Middleton Class-A or Bernoulli-Gaussian: which model to choose?

Weiler Alves Finamore, Marcelo da Silva Pinho, & Manish Sharma

***Abstract*— The performance of communication systems impaired by noise has been extensively and successively examined under the assumption that the noise impairing the system performance has a given probabilistic model. Specifying a model for the noise is a matter still under investigation. Choosing a good probabilistic model to represent the noise perturbing the transmission, when samples of the noise are known, is a task of high relevance when designing the communication system. Many approaches to specify the model have been proposed. In this paper, we assume that a sequence of samples, whether synthetic (computer-generated) or natural (measured noise), is available and compare three common modeling techniques, namely, the Gaussian model, the Middleton Class-A model and the Bernoulli-Gaussian model. Finding the process parameters is an easy task if the noise randomness is well described by a Memoryless Gaussian process (this is in fact a single parameter model). Although this model is easy to specify, it represents the most difficult channel through which one could conceive transmitting information. The specifications of two other models (non-Gaussian models) are discussed in this paper. The choice of a good model is also discussed. The ideas presented provide guidance to choose a good model (better matched to the true noise) based on the performance of LDPC coded transmission over BPSK systems.**

***Keywords*— Bernoulli-Gaussian channel, Channel mismatch, Impulsive noise, LPDC, Middleton Class-A channel.**

I. INTRODUCTION

This paper examines the problem of choosing a probabilistic model to represent a noise known through a sequence of samples. Three memoryless noise models will be considered, namely the Gaussian (G), the Bernoulli-Gaussian (BG), and the Middleton Class-A (MDD) stochastic processes. The mathematical description of these models is well known. Theses descriptions are discussed in detail in [1]. We emphasize, at this point, that while the memoryless Gaussian process is fully specified by choosing a single parameter (σ_0, the standard deviation of a Gaussian probability density function), the other two models description is more sophisticated and are based on three parameters. The BG stochastic process specification relies on two probability density functions specified by the standard deviations σ_0 and σ_1, of two Gaussians and on a third parameter (probability) specified by p_1. The stochastic process description corresponding to the MDD model is based on an infinite sequence of Gaussians probability density functions, all with standard deviation depending on a pair of parameters, (σ_0, Γ), and on an infinite sequence of probabilities (p'_ℓ, ℓ an integer index) taken from a Poisson distribution with parameter A.

Communication Department of the Instituto Tecnológico de Aeronáutica, S. José dos Campos, SP. This work was partially supported by FAPESP (2022/14386-7)

This paper does not focus on direct "parameter estimation" but on "moments estimation" which, once obtained, are then used to calculate the parameters.

Calculating the parameters of a memoryless Gaussian model, given that a sequence of samples of the noise is known, is a simple matter. The usual approach amounts to estimating the variance (assumed to be well approximated by the average value of the squared samples) which characterizes the probability density function. The variance of a Gaussian random variable (assumed to be zero mean) is the sole parameter specifying the process[1] and once the variance is estimated the sought parameter is obtained. In all cases the stochastic process modeling the noise will be represented by the infinite sequence of random variables $\{Z_i\}$ (i is an integer index belonging to the integer set $\mathbb{Z}$).

The parameters of a BG process can also be easily calculated once the 1^{st}, 2^{nd} and 3^{rd} moments (respectively $\widehat{\mathbb{E}}[|Z|]$, $\widehat{\mathbb{E}}[|Z|^2]$ and $\widehat{\mathbb{E}}[|Z|^3]$) of the absolute value of the r.v. Z_i of the BG-process are estimated. We use a recent published result by Finamore *et alli* [1] which introduces the solution to the problem of calculating the parameters of the BG-process which model the noise given that a finite sequence of N noise samples, say $[z_1, \ldots, z_n, \ldots, z_N]$, is known. The solution to the problem of calculating the parameters of the MDD-process by using estimated values $\widehat{\mathbb{E}}[Z^2]$, $\widehat{\mathbb{E}}[Z^4]$ and $\widehat{\mathbb{E}}[Z^6]$ as proposed by Middleton [2] is also used. The results are discussed in Section III.

