# A Robust MERRY Channel Shortening Technique for OFDM Systems in PLC Environment Corrupted by Impulsive Noise

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***Abstract*— This contribution introduces a robust version of the multicarrier equalization by restoration of redundance (MERY) algorithm for data communication through power line communication (PLC) channel corrupted by impulsive noise. The novelty is the introduction of a nonlinear and differentiable function into the cost function of MERRY algorithm. As a result, the proposed and named robust (R)-MERRY algorithm is less sensitive to the presence and hardness of impulsive noise. Simulation results reveal that bit error rate (BER) performance improvements can be attained in PLC scenarios.**

## I. Introduction

Broadband access in last miles is a great challenge in developing and underdeveloped countries due to the limited telecom infrastructures. In this scenario, *power line communications* (PLC) as well as wireless communication appear as interesting solutions to overcome the lack of capillarity.

The cost for PLC deployment can be reduced because the infrastructure available for transmission and distribution of electric energy can be used by PLC technology as medium for data communication [1]. However, as the power grids were not dimensioned for data communication, the impairments found in these mediums requires sophisticated digital signal techniques.

The main modulation scheme that has been chosen to overcome impairments, such as frequency selectivity, is a multicarrier one named *orthogonal frequency-division multiplexing* (OFDM). Besides the robustness to multipath channel, the OFDM presents low-cost if implemented in a *very large scale integrated circuit* (VLSI) devices. As a result, many standards for broadband communications, such as WiFi (IEEE802.11a/g/n), WiMAX (IEEE 802.16e), Digital Subscriber Loop (xDSL), PLC (HomePlug) and so on are employing this kind of multicarrier modulation scheme.

The PLC channel is composed of several branches in low voltage circuits, each one with a different terminal impedance. Therefore, the signal transmitted through the channel might suffer reflections. Therefore, the PLC channel is very dispersive in time domain or has very long impulse response as showed in the model proposed in [2]. In order to mitigate *inter-symbol interference* (ISI) at dispersive PLC channel, OFDM modulation employs the *cyclic prefix* (CP), which is a copy of the last $\nu$ samples of $N$-size OFDM symbol that is added at the beginning of the OFDM symbol so that the final block has $M = N + \nu$ samples. If $\nu \geq L_h - 1$, in which $L_h$ is the length of the channel impulse response, the convolution between the channel and the OFDM symbol become circular. Therefore, the effect of ISI caused by PLC channel channel become a simple complex gain in the frequency domain [3]. Thus, the ISI effect can be eliminate by using a *frequency equalizer* (FEQ) technique.

There is a trade-off between the length of CP and the length of OFDM symbol. Given that the CP is a redundant information, the efficiency of OFDM-based system can be severely reduced if the length of the CP tends to be equal to the length of OFDM symbol. One possibility to overcome this drawback is the use of *time-domain equalizer* (TEQ) for shortening the equivalent impulse response, which is a result of the convolution between the TEQ and the PLC channel.

There are a lot of techniques designed so far to shorten the time response of equivalent channel [3]. Among them, blind ones are attractive because any training data is required. Taking into account only blind and adaptive techniques, *sum-squared auto-correlation minimization* (SAM), which attempts to minimize the auto-correlation at the output of the TEQ was introduced in [4]. Recently, the *multi-carrier equalization by restoration of redundance* (MERRY) technique was introduced in attempt to restore the redundance of CP [5]. In general, techniques applied for channel shortening are designed for dispersive channels in the presence of the white Gaussian noise and therefore are not robust to impulsive noise, as found in PLC channels.

To offer new directions for the design of channel shortening techniques capable of working under the presence of impulsive noise, this contribution presents a robust version of MERRY (R-MERRY) algorithm. Basically, a nonlinear and differentiable function is introduced into the criterion of the MERRY algorithm. As a result, the new deduced algorithm is less sensible to the presence of impulsive noise. Computational simulations reveal that the R-MERRY surpasses, in terms of *bit error rate* (BER), the performance of the MERRY algorithm when the PLC channels is corrupted by impulsive noise.

This paper is organized as follows. Section II discusses the system model. Section III presents MERRY and the proposed R-MERRY algorithms. Section IV assesses the algorithms performance regarding convergence rate and BER. Finally, Section V points out some conclusions and outline further works.

