail the separation method based on linearization. In Section III, we elaborate on the specific modeling used for sources and nonlinear mixtures and algorithms to be considered in the simulations. In Section IV, we present and discuss the results of the simulations, concluding with some final considerations in Section V.

II. Nonlinear Blind Source Separation

The general formulation for the Nonlinear BSS (NLBSS) model can be written as

$$\mathbf{x}(t) = \mathbf{f}(\mathbf{s}(t)), \tag{1}$$

where $\mathbf{s}(t)$ is a time-varying vector of the source signal $\mathbf{s}(t) = [s_1(t), ..., s_n(t)]^\top$, $\mathbf{f}(.)$ is a nonlinear function from $\mathbb{R}^n$ to $\mathbb{R}^m$, and $\mathbf{x}(t) = [x_1(t), ..., x_m(t)]^\top$ is the resulting nonlinear mixture. Ideally, the separation in this case is to find a function $\mathbf{g}(.) = \mathbf{f}^{-1}(.)$ that is capable of reversing the effects caused by $\mathbf{f}(.)$, leading to estimated sources $\mathbf{y}(t)$ as in

$$\mathbf{y}(t) = \mathbf{g}(\mathbf{x}(t)). \tag{2}$$

Although the general formulation for the NLBSS model seems straightforward, there are characteristics regarding $\mathbf{f}(.)$ that need to be considered (e.g. invertible) for the source estimation to be possible, since nonlinear functions can appear in many forms. This may lead to different assumptions on $\mathbf{f}(.)$, resulting in different approaches depending on the kind of nonlinearity. This means that it might be difficult to propose a general algorithm for achieving source separation for all nonlinear mixing models.

One approach to the general NLBSS problem is to transform the model into a time-varying linear one. This is proposed and explained in detail in [8] and summarized in the sequence.

Alexandre Miccheleti Lucena, Kenji Nose-Filho and Ricardo Suyama are with the Engineering, Modeling and Applied Social Sciences Center, Federal University of ABC, Santo André, SP, Brazil, E-mails: alexandre.lucena@ufabc.edu.br, kenji.nose@ufabc.edu.br, ricardo.suyama@ufabc.edu.br. This work was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001, the National Council for Scientific and Technological Development (CNPq) grant #311380/2021-2, and the São Paulo Research Foundation (FAPESP) - grant #2020/09838-0 (BI0S - Brazilian Institute of Data Science).

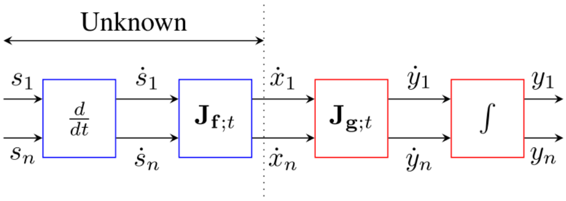

Fig. 1. Transforming the nonlinear BSS problem model to the linear time-variant one. [8].

A. Local linear approximation

Assuming $\mathbf{f}(.)$ is time-invariant, the mixing process is a fixed nonlinear mapping that transforms the sources into the observed mixture. Additionally, assuming it is a smooth mapping, one may approximate $\mathbf{f}(.)$ via Taylor expansion as:

$$\begin{aligned} &\mathbf{x}(t) = \mathbf{f}(\mathbf{s}(t)) \Rightarrow \\ &\forall t \;\; \mathbf{x}(t+\epsilon) = \mathbf{x}(t) + \frac{\partial \mathbf{f}}{\partial \mathbf{s}}(\mathbf{s}(t+\epsilon) - \mathbf{s}(t)) + \mathbf{o}(\epsilon) \end{aligned} \tag{3}$$

$$\Rightarrow \mathbf{x}(t+\epsilon) - \mathbf{x}(t) \approx \mathbf{J}_{\mathbf{f};t}(\mathbf{s})\Big|_{\mathbf{s}=\mathbf{s}(t)} (\mathbf{s}(t+\epsilon) - \mathbf{s}(t)) \tag{4}$$

$$\Rightarrow \Delta_{\mathbf{x}}(t) \approx \mathbf{J}_{\mathbf{f};t}(\mathbf{s})\Big|_{\mathbf{s}=\mathbf{s}(t)} \Delta_{\mathbf{s}}(t) \tag{5}$$

where $\mathbf{J}_{\mathbf{f};t}(\mathbf{s})$ is the Jacobian matrix of the nonlinear mixing function, $\mathbf{o}(\epsilon)$ represents Higher-Order Terms and $\Delta_{\mathbf{x}}(t)$ and $\Delta_{\mathbf{s}}(t)$ are the differences (increments) of the observation and source vectors respectively [8].