The model of the transmission medium, obtained from a sequence of available samples, is examined with Information Theory tools in order to assess the achievement of transmission at the highest possible rate (near the calculated capacity of the model).

Our findings are simple but the results are, besides original, of interest to many readers. They are particular useful to those dealing with communications over Power Line Communications or Sonar Communications [4], [5].

The reader should be attentive to the fact that finding the parameters of the stochastic process that mathematically represent the noise is not a guarantee that the chosen model is the best representation of the random phenomenon (noise). If the assumption that the noise has a Bernoulli-Gaussian behavior is true then the three estimated moments $\widehat{\mathbb{E}}[|Z|^\ell]$, $\ell \in \{1, 2, 3\}$ are all one needs to specify the stochastic process (all other moments estimate are, supposedly, close to their respective expected values $\mathbb{E}[|Z|^\ell]$, ℓ a positive integer). If this assumption is not met a discrepancy between the true

[1]As it is well known a memoryless Gaussian stochastic process $\{Z_i\}$ is fully specified by a unique Gaussian probability density function with variance σ_Z^2.

Cumulative Distribution Function (C.D.F.) and the empirical C.D.F. will exist (even if an optimal technique is employed to estimate the three moments). When choosing between two models the one with higher capacity[2] would be preferable. This approach can be justified by a recent result [6] which states that under channel mismatch decoding (not using the true likelihood function) the "mismatched capacity" is an upper bound to the maximum achievable transmission rate. We conjecture thus that a decoder designed with the less mismatched channel/model can produce a better probability of error performance and would thus be preferred. Our research aims at "quantifying the loss from choosing an ill set up model." It is a very simple (and obvious) idea but which has been ofttimes neglected. Our results quantifies how harmful it is to neglect information theory notions.

This work is organized as follows. Section II presents the three models used to describe the channel. The methods used to calculate the parameters of the models are detailed in Section III-C. Some comments on the choice of the model are posted in Section IV. The results of the work are presented in Section V, and the conclusion closes the paper in Section VI.

II. PROBLEM FORMULATION

Our results are set up using the well established assumption to describe mathematically the communication system: a sequence of samples $\{x_i\}$ (the index i is an integer usually identifying a time instant), is sent through a transmission medium perturbed by additive noise. To each channel input x_i belonging to the real set $\mathbb{R}$ there corresponds thus a channel output $y_i = x_i + z_i$ such that a sequence of real samples $\{y_i\}$, which is the addition of the value x_i to a noise component $z_i \in \mathbb{R}$ is delivered at the destination. In our investigation three models (three stochastic processes) are used to describe the random behavior of the perturbing noise $\{z_i\}$.

A. Gaussian Model

The Gaussian model (which specifies what is called the Gaussian Channel) considers that the noise sequence $\{z_i\} = \sigma_0\{r_i^{[0]}\}$ is modeled as the Stoc. Proc. $\{Z_i\} = \sigma_0\{R_i^{[0]}\}$ in which $\{R_i^{[0]}\}$ is a collection of i.i.d. zero mean, unity variance random variables such that $\mathsf{f}_{Z_i}(z)$ the probability density function of all the random variables Z_i are given by the same function

$$\mathsf{f}_Z(z) = \frac{1}{\sigma_0\sqrt{(2\pi)}} \exp\left(-\frac{z^2}{2\sigma_0}\right) \tag{1}$$