## II. System Model

The baseband model for a OFDM-based system is shown in Fig. 1. As we can see, it is a *single input single output* (SISO) system. Each of the $N$ sub-carriers modulates a *binary phase shift key* (BPSK) symbol. The modulation is performed via *inverse fast Fourier transform* (IFFT) at the transmitter side while the demodulation is accomplished via *fast Fourier transform* (FFT) at receiver side.

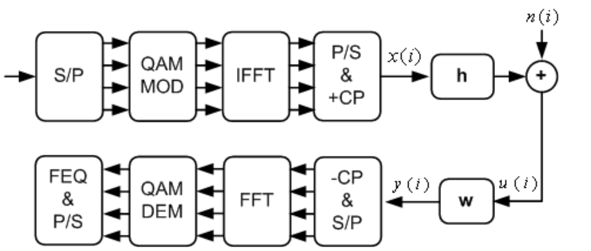


Fig. 1: The diagram of the baseband system model.

After the CP insertion, the last $\nu$ samples are identical to first $\nu$ samples in the $k$th symbol, i.e.:

$$x\left(Mk+i\right) \;=\; x\left(Mk+i+N\right) \quad i \in \{0,\, ...,\, \nu-1\}\,. \tag{1}$$

The $i$th received data is modeled by

$$u(i) = \mathbf{h}^T\mathbf{x}(i) \;+\; n(i), \tag{2}$$

where $\mathbf{h} = [h(0)\;\; h(1)\;\; \ldots\;\; h(L_h-1)]^T$ is the impulse response of the PLC channel, $\mathbf{x}(i) = [x(i)\;\; x(i-1)\;\; \ldots\;\; x(i-L_h+1)]^T$ is the transmitted signal and $n(i)$ is the additive noise, which is added to the channel output.

The channel model of a deterministic and time-invariant PLC channel based on multipath propagation model [2], a kind of top-down modeling approach, has its frequency response expressed by

$$H(f) = \sum_{i=1}^{P} \underbrace{|g_i\left(f\right)| \cdot e^{\varphi g_i(f)}}_{Weight} \cdot \underbrace{e^{-\left(a_0+a_1 f^k\right)d_i}}_{Attenuation} \cdot \underbrace{e^{-j2\pi f\tau_i}}_{Delay}, \tag{3}$$

in which $f \in \Re^+$, $P$ is the number of multipaths, $g_i\left(f\right)$ is the complex gain of the $i$th path, $a_0$ and $a_1$ are the attenuation parameters, $d_i$ and $\tau_i$ are the distance and delay parameters associated with the $i$th path. The attenuation profile of three PLC channels, which are modeled by (3) and presented in [6] are illustrated in Fig. 2. These PLC channels are typical for outdoor low-voltage electric power grids.The PLC channels #1, #2 and #3 represent a weakly frequency

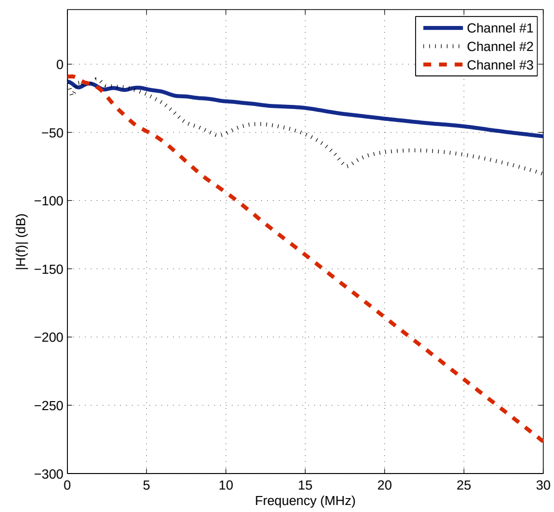


Fig. 2: Spectrum attenuation profile of three PLC channels.

attenuated, medium frequency attenuated, and heavily frequency attenuated electric power grids, respectively.

