# Dynamic Clustering and Pilot Assignment for Optimizing Cell-Free Massive MIMO Uplink

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*Abstract*— The increasing demand for higher rates and uniformity of rates among users has given rise to one new system in which the user is served by more than one Access Point. In such systems, one challenge is to find the optimal cluster of Access Points to serve the user. One way of doing that has been proposed by greedy pilot assignment and clustering, however such a way resorts to the knowledge of the large-scale coefficients of the channel between users and Access Points. This paper aims to find an algorithm that not only keeps system performance stable but also can be performed on more realistic systems, without knowledge of the channel and the large-scale coefficients besides having a low complexity, reduced to computing correlation matrices.

*Keywords*— Correlated Rayleigh fading, Cell Free, Dynamic Cooperation Clustering, Collective Channel Correlation Matrix, Repulsive Clustering Pilot Assignment, Greedy Access Point Assignment

# I. INTRODUCTION

Telecommunications networks are constantly facing evergrowing demands for higher transmission rates with greater quality. In order to address this demand various technologies were proposed. By use of spatial multiplexing methods and the statistics of the communication channel, [1] suggested that an increased number of antennas at an access point (AP) could lead to improved performance by coherent combining of the signals transmitted to an user equipment (UE) provided that the channels of each AP antenna to the UE are independent between each other. Such independence would result in the vanishing of the small-scale fading coefficients with only the large-scale coefficients remaining. Although this proposition is capable of reducing or even removing the small-scale fading interference, it is necessary for the phenomena of channel hardening to occur, that is, the channel of the UEs to their respective AP should be uncorrelated, which is not guaranteed to happen into real-life communication networks. This hypothesis also assumes perfect channel state information (CSI) knowledge.

In order to acquire CSI, the channel coefficients must be estimated. Various estimation methods such as the ones of [2] and [3] have been proposed, but they consider frequency selective channels and non-linear methods of estimation. A linear channel estimation method is used in [4] by linear minimum mean square error estimator (MMSE) using pilot signals. [4] also showed that in real-life applications, the same pilot signal must be reused by multiple UEs, impacting interference from the channel of the same pilot UEs into the channel estimate of the desired UE. Trying to address this limitation, authors such as [5] have proposed methods of removing pilot interference by singular value decomposition (SVD) of the received uplink signal. Such methods, however, resort to systems with controlled interference and few UEs, which may not be guaranteed in real-life applications. This work uses the proposed algorithm by [6] to assign pilots to UEs that are possibly far apart from each other and would result in lesser interference from pilot contamination.

The use of traditional cell systems, in which the UE is connected to only one AP, has shown that the transmission rates from each UE suffer great variation, especially at the cell edge. Recent works [7] propose the use of cell free (CF) networks, that is, networks in which each UE connects to a subset of APs instead of only one. [8] has also shown that the system must be scalable, therefore the UE should not be served by all APs in the network, but only by a subset. Thus, it is important to select the most suitable APs to compose the subset. By selecting the appropriate APs, [8] has shown that the transmission rates of UEs tend to suffer fewer variations compared to traditional cellular MIMO.

The authors in [8] have assigned the pilots to the UEs by a greedy algorithm. They have also used this assumption to select the APs to each UE. The pilot assignment and the AP clustering are based on the information gathered by transmissions of known pilot signals to the APs and the processing of this information to generate channel estimates over various coherence blocks. Therefore, this work aims to establish an algorithm that reduces the pilot contamination and assigns the UE to the APs with reasonable channel quality, using the estimated channel gains.

# II. SYSTEM MODEL

# A. Spatial Setup Model

For this work, we consider the square grid scenario, as seen in Fig. 1, in which the K UEs and the L APs are uniformly distributed over a square of area A. The APs possess each N antennas and the UEs are single antennas. The channel between a given UE k and an antenna n of an AP l is denoted  $h_{nlk}$ .