Notice that, under this assumption, the nonlinear time-invariant mixture can be seen as a linear time-variant one, as illustrated by Figure 1. Alternatively, the model becomes

$$\dot{\mathbf{x}} = \mathbf{J}_{\mathbf{f};t}(\mathbf{s})\dot{\mathbf{s}}, \tag{6}$$

with $\dot{\mathbf{x}}$ and $\dot{\mathbf{s}}$ denoting the time (or sample) derivatives of $\mathbf{x}$ and $\mathbf{s}$ respectively. Note that, although $\mathbf{s}(t)$ may vary over time, the Jacobian matrix does not directly depend on t: it actually depends on the values of $\mathbf{s}$, and the changes observed in the mixture $\mathbf{x}$ are due to different $\mathbf{s}$ and $\mathbf{J}_{\mathbf{f};t}$ pairs, hence a local linear instantaneous mixture model.

From the described model, it is easily seen (Figure 1) that sources can be estimated by reversing this local mixing process, as in

$$\dot{\mathbf{y}} = \mathbf{J}_{\mathbf{g};t}(\mathbf{x})\dot{\mathbf{x}}, \tag{7}$$

where $\mathbf{J}_{\mathbf{g};t}$ is the Jacobian (separation) matrix of $\mathbf{g}(.) = \mathbf{f}^{-1}(.)$ and $\dot{\mathbf{y}}$ is the estimated sources derivatives. Nevertheless, some additional assumptions have to be considered to correctly recover the signals.

The first assumption is that the function $\mathbf{f}$ is invertible, so $\mathbf{g}$ exists. As discussed above, $\mathbf{f}$ needs to be time-invariant and also memoryless; otherwise, its Jacobian would also vary over time. Lastly, the nonlinear function $\mathbf{f}$ and the sources $\mathbf{x}$ need to be differentiable (first-order) and have continuous derivatives. Since the model is meant to allow separation via ICA, other common assumptions may be considered, such as the number of the sources being equal to the number of the observations, the source derivatives must be mutually independent, and, at most, one of the derivatives of the sources follows the Gaussian distribution.

B. Separation Algorithm

The time-varying interpretation of the nonlinear function as a local linear mixture allows the use of ICA-based methods to recover the sources. However, since $\mathbf{J}_{\mathbf{g};t}$ is time-variant due to source variations, the ICA method used needs to be able to follow those variations (i.e. to be adaptive).

In [8], two general separation algorithms are proposed and investigated. The first one, the Adaptive Algorithm for Time-Invariant Linear Mixtures (AATVL), is a direct application of an adaptive ICA algorithm over the derivative of nonlinear mixed signals, following the exact nonlinear model described in Section II-A Although any adaptive ICA method can be applied, in [8] the authors choose the N-EASI (Normalized Equivariant Adaptive Separation via Independence) [9] as the adaptive ICA algorithm for the $\mathbf{J}_{\mathbf{g};t}$ matrix estimation.

The N-EASI algorithm [9], is a method for blind source separation based on the mutual independence of the sources. One of its main features relates to the equivariant estimation, ensuring that performance is independent of the specific mixing matrix and depends only on the source signal distributions. The algorithm employs adaptive serial updates, given in (8).

$$\begin{aligned} \mathbf{J}_{\mathbf{g};t+1} = \mathbf{J}_{\mathbf{g};t} - \lambda_t \Bigg[&\frac{\mathbf{y}(t)\mathbf{y}(t)^\dagger - \mathbf{I}}{1 + \lambda_t \mathbf{y}(t)^\dagger \mathbf{y}(t)} \\ &+ \frac{\mathfrak{h}(\mathbf{y}(t))\mathbf{y}(t)^\dagger - \mathbf{y}(t)\mathbf{h}(\mathbf{y}(t))^\dagger}{1 + \lambda_t \left|\mathbf{y}(t)^\dagger \mathbf{h}(\mathbf{y}(t))\right|} \Bigg] \mathbf{J}_{\mathbf{g};t} \end{aligned} \tag{8}$$

where λ_t is a sequence of positive adaptation steps and $\mathfrak{h}(\cdot)$ is an arbitrary component-wise (n-dimensional) nonlinear function.