By taking [7] into account, the additive noise is given by

$$n(t) = n_{bkgr}(t) + n_{nb}(t) + n_{pa}(t) + n_{ps}(t) + n_{imp}(t), \tag{4}$$

where $n_{bkgr}(t)$ is the background noise, $n_{nb}(t)$ is a narrowband noise, $n_{pa}(t)$ is a periodical impulsive noise asynchronous to the fundamental component of power system, $n_{ps}(t)$ is a periodic impulsive noise synchronous to the fundamental component of power system, and, finally, $n_{imp}(t)$ is an asynchronous impulsive noise which is the hardest one. In this work, in order to assess the robustness of the algorithms we have considered only the background and asynchronous impulse noise.

In this contribution the background noise is modeled as a white Gaussian one with power spectrum density given by $N_0$. The asynchronous impulse is modeled as the sum of $I_s$ damped sinusoidal components that is expressed by [8]

$$\begin{aligned} n_{ps}\left(t\right) = &\sum_{i=1}^{I_s} A_i \sin\left(2.\pi f_i \left(t - t_{arr,s}\right) + \alpha_i\right) \times \\ &e^{-\frac{t-t_{arr,s}}{\tau_i}} \prod\left(\frac{t-t_{arr,s}}{t_{w,s}}\right), \end{aligned} \tag{5}$$

where $f_i$ is the pseudo-frequency of the sinusoid and $\alpha_i$ the phase of the $i$th damped sinusoid. $\prod(t)$ is defined as a square pulse of $t_{w,s}$ duration in second, $t_{arr,s}$ is arrival time which was modeled by bipartite Markov's chain [7], $A_i$ denotes the amplitude of the $i$-th sinusoid. The term $\tau_i$ denotes the damping factor which was modeled as an exponential random variable with mean equal to 100 ns.

The equalized data, fed to the demodulating FFT, is given by

$$y(i) = \mathbf{w}^T\mathbf{u}(i), \tag{6}$$

where $\mathbf{w} = [w(0)\ w(1)\ \ldots\ w(L_w-1)]^T$ is the TEQ, $\mathbf{u}(i) = [u(i)\ u(i-1)\ \ldots\ u(i-L_w+1)]^T$ is the output channel vector and $L_w$ is the TEQ length. The effective or equivalent channel is given by $\mathbf{c} = \mathbf{h} \star \mathbf{w}$, where $\star$ denotes linear convolution.

## III. Proposed Algorithm

The cost function of the MERRY algorithm is given by [5]

$$J_M(\mathbf{w},\Delta) = E\left\{|y(i+\Delta) - y(i+N+\Delta)|^2\right\}, \quad (7)$$

where $\mathbf{w}$ is the TEQ and $\Delta$ is a delay corresponding to the boundaries between successive OFDM blocks after equalization.

The MERRY algorithm is obtained by taking the stochastic gradient descent of cost function expressed by (7). Then, the adaptive MERRY algorithm is as follows:

For symbol $k = 0, 1, 2, \cdots$,

$$\begin{aligned}
\underline{\mathbf{u}}(k) &= \mathbf{u}(Mk+\nu-1+\Delta) - \mathbf{u}(Mk+\nu-1+\Delta+N),\\
e_M(k) &= \mathbf{w}^T(k)\underline{\mathbf{u}}(k),\\
\widehat{\nabla} J_M(k) &= e_M(k)\underline{\mathbf{u}}^*(k),\\
\mathbf{w}(k+1) &= \mathbf{w}(k) - \mu_M \widehat{\nabla} J_M(k),\\
\mathbf{w}(k+1) &= \frac{\mathbf{w}(k+1)}{\|\mathbf{w}(k+1)\|},
\end{aligned} \quad (8)$$

where $\mathbf{w}(\cdot)$ and $\mathbf{u}(\cdot)$ are defined in (6).

The MERRY algorithm is updated for each OFDM symbol. If a burst of impulsive noise corrupts the channel output, then the value of $e_M(k)$ will increase considerably. As a result, the convergence of the MERRY algorithm will be severally degraded as illustrated by the $e_M(k)$ curve depicted in Fig. 3. In fact, this curve shows that the $e_M(k)$ is very sensitive to the presence of a impulsive noise. The conclusion is that the MERRY algorithm can not be useful for PLC system.

This weakness of the MERRY algorithm is alleviated by the proposed R-MERRY algorithm discussed in Section III.A.