B. Bernoulli-Gaussian Model

The BG model (which specifies the BG Channel) considers that the noise sequence is mathematically represented by $\{\tilde{z}_i\} = \sigma_0\{r_i^{[0]}\} + \sigma_1\{s_i^{[1]} r_i^{[1]}\}$. A particular noise sample $\tilde{z}_i$ is viewed as the manifestation of a perturbation which is the addition of the background noise $\sigma_0\{r_i^{[0]}\}$, always present, and an impulsive component which is absent when the state variable value is $s_i^{[1]} = 0$ or present when the value of the state variable is $s_i^{[1]} = 1$ (the process is said to be, in this case, in the impulsive state). The state is modeled as a Bernoulli random variable $S_i^{[1]}$ with $\mathsf{Prob}\{S_i^{[1]} = 1\} = p_1$ which do not change with the time index i. In the first case $\tilde{z}_i = \sigma_0 r_i^{[0]}$ is the ubiquitous thermal noise modeled as a Gaussian random variable $\tilde{Z}_i = \sigma_0 R_i^{[0]}$ and, when $s_i = 1$ the sample values are expressed as $\tilde{z}_i = \sigma_0 r_i^{[0]} + \sigma_1 r_i^{[1]}$, the addition of the always present background noise plus an intermittent component $s_i r_i^{[1]}$. The values $(\sigma_0, \sigma_1, p_1)$ constitute the triple of parameters of interest. The noise which is modeled by the stochastic process $\tilde{Z}_i = \sigma_0\{R_i^{[0]}\} + \sigma_1\{S_i R_i^{[1]}\}$ is fully characterized [1] by the probability density function

$$\begin{aligned}\mathsf{f}_{\tilde{Z}}(z) &= \frac{1-p_1}{\sigma_0\sqrt{2\pi}} \exp\left(-\frac{z^2}{2\sigma_0^2}\right) + \\ &+ \frac{p_1}{\sqrt{2\pi(\sigma_0^2+\sigma_1^2)}} \exp\left(-\frac{z^2}{2(\sigma_0^2+\sigma_1^2)}\right)\end{aligned} \tag{2}$$

(for every i we have $\mathsf{f}_{\tilde{Z}_i} = \mathsf{f}_{\tilde{Z}}$).

C. Middleton Class-A Model

A well known probabilisitic model for impulsive noise, proposed by Middleton [2], the three parameter model that can be written as

$$\check{Z}_i = \sum_{\theta=0}^{\infty} \sigma_\theta\, R_i^{[\theta]}\, \mathbb{I}\{S_i = \theta\}, \tag{3}$$

with the channel states running over an infinite set, namely, $\theta \in \{0, 1, \ldots, \infty\}$, and where $\mathbb{I}$ represents the indicator function. Middleton model [8] imposes also that Poisson be the probability law which is to rule the probability $p_\theta = \mathsf{Prob}\{S_i = \theta\}$ or, specifically,

$$p_\theta = \frac{A^\theta}{\theta!} e^{-A}.$$

In this case, it should be noticed, the probability of observing a noise sample in the background state is $p_0 = e^{-A}$. The power of the sequence of samples in state θ, $(\theta > 0)$ can be expressed through

$$\sigma_\theta^2 = \sigma_0^2\left(1 + \frac{\theta}{A\Gamma}\right), \tag{4}$$

where σ_0^2 is the power of the background noise, and Γ is a power ratio, given by $\Gamma = \frac{\sigma_0^2}{\sigma_I^2}$, $\sigma_I^2 = \sigma_{\check{Z}}^2 - \sigma_0^2$.

The probability density function of the random variable $\check{Z}$ is,

$$\mathsf{f}_{\check{Z}}(\zeta) = \sum_{\theta=0}^{\infty} p_\theta \left(\frac{1}{\sigma_\theta\sqrt{2\pi}} e^{-\zeta^2/2\sigma_\theta^2}\right).$$

Since $\sigma_{\check{Z}}^2 = \sum_{\theta=0}^{\infty} p_\theta \sigma_\theta^2$ one need thus, to specify the process, just to know the three parameters, A, σ_0, and $\sigma_{\check{Z}}$ (or Γ). Techniques to estimate the parameters of the Middleton Model have been presented in [2].

The Middleton model yet admitting that the noise goes through a very large number of states is however a model which, restricts the states to obey a Poisson distribution and depends on only three parameters.

