Highly Accurate Approximations for the Sum of Double-Nakagami-m Envelopes

Maria Cecilia Luna Alvarado, Lenin Patricio Jiménez Jiménez, Gustavo Fraidenraich and Michel Daoud Yacoub

Abstract— **The sum of cascade envelopes is prevalent in a diverse range of applications, techniques, and scenarios within wireless communication systems. However, the intricate nature of the sum renders it challenging to obtain an exact statistical characterization, fostering a growing interest in the pursuit of precise yet approximate formulations. In this work, we provide simplified yet accurate approximations for the probability density function and cumulative distribution function of the sum of independent but not identically distributed (i.n.i.d.) double-Nakagami-m envelopes. Furthermore, leveraging these statistical insights, we investigate the performance of an equal gain combining receiver operating over i.n.i.d. double-Nakagami-m fading channels and propose performance metrics accordingly. We also derive asymptotic expressions and quantify the diversity and coding gains. Finally, we validate our findings through Monte-Carlo simulations.**

Keywords— **Approximations, cumulative distribution function, double-Nakagami-m, probability density function, sum of cascade envelopes.**

I. INTRODUCTION

The sum of cascade envelopes plays a pivotal role in wireless communication systems particularly when characterizing the fading phenomena across diverse applications, techniques and scenarios [1]–[4]. However, its exact statistical characterization, specifically the probability density function (PDF) and cumulative density function (CDF), can be cumbersome to obtain. The complexity involved in the search for exact formulations can be alleviated by the use of closed-form approximations that may ultimately yield highly accurate results.

Lately, the double-Nakagami distribution has been extensively studied as means of describing the fading experimented by links in non-orthogonal multiple access (NOMA) systems as well as in vehicle-to-vehicle (V2V) and reconfigurable intelligent surface (RIS) assisted communications [5]–[8]. While the derived statistics and performance metrics are novel, they often entail complexity or limitations. For instance, in [7] formulations for the statistics of a RIS-assisted communication system over Nakagami-m fading are presented in terms of the Hankel transform and non-elementary functions. Also, in [6] the distribution of the sum of double-Nakagami-m is introduced with an application to a RIS-aided communication system. Although the PDF and CDF of the instantaneous signal-to-noise ratio (SNR) are given in closed-form, they are restricted to integer values of the shape parameter. In light of this, accurate approximations have been introduced in the literature allowing for simpler closed-form formulations of the PDF, CDF and performance metrics in various communication systems experimenting double-Nakagami-m fading. Specifically, the statistics of the sum of double-Nakagami-m envelopes were approximated by means of the central limit theorem (CLT), along with Generalized-K and Gamma distributions in [9]–[11], which allowed for a comprehensive analysis of NOMA and RIS-assisted communication systems. Note, however, that such an approximation is reasonable in case a significant number of elements in the sum is used, i.e., when the CLT is applicable.

The preceding discussion highlights the importance of developing more accurate approximations for the sum of double-Nakagami-m envelopes while maintaining simplicity and tractability. Numerous approaches and distributions have been introduced in the literature as means for approximating the sum of cascade RVs (refer to [9]–[15] and the references therein). However, to the best of our knowledge, no studies have yet endeavored to approximate the sum of double-Nakagami-m envelopes using a general fading model such as α-μ [16]. The adoption of the α-μ model as an approximation for the sum statistics of double-Nakagami-m RVs is founded upon two fundamental factors: (i) the model's inherent mathematical simplicity and tractability, and (ii) the extensive versatility and suitability exhibited by the distribution across a diverse array of real-world scenarios [17]–[21]. These attributes render the α-μ model as a highly valuable choice for approximating the sum statistics in various practical applications.