### A. Robust-MERRY Algorithm

To make algorithms based on minimization of squared error more robust against to impulsive noise, some authors have applied nonlinear functions to limit or saturate the value of instantaneous squared error [9]. Among these nonlinear functions, the hyperbolic tangent ($\tanh(\cdot)$) seems to be very reasonable one because it is differentiable and can return a value in the interval $-1 < x < 1, \forall x \in \Re$. Applying $\tanh(\cdot)$ in (7), one obtain,

$$J_R(\mathbf{w},\Delta) = E\left\{\tanh(|y(i+\Delta) - y(i+\Delta+N)|^2)\right\}, \quad (9)$$

which is the cost function from which the R-MERRY algorithm is derived.

The adaptive algorithm is obtained by applying the stochastic gradient descent in (9) and a power constraint into the equalizer taps in order to avoid trivial solution ($\mathbf{w} = \mathbf{0}$) [3]. The R-MERRY algorithm is as follows:

For symbol $k = 0, 1, 2, \cdots$,

$$\begin{aligned}
\underline{\mathbf{u}}(k) &= \mathbf{u}(Mk+\nu-1+\Delta) - \mathbf{u}(Mk+\nu-1+\Delta+N),\\
e_R(k) &= \mathbf{w}^T(k)\underline{\mathbf{u}}(k),\\
\widehat{\nabla} J_R(k) &= \tanh(|e_R(k)|)\left[1-\tanh(|e_R(k)|)\right]^2 e_R(k)\underline{\mathbf{u}}^*(k),\\
\mathbf{w}(k+1) &= \mathbf{w}(k) - \mu_R \widehat{\nabla} J_R(k),\\
\mathbf{w}(k+1) &= \frac{\mathbf{w}(k+1)}{\|\mathbf{w}(k+1)\|}.
\end{aligned} \quad (10)$$

The improved offered by R-MERRY algorithm is revealed by $e_R(k)$ curve illustrated in Fig. 3. Note that the $|e_R(k)| \ll |e_M(k)|$ when impulsive noise occurs. Additionally, the value of $\widehat{\nabla} J_R(k) \to 0$ when the channel output is corrupted by the impulsive noise. It means that the R-MERRY algorithm will not be updated during impulsive noise occurrence. As a result, we can conclude that the R-MERRY algorithm is more robust then MERRY one.

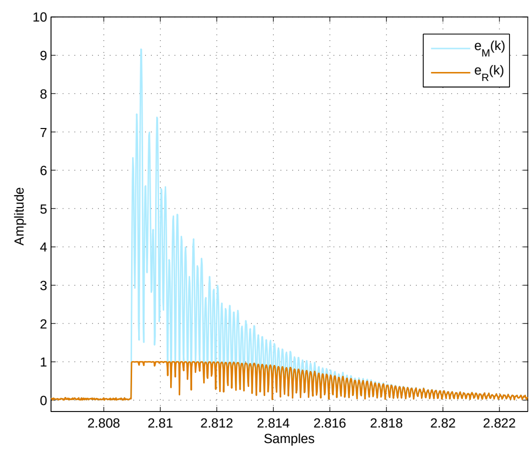


Fig. 3: Behavior of $e_M(k)$ and $e_R(k)$ when impulsive noise is added to the output of PLC channel during data communication by using OFDM system discussed in Section II.

## IV. Simulations Results

The performance comparisons between MERRY and R-MERRY algorithms are in terms of convergence rate and BER versus $E_b/N_0$ curves. Perfect synchronization and complete channel knowledge at the receiver are assumed. PLC channels and additive noise are described in Section II. Table I lists the main parameters considered in the simulations. The TEQ of MERRY and R-MERRY were initialized with the center spike approach, $\mathbf{w} = [0\ 0\ \cdots\ 1\ \cdots 0\ 0]^T$. The $E_b/N_0$ refers to the ratio between bit energy and $N_0$, which stand for the power spectrum density of the background noise.