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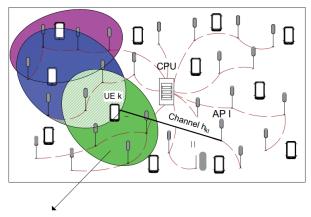




Fig. 1. Cell Free setup with APs and UEs randomly deployed. Each colored oval corresponds to the AP cluster of an UE. The APs are connected to a CPU

The model adopted for the channel is the block fading model, in which the communication channel coefficients are constant over a time interval defined as coherence time  $T_c$ , and the frequency range at which the channel is said to be flat, that is, not distorted is defined as coherence bandwidth  $B_c$ . Thus the coherence block is the number of symbols that can be transmitted over a flat time invariant channel and is given by  $\tau_c = T_c B_c$ .

Additionally, the adopted channel model is the correlated Rayleigh fading model taken from [8], in which the AP antennas propagation waves are more likely to be dispersed over a certain direction, thus being correlated, but the UE location favors multipath and scattering, and thus the small scale fading is still modeled as Rayleigh.

The channel vector  $\mathbf{h}_{kl} \in \mathbb{C}^{N \times 1}$  between UE k and AP l is a complex normal variable of zero mean and variance  $\mathbf{R}_{kl}$ , where  $\mathbf{R} \in \mathbb{C}^{N \times N}$  is the spatial covariance matrix The model adopted for the spatial covariance matrix is the local scattering model used by authors like [9] and [10], in which

$$[\mathbf{R}]_{lm} = \beta \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{j2\pi(l-m)(\cos(\varphi+\epsilon)\sin(\theta+\delta))} p(\epsilon,\delta) d\epsilon d\delta,$$
(1)

where

$$p(\epsilon, \delta) = \frac{1}{\sqrt{2\pi}\sigma_{\varphi}} e^{-\epsilon^2/2\sigma_{\phi}^2} \frac{1}{\sqrt{2\pi}\sigma_{\theta}} e^{-\delta^2/2\sigma_{\theta}^2}.$$
 (2)

The terms  $\varphi$  and  $\theta$  are the azimuth angle and the elevation angle respectively between the AP and the UE. The terms  $\sigma_{\varphi}$  and  $\sigma_{\theta}$  are the angular standard deviation (ASD) of the azimuth and elevations angles, that is the measure of the dispersion between the possible angles between the UE and the AP.

The coefficients  $\beta_{kl} = \frac{\operatorname{tr}(\mathbf{R}_{kl})}{N}$  are the large scale coefficients between UE k and AP l, which are also defined accordingly to [9], in decibels and tr(.) is the trace operator:

$$\beta = P(d_0) - 10\alpha \log(d/d_0) - F_{dB},$$
(3)

where the term  $P(d_0)$  is the pathloss at the reference distance  $d_0$  and is usually dependent on the carrier frequency, the an-

tenna's characteristics, as well as propagation properties. The term  $\alpha$  is the attenuation coefficient reflecting the propagation medium, and  $F_{dB}$  is the shadow fading term, obtained from [8].

## B. Uplink Transmission

On CF systems, multiple APs serve one UE. Define  $\mathcal{M}_k$ , the AP cluster containing all of the APs that serve the UE. Thus, we can define the matrix that selects the clusters from UE k,  $\mathbf{D}_{kl}$  as

$$\mathbf{D}_{kl} = \begin{cases} \mathbf{I}_N & , l \in \mathcal{M}_k \\ \mathbf{0}_{N \times N} & , \text{ otherwise} \end{cases}, \tag{4}$$

where  $I_N$  is the identity matrix of size N. In the uplink, each AP receives a superposition of the signals from the UEs, given by:

$$\mathbf{y}_l = \sum_{k=1}^K \mathbf{h}_{kl} s_k + \mathbf{z},\tag{5}$$

where  $s_k$  is the symbol transmitted by UE k at a given time. The channel vector  $\mathbf{h}_{kl}$  follows a complex normal distribution  $\mathbf{h}_{kl} \sim \mathcal{N}_C(0, \mathbf{R}_{kl})$ . The noise received at each AP is a complex normal random vector  $\mathbf{z} \sim \mathcal{N}_C(0, \sigma_n^2 \mathbf{I})$ , of addictive white gaussian noise (AWGN) coefficients, where  $\sigma_n^2$  is the average noise power.