The second algorithm is a modification of the AATVL algorithm, that includes a nonlinear regression step after the adaptive ICA. This is motivated by the fact that, since $\mathbf{f}$ and its inverse $\mathbf{g}$, are assumed to be time-invariant and smooth, the estimates of $\mathbf{J}_{\mathbf{g};t}$ obtained from the ICA algorithm can be used to approximate the *true* mapping that underlies the elements of $\mathbf{J}_{\mathbf{g};t}$, although it might be affected by estimation errors. In this sense, the Batch Algorithm for Time-Invariant Nonlinear mixtures (BATIN) apply a nonlinear regression step after all samples are processed by the adaptive ICA step (hence batch) to better approximate the $\mathbf{J}_{\mathbf{g};t}$ estimates. The nonlinear regression algorithm used by the authors in [8] at this step is the smoothing splines [10]. However, it is argued that the nonlinear regression can be replaced by different algorithms.

In [8] both algorithms (AATVL and BATIN) are presented as a proof of concept alongside simulation results. Nonetheless, there are still possible scenarios and modifications to be tested for a better understanding of this approach, which are investigated in this paper.

III. NLBSS MODEL INVESTIGATION

The local linear approximation and separation algorithm described in Sections II-A and II-B open space to several questions about the conditions under which the proposed approach is valid. Even though the required assumptions are satisfied, there are changes regarding the sources, mixture model, and the separation algorithm (such as the nonlinear regression step) that can impact the overall performance of the separation and must be investigated.

A. Source signal model

The smoothness assumption applies not only to the nonlinear mixing function but also to the source signals. This is due to the fact that the ICA algorithm must track variations in the local linear approximation model. Considering that the sources must be differentiable and have mutually independent derivatives, for the simulations it is easier to generate derivatives of the signals and then integrate them using the cumulative summation approximation. The sources used in the simulations were inspired in [8] following a sine wave

$$\dot{s}_1(t) = \sin(\sqrt{3}\omega t) \Rightarrow s_1(t) \propto \int \dot{s}_1 \; dt \;, \tag{9}$$

and a triangle (sawtooth) wave

$$\dot{s}_2(t) = \text{saw}(\omega t) \Rightarrow s_2(t) \propto \int \dot{s}_2 \; dt \;, \tag{10}$$

representing simple signals that satisfy the proposed assumptions needed for the separating model.

As an alternative to this model, we propose the usage of autoregressive (AR) source signals. To satisfy the smoothness condition imposed on the sources, similarly to the previous source, one may produce the derivatives of the sources following an AR(p) model (where p is the number o coefficients) of a slow varying signal, and then approximate the resulting source by cumulative summation. Although the result is not explicitly differentiable, a high enough coefficient can result in a smooth approximation of a random signal. In this sense, the AR source signals were generated using AR(1) following

$$\dot{s}_i(t) = a_i \dot{s}_i(t-1) + \epsilon(t) \Rightarrow s_i(t) \propto \int \dot{s}_i \; dt \;, \tag{11}$$

where a_i is the AR coefficient $\epsilon(t)$ is a random process that follows a normal distribution $p(\epsilon) \sim \mathcal{N}(0,1)$.

B. Nonlinear mixture model

For the application of the proposed method, the chosen nonlinear models must respect the previously established assumptions, i.e. time-invariant, memoryless, differentiable and invertible. In that sense, two nonlinear mixture models are explored in this work and described in the following sections.

1) Example 1: The first mapping is a nonlinear function that mixes the sources with a rotation. However, it becomes nonlinear as the rotation angle depends on the magnitude of the input vector. This mixture model is proposed in [11] and describes a nonlinear function of the sources based on a rotation matrix, and can be described as

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \cos\alpha\left(\mathbf{s}(t)\right) & -\sin\alpha\left(\mathbf{s}(t)\right) \\ \sin\alpha\left(\mathbf{s}(t)\right) & \cos\alpha\left(\mathbf{s}(t)\right) \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix}, \tag{12}$$

being the rotation angle determined by the magnitude of the source vector:

$$\alpha\left(\mathbf{s}(t)\right) = \sqrt{s_1(t)^2 + s_2(t)^2}. \tag{13}$$

Notice that, in this case, although the mixing process is nonlinearly related to the sources, the mixing model can be essentially understood as a time-varying linear mixture, i.e.,

$$\mathbf{x}(t) = \mathbf{A}_1(t)\mathbf{s}(t) \tag{14}$$

2) Example 2: The second mapping is based on the combination of exponential functions over the input sources, defining $\mathbf{f}(\mathbf{s}(t))$ in a way that