For evaluation of the convergence rate of R-MERRY and MERRY algorithms, the channel #1 was considered, whose

TABLE I: System parameters of the OFDM system.

| Parameters | Values |
|---|---|
| Number of tones | 512 |
| Sampling frequency | 60 MHz |
| Subcarrier spacing | 59.594 kHz |
| Channel bandwidth | 30 MHz |
| Cyclic prefix | 1/32 |
| Modulation | BPSK |
| Channel knowledge at the receiver | Available |

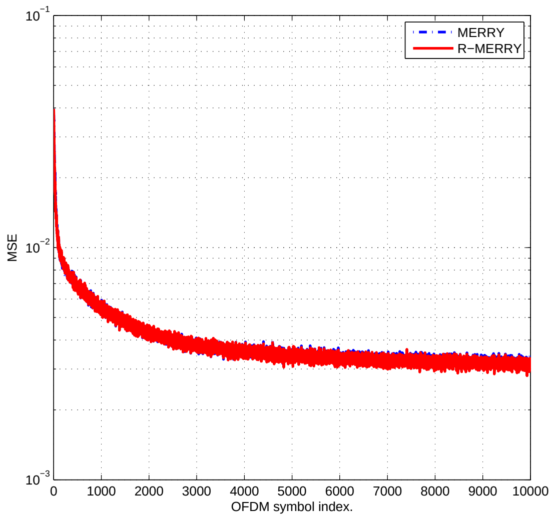


Fig. 4: MSE estimations.

amplitude spectrum is shown in Fig. 2. The parameters such as length $L_w$ of TEQ, delay $\Delta$ as well as the step-size, $\mu$, were setting such that the algorithms converged to the same level of the *mean squared error* (MSE). The values of these parameters are in Tab. II. It is assumed that the additive noise is only the background one, which is a white Gaussian one, $\mathcal{N}(0, \sigma^2)$.

As we can see in Fig. 4, both algorithms present the same convergence rate whether suitably initialized. For getting this results, it was considered that additive noise is white Gaussian one, $\mathcal{N}(0, \sigma^2)$, so that $E_b/N_0 = 35$ dB. In this scenario, the algorithms took roughly $4000$ OFDM symbols to achieve the convergence. After their initial convergence period, both algorithms are capable of tracking channel variations. One can point out that the slow convergente behavior can become a problem in fast time-variant channel. However, the authors have assessed that it is possible to achieve the convergence rate with both algorithms using as few as $50$ OFDM symbols.

The performance assessment in terms of BER considered the three PLC channels in Fig. 2. The additive noise employed in this evaluation were a background noise modeled by the white Gaussian noise and the asynchronous impulsive

TABLE II: Parameters for convergence analysis.

| Parameters | R-MERRY | MERRY |
|---|---|---|
| $Eb/N_0$(dB) | 30 | 30 |
| $L_w$ | 39 | 39 |
| $\Delta$ | 25 | 25 |
| $\mu$ | 0.1 | 0.011 |

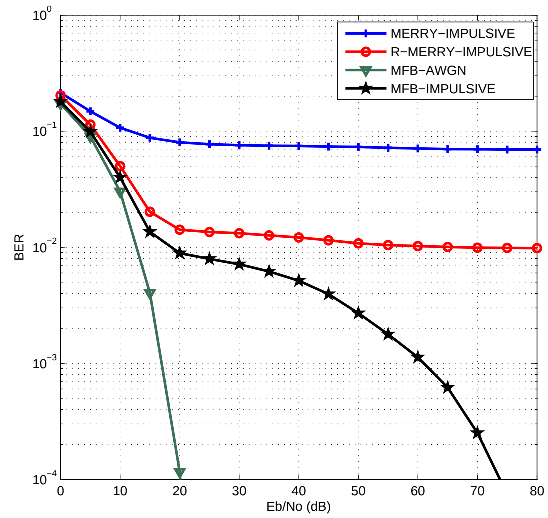


Fig. 5: BER for PLC channel #1.

noise modeled as in (5). Both algorithms were training with PLC channel with $E_b/N_0 = 35$ dB on the presence of impulsive noise with power $40$ dB above the background noise. After the convergence, the TEQ was employed as a channel shortening and, then, BER was measured. The power of impulsive noise as high as $40$ dB characterize a very hostile environment. This was chosen to highlight the robustness of the propose technique in PLC channel corrupted by impulsive noise. Around $10000$ OFDM symbols were transmitted for BER measurement.