To correctly decode the signal from each UE, each AP multiplies the received signal vector by the combining vector,  $\mathbf{v}_{kl} \in \mathbb{C}^{N \times 1}$ . Since not all the users will connect to the AP, the resulting decoded signal will be,

$$\hat{s}_k = \mathbf{v}_{kl}^* \mathbf{h}_{kl} s_k + \sum_{i=1, i \neq k}^K \mathbf{v}_{kl}^* \mathbf{h}_{il} s_i + \mathbf{v}_{kl}^* \mathbf{z}_l$$
(6)

assuming that  $l \in \mathcal{M}_k$  and  $\mathbf{v}_{kl}^* = \mathbf{D}_{kl}\mathbf{v}_{kl}$ . In order to do so, the receiver must have CSI. To obtain CSI, the receiver must estimate the channel so that the combining vector is obtained accordingly. In the next section methods of estimating the channel and consequently allocating pilots in order to reduce interference and selecting the APs with reasonable channel quality will be discussed.

Defining the collective channel  $\mathbf{h} \in \mathbb{C}^{NL \times 1}$ , the collective  $\mathbf{D} \in \mathbb{C}^{NL \times NL}$  matrix and the collective combiner  $\mathbf{v} \in \mathbb{C}^{NL \times 1}$  for UE *k*:

$$\mathbf{h}_{k} = \begin{vmatrix} \mathbf{h}_{k1} \\ \mathbf{h}_{k2} \\ \vdots \\ \mathbf{h}_{kL} \end{vmatrix}, \quad \mathbf{v}_{k} = \begin{vmatrix} \mathbf{v}_{k1} \\ \mathbf{v}_{k2} \\ \vdots \\ \mathbf{v}_{kL} \end{vmatrix}, \quad (7)$$

with  $\mathbf{D}_k = \text{diag}(\mathbf{D}_{k1}, \mathbf{D}_{k2}, \dots, \mathbf{D}_{kL})$ . CF systems must have CSI in order to be able to correctly decode the signals. One way to acquire it is by estimating the channel coefficients by use of linear MMSE channel estimation, given by

$$\hat{\mathbf{h}}_{kl} = \sqrt{p_k} \mathbf{R}_{kl} \Psi_{t_k l}^{-1} \mathbf{y}_{t_k l}^{\text{pilot}}.$$
(8)

The term  $\mathbf{y}_{t_k l}^{\text{pilot}} \in \mathbb{C}^{N \times 1}$  refers to the transmitted uplink signal received from the transmitted UE pilot  $t_k$ , that is

$$\mathbf{y}_{t_k l}^{\text{pilot}} = \sqrt{\eta_k} \mathbf{h}_{kl} + \sum_{i \in \mathcal{P}_k \setminus k} \sqrt{\eta_i} \mathbf{h}_{il} + \mathbf{Z}_l \phi_{t_k}, \qquad (9)$$

where  $\eta_i$  is the power of the pilot transmitted by the *i*-th UE and  $\phi_{t_k}$  is a pilot sequence of length  $\tau$  from a pilot book  $\mathcal{P}_k$  such that it's vectors obey the relationship  $\phi_k \phi_i^H = \delta_{ik}$ , where  $(.)^H$  is the transpose conjugate operator. The matrix  $\Psi_{t_k l} \in \mathbb{C}^{N \times N}$  is the uplink signal correlation matrix, that is

$$\Psi_{t_k l} = \mathbb{E}\left\{\mathbf{y}_{t_k l}^{\text{pilot}}(\mathbf{y}_{t_k l}^{\text{pilot}})^H\right\} = \sum_{i \in \mathcal{P}_k} \eta_i \mathbf{R}_{il} + \sigma_n^2 \mathbf{I}, \qquad (10)$$

where  $\mathbb{E}\{.\}$  is the expected value operator. From Eq. (9) it is possible to see that the channel estimate by least squares (LS) from UE k will be contaminated by the UEs using the same pilot  $t_k$  from the pilot book  $\mathcal{P}_k$ . That is why it is important to find a pilot assignment algorithm such that this interference is reduced. Since the estimated CSI is not perfect, the correlation error matrix  $\mathbf{C} \in \mathbb{C}^{N \times N}$  is defined as