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \mathbf{f}(\mathbf{s}(t)) = \begin{bmatrix} e^{s_1(t)} - e^{s_2(t)} \\ e^{-s_1(t)} + e^{-s_2(t)} \end{bmatrix}. \tag{15}$$

Considering this mixing process, it is clear that it is not possible to consider it as a time-varying linear mixture, as in (14), but as a time-varying linear mixture of the derivatives:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} e^{s_1(t)} & -e^{s_2(t)} \\ -e^{-s_1(t)} & -e^{-s_2(t)} \end{bmatrix} \begin{bmatrix} \dot{s}_1(t) \\ \dot{s}_2(t) \end{bmatrix}, \tag{16}$$

$$\dot{\mathbf{x}}(t) = \mathbf{A}_2(t)\dot{\mathbf{s}}(t). \tag{17}$$

C. Nonlinear regression

The nonlinear regression step appears only in the BATIN algorithm and is a strategy to recover better overall estimates of the $\mathbf{J}_{\mathbf{g};t}$ matrix. The choice for the nonlinear regression algorithm is arbitrary, since there are many factors that can motivate this decision (e.g. complexity, number of parameters, etc.), and there is no discussion or comparison about this topic.

Considering its simplicity, we chose to apply the General Regression Neural Networks (GRNN) [12], since it have only one smoothing parameter that can be adjusted and control how fitted to the data the estimated function is. This is an interesting characteristic that was not explored in the original work. In the context of the BATIN algorithm, we can write the GRNN formulation as

$$[\mathbf{J}_{\mathbf{g}}(\mathbf{x})]_{ij} = \frac{\sum_{k=1}^{N} [\mathbf{J}_{\mathbf{g};k}]_{ij} K(\mathbf{x}, \mathbf{x}_k)}{\sum_{k=1}^{N} K(\mathbf{x}, \mathbf{x}_k)}, \tag{18}$$

where, $[\mathbf{J}_{\mathbf{g};k}]_{ij}$ is the ij coefficient of the separating Jacobian matrix estimates, $K(\mathbf{x}, \mathbf{x}_k)$ is a radial basis function kernel, here chosen as

$$K(\mathbf{x}, \mathbf{x}_k) = e^{-\mathbf{d}_k / 2\sigma^2}, \tag{19}$$

that is a Gaussian transfer function (Gaussian kernel), where σ controls the standard deviation (width) of the Gaussian and defines the neighborhood of influence on the data, and therefore becomes a smoothing parameter. Also, $\mathbf{d_k}$ is the squared Euclidean distance between a specific mixture sample vector $\mathbf{x}_k$ that needs to be evaluated and the mixture data expressed as

$$\mathbf{d}_k = (\mathbf{x} - \mathbf{x}_k)^\top (\mathbf{x} - \mathbf{x}_k). \tag{20}$$

IV. Simulation Results

To assess the method, we perform different simulations to investigate the proposed method concerning the source signals, the mixing model, the smoothness of the nonlinear regression, and also how the ICA on the source derivative performs in comparison to the direct ICA algorithm, i.e., ignoring the nonlinearity.

For this purpose, two sets of sources were generated with 5000 samples each. The first set, used in results of Section IV-A and Section IV-C follows the sine and triangle source derivatives described in (9) and (10) with $\omega = 1/100$. The second set follows the AR model described in (11), and is used in Section IV-B.

In this context, it becomes important to not only evaluate the specific local linear approximation method for nonlinear mixtures but also evaluate the direct application of ICA methods (i.e. N-EASI and fastICA [13]) on the nonlinear task as a reference. Results for the linear algorithm are shown for all simulations.

As the BATIN algorithm benefits from a smoothing regression step, we considered applying the GRNN algorithm on the N-EASI estimates. The results of this method will be reffered to as N-EASI$_G$ differing from the regular N-EASI (i.e. direct application of a linear adaptive ICA to the nonlinear mixture). Also, to distinguish the original BATIN that uses the smoothing spline from our approach using the GRNN as the nonlinear separation step, the proposed variation will be referenced as (BATIN$_G$).