This research focuses on deriving approximate yet accurate formulations for the PDF and CDF of the sum of i.n.i.d. double-Nakagami-m envelopes. The derivation builds upon the versatile α-μ fading model and is detailed in the subsequent sections. Furthermore, we analyze the performance of a pre-detection EGC receiver operating in a double-Nakagami-m fading environment by deriving key metrics such as outage probability (OP) and average symbol error rate (ASER). Additionally, we propose asymptotic expressions to further enhance the understanding of system performance. The proposed approximations provide a valuable contribution by establishing a tractable analytical framework, greatly facilitating the analysis of communication systems. Note that, differently from [9] our approach allows for every double-Nakagami-m envelope involved in the sum to be i.n.i.d. Furthermore, our proposed approximation does not rely on a large number of sums to

M. L. Alvarado, L. P. J. Jiménez, G. Fraidenraich, and M. D. Yacoub are with the Wireless Technology Laboratory, Department of Communications, School of Electrical and Computer Engineering, State University of Campinas (UNICAMP), Campinas, SP 13083-852, Brazil (e-mail: m264371@dac.unicamp.br; l264366@dac.unicamp.br; gf@decom.fee.unicamp.br; mdyacoub@unicamp.br).

This work was partially supported by Rede Nacional de Ensino e Pesquisa (RNP)-Projeto Brasil 6G Fase 2, Eldorado Research Institute, and the São Paulo Research Foundation (FAPESP) under grant 2023/02578-1.

achieve a highly accurate fitting, as observed in [9], [11].

The rest of this paper is organized as follows. Section II introduces the problem formulation. Section III presents the approximated sum statistics leveraging on the general α-μ distribution. Section IV analyzes the performance of an EGC diversity receiver. Section V discusses the numerical results. Finally, Section VI summarizes the conclusions.

In what follows, $\Pr[\cdot]$ denotes probability; $f_{(\cdot)}(\cdot)$, PDF; $F_{(\cdot)}(\cdot)$, CDF; $\mathbb{E}[\cdot]$ expectation; $\mathbb{V}[\cdot]$, variance; $\Gamma(\cdot)$, the gamma function [22, eq. (06.05.02.0001.01)]; $\gamma(\cdot,\cdot)$, the lower incomplete gamma function [23, eq. (8.2.1)]; and $\simeq$ denotes "asymptotically equal to".

II. Problem Formulation

Let R be the sum of Z_l RVs, i.e.,

$$R = \sum_{l=1}^{L} Z_l, \tag{1}$$

where Z_l are double-Nakagami-m variates with PDF and CDF given in [24], respectively, as

$$f_{Z_l}(z_l) = \frac{4\, z_l^{(m_1+m_2-1)} K_{m_1-m_2}\left(2 z_l \prod_{i=1}^{2}\sqrt{\frac{m_i}{\Omega_i}}\right)}{\prod_{i=1}^{2}\left(\frac{\Omega_i}{m_i}\right)^{\frac{1}{2}(m_1+m_2)}\Gamma\left(m_i\right)} \tag{2}$$

$$F_{Z_L}(z_l) = \frac{1}{\Gamma(m_1)\Gamma(m_2)} G_{1,3}^{2,1}\left(\frac{m_1 m_2 z_l^2}{\Omega_1\Omega_2}\middle|\begin{matrix}1\\ m_1, m_2, 0\end{matrix}\right) \tag{3}$$

in which $\{m_i\}_{i=1}^2$ and $\{\Omega_i\}_{i=1}^2$ are the shape and spread parameters of each Nakagami-m RV respectively, $K_v(\cdot)$ is the modified Bessel function of the second kind of v-th order [22, eq. (03.04.02.0001.01)], and $G_{c,d}^{a,b}[\cdot|\cdot]$ is the Meijer's G-function [25, eq. (9.301)].

The n-th moment of cascaded Nakagami-m is given by

$$\mathbb{E}\left[Z_l^n\right] = \prod_{i=1}^{2}\frac{\left(\frac{\Omega_i}{m_i}\right)^{n/2}\Gamma\left(\frac{n}{2}+m_i\right)}{\Gamma(m_i)}. \tag{4}$$

Our primary aim is to derive approximate formulations for the PDF and CDF of the sum presented in (1). The derivation is addressed in the following sections. The proposed framework enables the development of simplified yet precise formulations for the sum statistics and performance metrics, which hold great importance in understanding and analyzing various wireless communication systems.