$$\mathbf{C}_{kl} = \mathbb{E}\left\{\tilde{\mathbf{h}}_{kl}\tilde{\mathbf{h}}_{kl}^{H}\right\} = \mathbf{R}_{kl} - \eta_{k}\mathbf{R}_{kl}\boldsymbol{\Psi}_{t_{k}l}^{-1}\mathbf{R}_{kl}, \quad (11)$$

where  $\hat{\mathbf{h}}_{kl} = \mathbf{h}_{kl} - \hat{\mathbf{h}}_{kl}$ , and thus the estimated channel vector is distributed as  $\hat{\mathbf{h}}_{kl} \sim \mathcal{N}_C(\mathbf{0}_N, \mathbf{R}_{kl} - \mathbf{C}_{kl})$ 

1) Spectral Efficiency for Centralized Uplink Operation: In centralized networks, the CPU processes the received signals at each AP and selects the appropriate combining vector for each UE-AP pair. A lower bound for the spectral efficiency (SE) of the centralized uplink operation is given by

$$\mathbf{SE}_k = \frac{\tau_u}{\tau_c} \mathbb{E} \left\{ \log_2(1 + \mathbf{SINR}_k) \right\},\tag{12}$$

where  $\tau_u$  is the number of transmitted uplink symbols, with the instantaneous effective signal to interference to noise ratio (SINR) given by

$$\operatorname{SINR}_{k} = \frac{p_{k} |\mathbf{v}_{k}^{H} \mathbf{D}_{k} \hat{\mathbf{h}}_{k}|^{2}}{\sum_{\substack{i=1\\i \neq k}}^{K} p_{i} |\mathbf{v}_{k}^{H} \mathbf{D}_{k} \hat{\mathbf{h}}_{i}|^{2} + \mathbf{v}_{k}^{H} \mathbf{E}_{k} \mathbf{v}_{k} + \sigma^{2} ||\mathbf{D}_{k} \mathbf{v}_{k}||},$$
(13)

where ||.|| is the norm and

$$\mathbf{E}_{k} = \sum_{i=1}^{K} p_{i} \mathbf{D}_{k} \mathbf{C}_{i} \mathbf{D}_{k}.$$
 (14)

#### 2) Spectral Efficiency for Distributed Uplink Operation:

In the distributed case, each AP estimates the channels of its connected UEs and selects the appropriate combining vector. Opposed to the centralized case, the CPU is only required to process the combined signals. Since the CPU has knowledge of all of the estimated channels, it should be able to assign larger weights to the channels of APs that have larger signal to noise ratio (SNR)s. Those weights, denoted by  $a_{kl}$  are known as large scale fading decoding (LSFD) coefficients, and were used in works such as [11]. Also, by defining

$$\mathbf{g}_{ki} = \begin{bmatrix} \mathbf{v}_{k1}^{H} \mathbf{D}_{k1} \mathbf{h}_{il} \\ \vdots \\ \mathbf{v}_{kL}^{H} \mathbf{D}_{kL} \mathbf{h}_{iL} \end{bmatrix}, \qquad (15)$$

we can define a lower bound for the SE of the distributed uplink operation as

$$\mathbf{SE}_{k} = \frac{\tau_{u}}{\tau_{c}} \mathbb{E}\left\{\log_{2}(1 + \mathbf{SINR}_{k})\right\},\tag{16}$$

with the instantaneous effective SINR given by

$$SINR_{k} = \frac{p_{k}|\mathbf{a}_{k}^{H}\mathbb{E}\{\mathbf{g}_{kk}\}|^{2}}{\mathbf{a}_{k}^{H}(\sum_{i=1}^{K}p_{i}\mathbb{E}\{\mathbf{g}_{ki}\mathbf{g}_{ki}^{H}\} - p_{k}\mathbb{E}\{\mathbf{g}_{kk}\}\mathbb{E}\{\mathbf{g}_{kk}^{H}\} + \mathbf{F}_{k})\mathbf{a}_{k}},$$
(17)  
where  $\mathbf{F}_{k} = \sigma_{n}^{2} \text{diag}(\mathbb{E}\{||\mathbf{v}_{k1}\mathbf{D}_{k1}||^{2}\}, \dots, \mathbb{E}\{||\mathbf{v}_{kL}\mathbf{D}_{kL}||^{2}\}).$ 