The quality of the recovered source signals was calculated using the Signal-to-Interference Ratio (SIR) a quantitative performance measurement. In summary, the SIR evaluates the ratio between the energy of the target source s_i and the energy of the residual interference signal (i.e. difference between estimated and true signal) $e_i[k] = \hat{s}_i[k] - s_i[k]$ as in

$$\text{SIR}_i := 10 \log_{10} \frac{\sum_k s_i^2[k]}{\sum_k e_i^2[k]} . \tag{21}$$

A. Smoothing parameter evaluation

The nonlinear regression step plays an important role in the local linear approach as the BATIN algorithm has a better performance compared to AATVL. However, many algorithms can achieve nonlinear regression using different strategies, resulting in more or less proximity to the reference data. As a representative algorithm that can be easily implemented and has simple interpretation we chose to evaluate the GRNN algorithm to test different degrees of data fitting. The smoothing parameter of the GRNN algorithm using a Gaussian kernel is defined by σ, and it can be adjusted to set the amount that the regression model is fitted to the adaptive ICA estimates.

The sine and triangle sources ((9) and (10)) were mixed using the nonlinear model (12). Different algorithms were used to separate the sources and the resulting signals were evaluated in terms of SIR. In this scenario, as discussed in Section III-B.1, the nonlinear mapping (12) can be seen as a time-varying linear mixture, so the direct application of the N-EASI algorithm becomes relevant.

Figure 2 shows the resulting SIR calculated for different smoothing parameters for the methods using the GRNN nonlinear regression. Other algorithms that do not rely on the GRNN and hence don't have a smoothing parameter are shown as constants. It is possible to notice that both methods relying on the GRNN regression have a better overall performance than the other methods. Although only BATIN achieved relevant SIR values, in this scenario it is evident that adaptive ICA combined

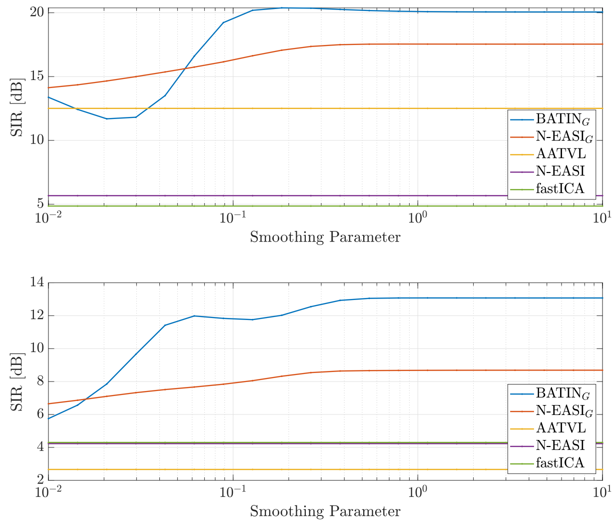

Fig. 2. SIR values for the separated smooth sinusoidal sources ($\omega = 1/100$) s_1 (top) s_2 (bottom), mixed with nonlinear model (12), for different smoothing parameters of the GRNN algorithm.

with the nonlinear regression obtained better results than the direct adaptive ICA (N-EASI) and the pure local approximation (AATVL). This evidence that smoothing estimates is a relevant step. As expected, the regular ICA algorithms performed poorly on the separation task.

Another relevant result was obtained by repeating the same experiment by only changing the frequency of the sources to $\omega = 1/25$. This change also indirectly affects the smoothness of the sources. In this scenario the SIR values obtained for the BATIN$_G$ and N-EASI$_G$ for the s_1 where of the order of 25 dB and 31 dB respectively and for s_2 both achieved the order of 21 dB, almost constant for all the smoothing parameter range, indicating that source smoothness is a factor to be taken into account. As the values are all constant, and for saving space on this paper, this scenario will not be illustrated. In this case, the initially proposed method based on deriving the observed signals, performing an adaptive linear blind source separation, smoothing the coefficients of the separation matrices, and integrating the results to obtain the estimated signals, can be reduced by simply performing an adaptive linear blind source separation and smooth the coefficients of the separation matrices.