III. Approximated Sum Statistics

The statistics of R can be accurately approximated by the α-μ fading model with PDF and CDF given in [16], respectively, by

$$f_R(r) = \frac{\alpha\,\mu^{\mu} r^{\alpha\mu-1}}{\hat{r}^{\alpha\mu}\Gamma(\mu)}\exp\left(-\frac{\mu\, r^{\alpha}}{\hat{r}^{\alpha}}\right) \tag{5}$$

$$F_R(r) = \frac{\gamma\left(\mu, \mu\, r^{\alpha}/\hat{r}^{\alpha}\right)}{\Gamma(\mu)}, \tag{6}$$

where $\alpha > 0$ is the shape parameter, $\hat{r} = \sqrt[\alpha]{\mathbb{E}[R^{\alpha}]}$ is the scale parameter, $\mu = \mathbb{E}^2\left[R^{\alpha}\right]/\mathbb{V}\left[R^{\alpha}\right] > 0$ is the inverse of the normalized variance of R^{α}, and the k-th moment is found as

$$\mathbb{E}\left[R^k\right] = \frac{\hat{r}^k\,\Gamma\left(\mu+\frac{k}{\alpha}\right)}{\Gamma(\mu)\mu^{k/\alpha}}. \tag{7}$$

To attain a high level of accuracy, we employed a moment-based technique to estimate the parameters of the α-μ PDF. More specifically, we estimated the values of α, μ, and $\hat{r}$ by utilizing the precise moments of R. By numerically solving the following system of transcendental equations we determine the values of α, μ, and $\hat{r}$

$$\frac{\Gamma^2\left(\mu+\frac{1}{\alpha}\right)}{\Gamma(\mu)\Gamma\left(\mu+\frac{2}{\alpha}\right)-\Gamma^2\left(\mu+\frac{1}{\alpha}\right)} = \frac{\mathbb{E}^2[R]}{\mathbb{E}\left[R^2\right]-\mathbb{E}^2[R]} \tag{8}$$

$$\frac{\Gamma^2\left(\mu+\frac{2}{\alpha}\right)}{\Gamma(\mu)\Gamma\left(\mu+\frac{4}{\alpha}\right)-\Gamma^2\left(\mu+\frac{2}{\alpha}\right)} = \frac{\mathbb{E}^2\left[R^2\right]}{\mathbb{E}\left[R^4\right]-\mathbb{E}^2\left[R^2\right]} \tag{9}$$

$$\hat{r} = \frac{\mu^{1/\alpha}\,\Gamma(\mu)\,\mathbb{E}[R]}{\Gamma\left(\mu+\frac{1}{\alpha}\right)}, \tag{10}$$

where the moments of R can be calculated using

$$\mathbb{E}\left[R^n\right] = \sum_{n_1=0}^{n}\sum_{n_2=0}^{n_1}\cdots\sum_{n_{L-1}=0}^{n_{L-2}}\binom{n}{n_1}\binom{n_1}{n_2}\cdots\binom{n_{L-2}}{n_{L-1}} \times \mathbb{E}\left[Z_1^{n-n_1}\right]\mathbb{E}\left[Z_2^{n_1-n_2}\right]\cdots\mathbb{E}\left[Z_L^{n_{L-1}}\right]. \tag{11}$$

IV. Application to EGC

Capitalizing on the formulations derived in Section III, we are going to analyze in an approximated but precise manner the performance of an EGC receiver subject to double-Nakagami-m fading.

The instantaneous SNR at the output of an L branch EGC receiver experiencing double-Nakagami-m fading is given by

$$\Upsilon = \frac{\rho}{L}\left(\sum_{l=1}^{L} Z_l\right)^2 = \frac{\rho}{L}R^2, \tag{12}$$

where $\rho = E_s/N_0$ is the average SNR per symbol, E_s is the average energy per symbol and N_0 is the power spectral density of the noise.