The terms MERRY-IMPULSIVE, R-MERRY-IMPULSIVE, MFB-AWGN and MFB-IMPULSIVE stand for MERRY algorithm corrupted by impulsive plus background noise, R-MERRY algorithm corrupted by impulsive plus background noise, matched filter corrupted by background noise, and matched filter corrupted by impulsive plus background noise, respectively.

As we can see in Figs. 5, 6 and 7, the propose R-MERRY algorithm outperforms the MERRY one in all scenarios. For the channel $\#1$, the propose technique is close to *matched filter bound* (MFB) for the BER higher than $2 \times 10^{-2}$.

For the PLC channel $\#2$, the performance of MERRY algorithm is completely affected because in this scenario the burst of impulsive noise that hit the signal during the algorithms training process does not let the MERRY to converge. On the other hand, the propose technique is little affected by such a impulsive noise because during its

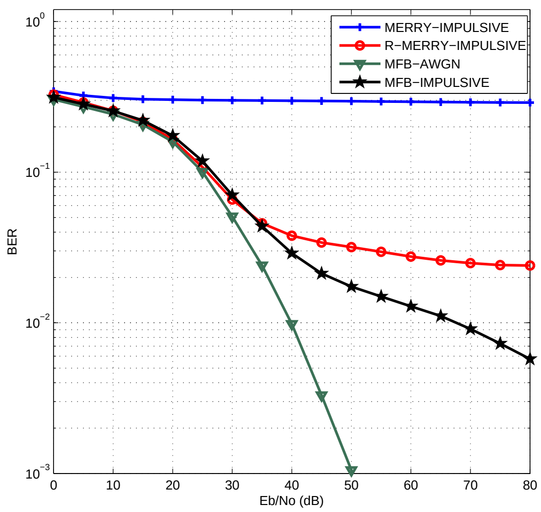


Fig. 6: BER for PLC channel #2.

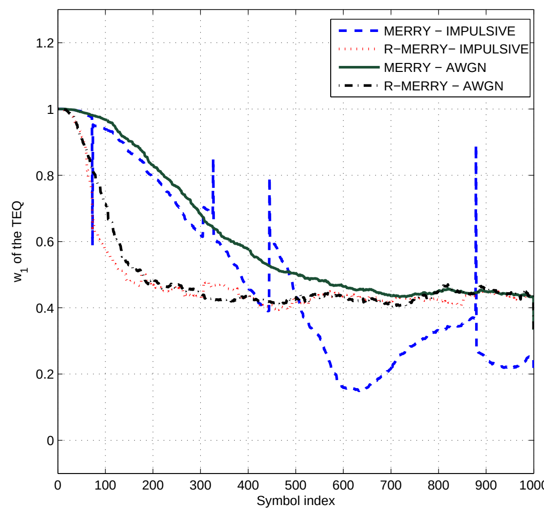


Fig. 8: Tap convergence.

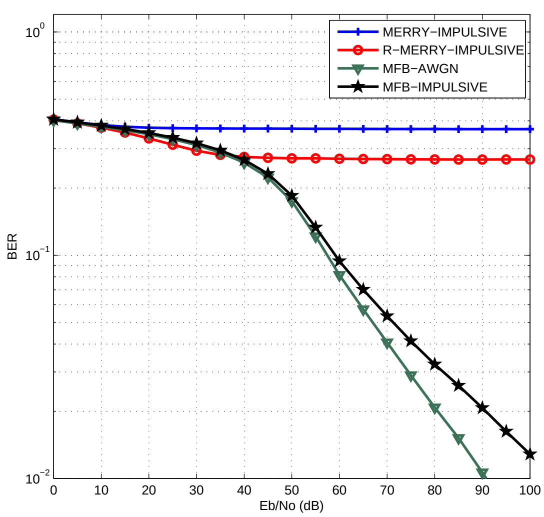


Fig. 7: BER for PLC channel #3.

occurrence, the gradient of the R-MERRY is equal to zero, and, therefore the adaptation of TEQ does not take place, as discussed at the end of Section III.A.