B. Autoregressive sources

In order to test the method with a different type of source, we replicate the previous experiment considering AR sources. As described by equation (11), the sources were generated considering an AR(1) model, with $a_1 = a_2 = 0.99$, to ensure a slowly varying signal. As in the previous section, the nonlinear mixture model used was (12). As for the sinusoidal sources, Figure 3 shows the resulting SIR calculated for different smoothing parameters for the methods that use the GRNN nonlinear regression. By analyzing the results, it is interesting to note that, even though most algorithms were not able to achieve high SIR values, the BATIN$_G$ algorithm could recover the sources. In this case, it is not possible to claim that the smoothing parameter influenced the algorithm's performance.

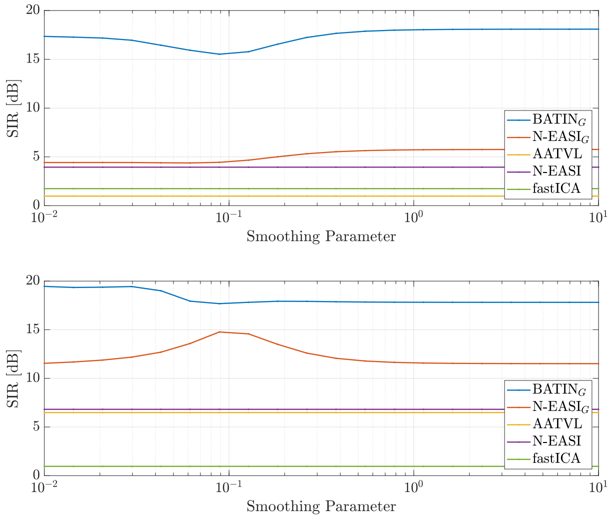

Fig. 3. SIR values for the separated smooth AR(1) sources s_1 (top) s_2 (bottom), mixed with nonlinear model (12), for different smoothing parameters of the GRNN algorithm.

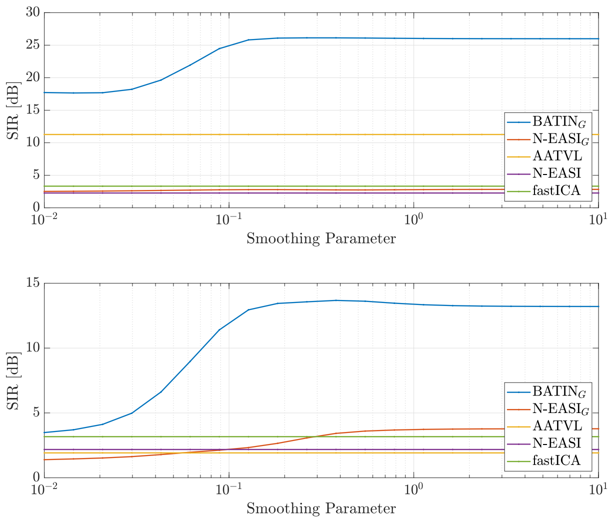

Fig. 4. SIR values for the separated smooth sinusoidal sources s_1 (top) s_2 (bottom), mixed with nonlinear model (15), for different smoothing parameters of the GRNN algorithm.

C. Exponential mixture model

In this set of simulations, the linear and nonlinear methods used in IV-A are tested in a similar manner on another nonlinear mixture model described in (15). As discussed in Section III-B.2, this model cannot be described as a time-variant linear mixture and the separation may rely solely on the local linear approach. As previously shown, Figure 4 shows the resulting SIR calculated for different smoothing parameters for the methods that use the GRNN nonlinear regression. Results confirm that the only method to achieve the separation with higher values of SIR is the BATIN_G. However, the local linear approximation is not sufficient since the AATVL algorithm also performed poorly. Other algorithms were expected to have bad performance as they are linear methods.

V. Conclusions

In this paper, we investigate a general approach to the NLBSS problem based on a local approximation of the model, also exploring time-varying techniques. On this matter, we propose a modification of the BATIN algorithm on its nonlinear regression step by using the GRNN algorithm (BATIN_G), leading to the best performance in most of the tested scenarios. Simulations showed that the usage of the GRNN as a smoothing algorithm improved the source separation performed even when applied without the local approximation in some cases. Moreover, a different type of smooth source based on AR(1) signals is considered, showing that BATIN_G was able to perform the separation, hence opening space to considering random processes in this kind of modeling.

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