The PDF and CDF of Υ can be readily obtain from (5) and (6) by performing a transformation of variables, yielding in

$$f_{\Upsilon}(\Upsilon) = \left(\frac{L\Upsilon}{\rho\,\hat{r}^2}\right)^{\frac{\alpha\mu}{2}}\frac{\alpha\,\mu^{\mu}}{2\,\Upsilon\,\Gamma(\mu)}\exp\left(-\mu\left(\frac{L\Upsilon}{\rho\,\hat{r}^2}\right)^{\frac{\alpha}{2}}\right) \tag{13}$$

$$F_{\Upsilon}(\Upsilon) = \frac{\gamma\left(\mu, \mu\,\hat{r}^{-\alpha}\left(\frac{L\Upsilon}{\rho}\right)^{\alpha/2}\right)}{\Gamma(\mu)}. \tag{14}$$

In the following, performance metrics such as OP and ASER are derived.

A. Outage Probability

The OP is defined as the probability that the instantaneous SNR falls below a specified threshold γ_{th}, i.e.,

$$P_{\text{out}} \triangleq \Pr\left[\Upsilon \leq \gamma_{\text{th}}\right] = F_{\Upsilon}(\gamma_{\text{th}}). \tag{15}$$

From (15) and (14) the OP can be found as

$$P_{\text{out}} = \frac{\gamma\left(\mu, \mu\,\hat{r}^{-\alpha}\left(\frac{L\,\gamma_{\text{th}}}{\rho}\right)^{\alpha/2}\right)}{\Gamma(\mu)}. \tag{16}$$

Moreover, we investigate the system performance in the high signal-to-noise ratio (SNR) regime, specifically as $\rho \to \infty$. With the aid of [26, eq. (8.2.6)] and [26, eq. (8.7.1)] we rewrite (16) using an infinite series representation. As the first term dominates the series the asymptotic OP can be obtained as

$$P_{out} \simeq (O_c\,\rho)^{-O_d}\,, \tag{17}$$

where $O_d = \alpha\,\mu/2$ is the diversity gain, and O_c is the coding gain and is given by

$$O_c = \frac{\hat{r}^2\,\Gamma(\mu+1)^{\frac{2}{\alpha\mu}}}{\mu^{2/\alpha}\,L\,\gamma_{\text{th}}}. \tag{18}$$

B. Average Symbol Error Rate

The ASER for an EGC receiver is given by [27, eq. (9.61)]

$$P_{\text{s}} = \frac{1}{2}\int_0^\infty \operatorname{erfc}\left(r\sqrt{\frac{\mathcal{G}\rho}{L}}\right) f_R(r)\,\mathrm{d}r, \tag{19}$$

where $\operatorname{erfc}(\cdot)$ is the complementary error function [26, eq. (7.1.2)] and $\mathcal{G}$ is a modulation dependent parameter. From (19) and (5) and after some algebraic manipulations the ASER can be obtained as follows

$$\begin{aligned} P_{\text{s}} =& \frac{1}{2\sqrt{\pi}\Gamma(\mu)}\left(\frac{\mathcal{G}\rho}{L}\right)^{-\alpha\mu/2} \\ &\times \sum_{i=0}^{\infty} \frac{(-1)^i\left(\frac{\mu}{\hat{r}^\alpha}\right)^{i+\mu}\Gamma\left(\frac{1}{2}(i\alpha+\mu\alpha+1)\right)}{i!(i+\mu)\left(\frac{\mathcal{G}\rho}{L}\right)^{(\alpha i/2)}}. \end{aligned} \tag{20}$$

The asymptotic ASER can be found by taking the first term of (20) since it dominates the series and is given as

$$P_{\text{s}} \simeq (G_c\,\rho)^{-G_d}\,, \tag{21}$$

where $G_d = \alpha\,\mu/2$ is the diversity gain, and G_c is the coding gain which is given by

$$G_c = \frac{\mathcal{G}\,\hat{r}^2}{L\mu^{2/\alpha}}\left(\frac{\Gamma\left(\frac{1}{2}(\alpha\mu+1)\right)}{2\sqrt{\pi}\,\Gamma(\mu+1)}\right)^{\frac{2}{\alpha\mu}}. \tag{22}$$

It is worth emphasizing that the formulations presented in Sections III and IV make a significant contribution to the existing literature, particularly in the analysis of diverse communication systems. Furthermore, the derivations are notably straightforward and easy to comprehend, adding to their inherent simplicity.