# III. PILOT ASSIGNMENT AND AP CLUSTERING FORMATION ALGORITHMS

# A. Dynamic Cooperation Clustering Formation

One important step for the implementation of CF systems is the assignment of the UEs to the APs. Such assignment is defined as dynamic cooperation clustering (DCC). It is dynamic because, since the large-scale coefficients vary with time (although many times such variation is only perceived after thousands of coherence blocks), the clustering must be performed once again.

Ideally, each UE should be served by all the available UEs in the grid. However, the authors of [8] have shown that such a method is computationally expensive, and for networks where there are thousands of APs and hundreds of UEs, it can become unfeasible. Therefore only some of the APs must be chosen by a specific UE. The above algorithms usually assume knowledge of the spatial covariance matrices, and consequently, the large scale coefficients  $\beta_{kl}$ .

In real-life applications, however, such parameters are not known and must be estimated, so that the estimated covariance matrix and the large-scale coefficients are given by  $\mathbf{R}_{\text{sample}}$ and  $\hat{\beta}$  respectively. The proposed algorithm uses the estimated large-scale fading coefficients, normalizes them and selects the APs for each UE based on a threshold value.

1) Correlation Clustering Algorithm: Considering the number of UEs a multiple of the number of orthogonal pilot sequences,  $\tau$  from the pilot book  $\mathcal{P}_k$ . Each AP receives the transmission from  $\tau_p$  UEs and estimates the channel by LS,

$$\hat{\mathbf{h}}_{kln} = \sum_{i \in \mathcal{P}_k \setminus k} \mathbf{h}_{il} \phi_{t_i}^H \phi_{t_k} + \mathbf{Z}_{kl} \phi_{t_k} / \sqrt{\eta_k}, \qquad (18)$$

where the subscript n is the transmission number of UE k. Since the pilots are orthogonal, Eq. (18) reduces to

$$\hat{\mathbf{h}}_{kln} = \mathbf{h}_{kl} + \mathbf{Z}_{kl}\phi_{t_k}/\sqrt{\eta_k}.$$
(19)

After all UEs have transmitted, the APs average the LS estimates to obtain the average estimated channel by LS,

$$\mathbf{h}_{kl}^{LS} = \mathbb{E}\left\{\hat{\mathbf{h}}_{kl}\right\}.$$
(20)

In order to estimate the spatial covariance matrix, the average LS estimate is obtained for a number  $n_C$  of coherence blocks, such that the estimated spatial covariance matrix is given by

$$\mathbf{R}_{kl}^{\text{sample}} = \frac{1}{n_C} \sum_{c=1}^{n_C} (\mathbf{h}_{ckl}^{LS}) (\mathbf{h}_{ckl}^{LS})^H$$
(21)

After obtaining the estimated large-scale coefficients for each UE-AP pair, they are normalized in such a way that

$$\tilde{\beta}_{kl}^{sample} = \frac{\beta_{kl}^{sample} - \bar{\beta}_{k}^{sample}}{\sqrt{\operatorname{Var}(\beta_{k}^{sample})}},$$
(22)

where  $\bar{\beta}_k^{\text{sample}}$  is the mean of the estimated large scale coefficients of UE k over all the APs and  $\text{Var}(\beta_k^{\text{sample}})$  is the variance, so that each coefficient  $\tilde{\beta}_{kl}^{\text{sample}}$  is distributed with 0 mean and unit variance. Each UE connects to the APs that possess the normalized large-scale coefficient above a certain threshold  $\gamma$ . That way the UE connects only to the APs with reasonable signal strength.