As expected, in the PLC channel #3, both techniques present weak performance due to the strong attenuation of this PLC channel. Nevertheless, the proposed technique outperforms the MERRY one.

In Fig. 8 is shown the convergence of one tap of the TEQ when PLC channel is corrupted by additive white Gaussian noise (AWGN) as well as additive impulsive plus white Gaussian noise (AIWGN). The propose R-MERRY is little affected by the burst of the impulsive noise. On the other hand, the MERRY algorithm leads the tap to another value and only some OFDM symbols after the impulsive noise occurrence, the tap cames back to its steady state value.

## V. Conclusion

This contribution introduced a robust MERRY algorithm for multicarrier system based on OFDM aiming at high-speed data communication through PLC channel, which is corrupted by impulsive noise. Regarding the convergence rate, both MERRY and R-MERRY algorithms present the same performance. It means that R-MERRY algorithms offers a slow convergence rate. Therefore, the R-MERRY algorithm demands many OFDM symbols to achieve its convergence, what can be a problem whether the coherence time of channel is shorten than the number of OFDM symbol needed for algorithm convergence. In PLC channel, it will not be a huge problem because only in a few situation the PLC channel suffers abrupt changes. Nevertheless, after achieved the convergence, the proposed algorithm is capable of tracking channel variations.

As far as the BER performance is concerned, the propose R-MERRY algorithm outperforms the MERRY in impulsive noise scenarios.

The improvement offered by R-MERRY is paid by increasing the algorithm complexity. Fortunately, this complexity can be reduced by approximating the $\tanh(\cdot)$ with a linear function for small values of the arguments and clipping the output for large values of arguments or using a look-up table with the values of the $\tanh(\cdot)$.

As a further study, the investigation of new functions for clipping the squared error and the derivation of new algorithms are important investigation questions. Another interesting direction of research is the design of algorithms based on second derivative information to find out fast adaptive algorithms for channel shortening.

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## REFERENCES

[1] A. Haidine, H. Hrasnica, R. Lehnert, *Broadband Powerline Communications Networks : Network Design*, John Wiley & Sons Inc., 2004.

[2] M. Zimmermann, and K. Dostert, "A Multipath model for the powerline channel," *IEEE Trans. on Communications*, vol. 50, no 4, pp. 553-559, Apr. 2002.

[3] R. K. Martin, C. R. Johnson Jr., "Adaptive equalization: transitioning from single-carrier to multicarrier systems," *IEEE Signal Processing Magazine*, vol. 22, no 6, pp. 108 - 122, Nov. 2005.

[4] Balakrishnan, J. and Martin, R.K. and Johnson, C.R., Jr., "Blind, adaptive channel shortening by sum-squared auto-correlation minimumization (SAM)," *IEEE Transactions on Signal Processing*, vol. 51, no 12, pp. 3086-3093, Dec. 2003.

[5] R. K. Martin, J. Balakrishnan, W. A. Sethares, and C. R. Johnson Jr., "Blind, adaptive channel shortening for multicarrier systems, in *Proc. IEEE Asilomar Conference on Signals, Systems & Computers*, vol. 1, pp. 372 - 376, Nov. 2002.

[6] P. J. Langfeld, "The capacity of typical powerline reference channels and strategies for system design," *in Proc. of the 5th International Symposium on Power-Line Communications and its Applications ISPLC-2001*, Malmö, Sweden, pp. 271-278, Apr. 2001.

[7] M. Zimmermann, and K. Dostert, "Analysis and modeling of impulse noise in broad-band powerline communications," *IEEE Trans. on Electromagnetic Compatibility*, vol. 44, no 1, pp. 249-258, May 2002.

[8] R. Hormis, I. Berenguer, and X. Wang, "A simple broadband transmission scheme for power line channels," *IEEE Selected Areas on Communications*, vol. 24, no 7, pp. 1351-1363, Jul. 2006.

[9] E. Soria-Olivias, J. D. Martín-Guerrero, A. J. Serrano-Lopez, J. Calpe-Maravilha, and J. Chanmbers, "Steady-state and tracking analysis of a robust adaptive filter with low computational cost," *Signal Processing*, vol. 87, pp. 210-215, 2007.

[10] S. Haykin, *Blind Deconvolution*, Prentice-Hall, 1994.