V. Numerical Results

In this section, we corroborate our analytical expressions through Monte-Carlo (MC) simulations.[1]

[1]The number of Monte-Carlo samples was set to 10^6. Also, we used a maximum of 500 terms in (20).

In the upcoming, each figure shows the scale and spread parameters corresponding to the double-Nakagami-m RVs involved in the sum. To provide a clearer understanding of what is illustrated in this section, please refer to the corresponding curve for $L = 5$ in Fig. 1. In this case, the first five values depicted in the figure (i.e., $\{m_i\}_{i=1}^5$ and $\{\Omega_i\}_{i=1}^5$) are set to be the parameters of the five double-Nakagami-m RVs involved in the sum. Then using equations (8), (9), (10), and (11) the parameters α, μ and $\hat{r}$ are estimated. The aforementioned steps are followed for every curve depicted in the following figures. Table I presents the estimated parameters α, μ and $\hat{r}$ for different values of L.

TABLE I

Estimated parameters α, μ and $\hat{r}$ for different values of L.

L	α	μ	$\hat{r}$
5	0.9628	17.9058	12.6614
6	0.9616	24.0591	16.4435
7	0.9611	31.0539	20.6332
8	0.9610	38.8819	25.2289
9	0.9613	47.5354	30.2295
10	0.9628	56.8830	35.6345

Considering an arbitrary number of L and different distribution parameters m_i and Ω_i, Fig. 1 and Fig. 2 show the approximated PDF and CDF of the sum of double-Nakagami-m envelopes, respectively, with parameters α, μ and $\hat{r}$ estimated by the method of moments described in Section III. The figures show an excellent agreement between the proposed approximation and the simulations hence validating our analytical approximations.

Fig. 3 illustrates the OP in terms of SNR varying the number of branches L in the receiver and considering different coefficients for each cascaded Nakagami-m channel. As expected, for any fixed value of SNR, the system's availability improves as L increases. Besides the perfect agreement between the approximation and the simulation, the figure also exhibits the asymptotic behavior, in a high SNR regime, of each curve.

Fig. 4 depicts the ASER considering a binary phase shift keying (BPSK) modulation scheme ($\mathcal{G}$=1), different L values, and distinct distribution parameters for each double Nakagami-m channel. The figure shows how the number of branches L benefits the reliability of the system. In other words, for a given value of SNR, the ASER diminishes as L increases. The MC simulations and the asymptotic curves validate again the proposed approximation.

Finally, it is worth noting that since we are approximating R by the α-μ distribution, the diversity gain derived in (17) and (21) solely depends upon the estimated parameters α and μ. This was expected and further corroborated during the execution of this numerical analysis since L is somehow embedded in the estimated parameters. Thus as L increases, α and μ increase as well.

VI. Conclusions

We have successfully derived precise closed-form approximations for the sum of double-Nakagami-m envelopes, yielding accurate results. Specifically, we showed that the sum

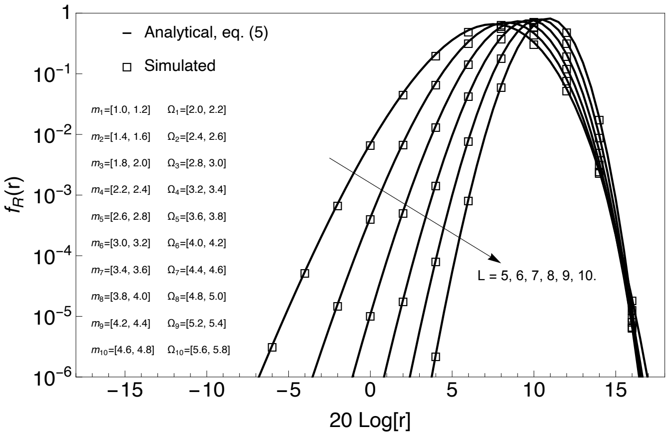

Fig. 1. PDF of R for multiple values of L and distribution parameters.