#### **B.** Pilot Assignment

1) Pilot Assignment Algorithm: In order to find the nearoptimal pilot assignment so that the same pilot signal is delegated to the UEs that are geographically apart, the present work uses the Repulsive Clustering Techniques discussed by [6], which consists on maximizing the following objective function, where x of dimensions  $K \times \tau_p$  is the binary matrix that assigns the pilot to each UE:

$$\max_{\mathbf{x}} \sum_{t=1}^{\tau_{p}} \sum_{k=1}^{K} \sum_{\tilde{k}=k+1}^{K} f(k, \tilde{k}) x[k, t] x[\tilde{k}, t]$$
s.t. 
$$\sum_{t=1}^{\tau_{p}} x[k, t] = 1$$

$$\frac{K}{\tau_{p}} \leq \sum_{k=1}^{K} x[k, t] \leq \frac{K}{\tau_{p}} + 1$$
(23)

The function  $f(k, \tilde{k})$  is defined as, by defining  $\mathbf{R}_{\beta_k \beta_{\tilde{k}}} = \mathbf{B}_k^T \mathbf{B}_{\tilde{k}}$ , where  $\mathbf{B}_k$  is the vector of large scale coefficients from UE k to all APs,

$$f(k,\tilde{k}) = \frac{\mathbf{R}_{\beta_k\beta_{\tilde{k}}}}{\sqrt{\operatorname{diag}(\mathbf{R}_{\beta_k\beta_{\tilde{k}}})\otimes\operatorname{diag}(\mathbf{R}_{\beta_k\beta_{\tilde{k}}})}}, \qquad (24)$$

where the  $a \otimes b$  operation is the outer product between a and b and k and  $\tilde{k}$  are arbitrary UE indices. The algorithm assumes that the users are separated into  $\tau_p$  disjoint clusters. Firstly the UEs are allocated pilots randomly and the value of the objective function is obtained. Then one element u from any cluster U is swapped with one element w from another cluster W. Then the overall score is calculated according to Eq. (23). If the score increases, then the elements are swapped. If not, the elements are swapped back. This is repeated for all of the elements of all clusters. Then the score is again compared to the score before the swapping of clusters and elements. If the score is greater then repeat the algorithm again. If however, the score is lower, then the allocation of pilots had reached a point such that the interference between non-orthogonal pilots is reduced, so the algorithm should stop.

# IV. NUMERICAL RESULTS

In order to compare the performance of algorithms, the proposed threshold algorithm with  $\gamma = -0.1$  is compared with the random pilot assignment, with  $\gamma = -0.1$ , and the greedy algorithm of [8]. The setup of [8] is used, in which K = 40 UEs are served by L = 100 APs, each with N = 4 antennas, who are randomly deployed in a square grid of area 1 km<sup>2</sup>. Each AP has an elevation of h = 10 m

to each UE. The received noise power is  $\sigma_n^2 = -94$  dBm, and the transmitter power is p = 20 dBm. The reference channel gain of the pathloss model is  $P(d_0) = -140.6 \text{ dB}$ at the reference distance  $d_0 = 1$  km. The shadowing standard deviation is  $\sigma_F = 4$  dB. The ASDs are all equal to 15 degrees,  $d_{decorr} = 9$  m, and the APs are deployed according to a wraparound topology as described by [9]. The mean LS channel estimate is performed after  $n_t = 20$  transmissions, and the spatial covariance matrix is estimated over  $n_C = 30$  coherence blocks. The channel coherence block length is  $\tau_c = 200$ . For the simulation, the used combiners will be the partial regularized zero forcing (PRZF), the partial mean square error estimator (PMMSE), the near optimal local partial minimum square error estimator (nopt-LPMMSE), and the optimal local minimum square error estimator (opt-LMMSE), defined by [8]. The SEs will be compared for the proposed threshold algorithm with  $\gamma = -0.1$ . The reason for this value is that, assuming that the large scale coefficients are a random variable dependent on uniformly distributed distances and log-normal shadowing coefficients, the majority of them will lie, after various simulations, above the -0.2 value. Since this value is such that almost every AP is selected, the value of -0.1 is chosen instead. The SEs will also be compared for the random pilot assignment (the same as the proposed algorithm, but instead of finding the sub-optimal pilot assignment, they are randomly chosen for the UEs), and the greedy algorithm of [8]. The cumulative distribution function (CDF) of the centralized and distributed uplink SEs are shown in Fig. 2 and Fig. 3:

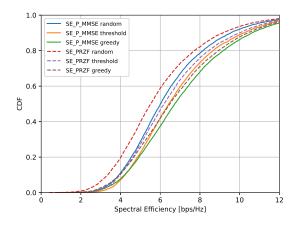


Fig. 2. CDF for the SE of different combining methods for centralized uplink operation. Continuous lines are the SEs obtained from PMMSE combining and dashed lines the SE obtained from PRZF combining

To better compare the systems, the 10-th, 50-th and 90-th percentile of each of the methods is shown in I: Comparing the values, it can be seen that the SEs obtained by PMMSE are greater than those obtained from PRZF. This is explained by the fact that the PRZF combiner is a simplification of the PMMSE since it neglects the interference between UEs that are connected to the same AP. The random pilot assignment algorithm has the lowest SEs in all cases, since it doesn't aim to reduce the co-pilot interference. The greedy and the threshold algorithm have a similar performance, with the

	random				greedy				threshold			
Percentile	PRZF	PMMSE	LP-MMSE	L-MMSE	PRZF	PMMSE	LP-MMSE	L-MMSE	PRZF	PMMSE	LP-MMSE	L-MMSE
90	3.35	3.91	1.69	1.84	3.95	4.20	2.31	2.50	3.98	4.22	2.35	2.59
50	5.52	6.02	3.78	4.03	6.49	6.73	4.39	4.59	6.19	6.43	4.33	4.59
10	9.16	9.55	7.48	7.68	10.41	10.62	8.12	8.28	10.01	10.22	8.07	8.28

TABLE I

PERCENTILES OF THE SE FOR RANDOM, GREEDY AND THRESHOLD ALGORITHMS

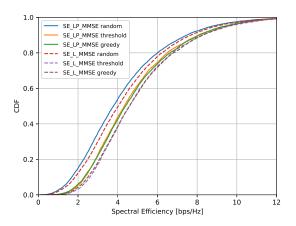


Fig. 3. CDF for the SE of different combining methods for distributed uplink operation. Continuous lines are the SEs obtained from nopt-LPMMSE combining and dashed lines the SE obtained from opt-LMMSE combining

threshold algorithm slightly higher in the 90-th percentile, and the greedy algorithm higher by a margin of 0.4 bps/Hz in the 50-th and 10-th percentiles. However, simulation time was 7 times larger for the analyzed system when using the greedy algorithm than when threshold algorithm was used. This happens because the greedy method selects  $\tau_p$  UEs to be served by each AP. On the other hand, the threshold method selects only the UEs with the desired channel quality to be served by the AP, which tend to be less than  $\tau_p$ . Thus, it is less computationally expensive than the greedy method, for similar performance.

The same considerations of the centralized case can be done for the distributed: however, one should notice that for a broader threshold  $\gamma$ , the performance gap between nopt-LPMMSE and opt-LMMSE combining is large, since few APs mean that the near-optimal LSFD coefficients will be computed based only on the connected APs channel estimates and not all of them. Although it is also possible to notice that the gap between the SEs of both setups is smaller in the distributed case than in the centralized. This happens because the assignment of LSFD coefficients for each AP attributes a higher weight to the AP with the greatest channel gain, which is probably the AP closest to a specific UE. Finally, we can see that the distributed case has overall smaller SEs than the distributed one. However, the distributed case makes it possible that more APs be included in the grid, as opposed to the centralized case, where computation must begin once again every time more APs are added to the grid.

# V. CONCLUSION

The proposed algorithm performs similarly to the one proposed by [8], if we consider a threshold  $\gamma$ , such that a sufficient number of APs is connected to each UE. For the analyzed system it can be seen that the proposed algorithm is viable since performance is satisfactory and the complexity is reduced to simply estimating the spatial covariance matrix and performing operations with the correlation between estimated large-scale coefficients. The proposed algorithm is heuristic, for the sake of reducing complexity. Future research might consider assigning the APs to the UEs and allocating the pilots by using more sophisticated methods such as Neural Networks. Optimization of the combining vectors can also be the object of future research, to further improve the SE of each UE.

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