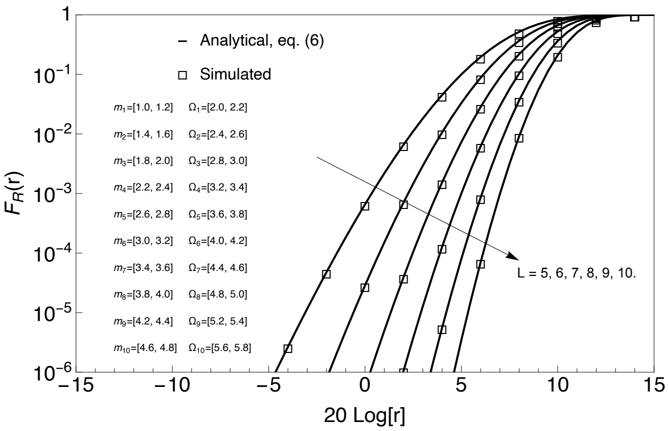

Fig. 2. CDF of R for multiple values of L and distribution parameters.

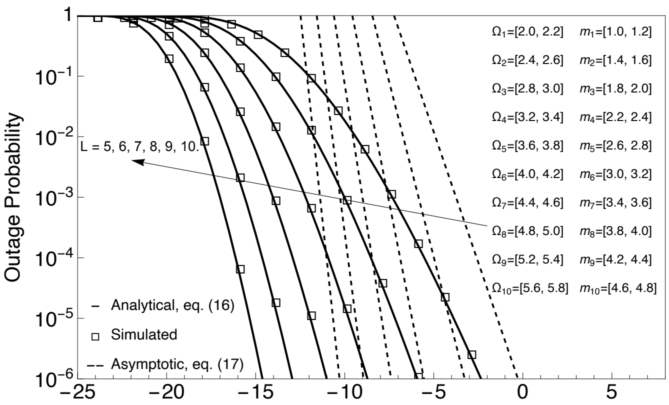

Fig. 3. OP versus SNR considering multiple values of L and $\gamma_{\text{th}} = 0$ dB.

statistics can be greatly approximated by a single α-μ RV. Additionally, we have thoroughly examined the performance of an EGC diversity receiver and introduced simplified yet accurate formulations for key metrics, including OP and ASER. Also, we derived asymptotic expressions for these performance metrics. The tractability and simplicity of our results provide researchers with a convenient framework for studying and evaluating the performance of advanced technologies such as

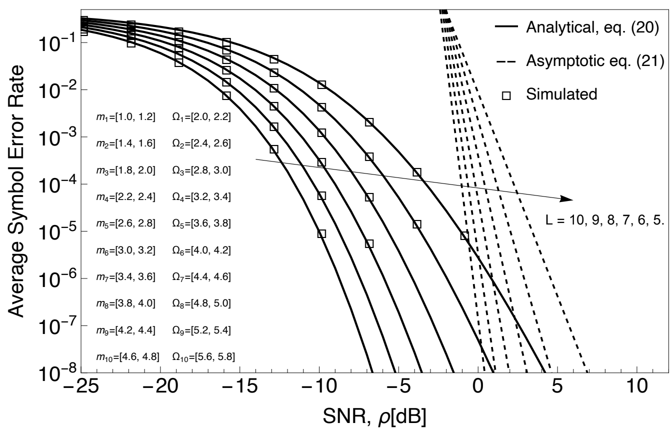

Fig. 4. ASER versus SNR with $\mathcal{G} = 1$ and considering multiple values of L.

NOMA and RIS. Lastly, our findings have been validated through extensive Monte-Carlo simulations.

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