[2]The capacity is a property of the model—to every noise probability density function there corresponds a channel capacity

III. Calculating the parameters of each model

Let us say that a block of N_s noise samples $[z_1, \ldots, z_n, \ldots, z_{N_s}]$ has been provided. The ℓ-th moment, v.i.z., $\mathbb{E}[|Z|^\ell]$ of the stochastic process $\{Z_i\}$ (considered to be stationary and ergodic) is estimated by a value $M_\ell = \widehat{\mathbb{E}}[|Z|^\ell]$ given by

$$M_\ell = \frac{1}{N_s}\sum_{n=1}^{N_s} |z_n|^\ell. \tag{5}$$

Without loss of generality we will assume that the samples $[z_1, \ldots, z_n, \ldots, z_{N_s}]$ have been normalized such that $M_2 = 1$. The parameters associated to each model are thus calculated as follows.

A. Gaussian model parameters

The estimated parameter $\hat{\sigma}_0$ specifying the G-model, as it is well known, is given by $\hat{\sigma}_0 = M_2 = 1$.

B. Bernoulli-Gaussian model parameters

Using the results in [1] and considering that the estimated moments M_1, M_3 (with, of course $M_2 = 1$) are known, the parameters specifying the BG-model, namely σ_0, σ_1, p_1, can be obtained by first finding the largest root ξ of the equation

$$\alpha^2 - (\kappa - 2)\alpha + 1 \quad = \quad 0, \tag{6}$$

in which κ is defined as

$$\kappa =: \frac{\left(M_3\sqrt{\pi/8} - M_1\sqrt{\pi/2}\right)^2}{(M_1 M_3 \pi/4 - 1)\,(1 - M_1^2\pi/2)}. \tag{7}$$

If the first three moments estimation fall in the BG-process feasible region (i.e., they obey the conditions $M_1 > \frac{4}{\pi}\frac{1}{M_3}$ and $M_1 < \sqrt{\frac{\pi}{2}}$) then the three parameters can be computed by using the relations

$$\begin{aligned}
\sigma_0 &= \frac{M_3\sqrt{\pi/8} - M_1\sqrt{\pi/2}}{(1 - M_1^2\sqrt{\frac{\pi}{2}})(\xi + 1)}, & (8)\\
\sigma_1 &= \sigma_0\sqrt{\xi^2 - 1}, \text{ and} & (9)\\
p_1 &= \frac{1 - \sigma_0^2}{\sigma_1^2}. & (10)
\end{aligned}$$

If the estimated moments fall outside the feasible region the designer should be aware that no BG-process with such values do exist. The BG process obtained by replacing the estimated moments by new values, within the feasible region, close to the current moments can however produce a BG-process which can be used as a model.

C. Middleton Class-A model parameters

As shown in [2], the three parameters of the Middleton Class A model can be obtained from M_2, M_4, and M_6. The mathematical deduction presented below shows how the parameters can be calculated. Given the state $S_i = \theta$, the moments are given by the Gaussian results [3], that is,

$$\begin{aligned}
\mathbb{E}[\check{Z}_i^2 | S_i = \theta] &= \sigma_\theta^2,\\
\mathbb{E}[\check{Z}_i^4 | S_i = \theta] &= 3\sigma_\theta^4, \text{ and}\\
\mathbb{E}[\check{Z}_i^6 | S_i = \theta] &= 15\sigma_\theta^6.
\end{aligned}$$

Using (4) in the above equations, and applying the total probability rule for expected value, it can be shown that

$$\begin{aligned}
\mathbb{E}[\check{Z}_i^2] &= \sigma_0^2\left(1 + \Gamma^{-1}\right),\\
\mathbb{E}[\check{Z}_i^4] &= 3\sigma_0^4\left(1 + 2\Gamma^{-1} + \Gamma^{-2} + \Gamma^{-2}A^{-1}\right), \text{ and}\\
\mathbb{E}[\check{Z}_i^6] &= 15\sigma_0^6\left(1 + 3\Gamma^{-1} + 3\Gamma^{-2} + 3\Gamma^{-2}A^{-1} + \right.\\
&\qquad \left. + \Gamma^{-3} + 3\Gamma^{-3}A^{-1} + \Gamma^{-3}A^{-2}\right).
\end{aligned}$$

The above results were reached using also expressions for the first, the second and the third moments of a Poisson random variable [3]. These three equations form a non linear system which can be solved mathematically. Replacing the moments by their estimated values and considering that $M_2 = 1$, the parameters are given by

$$\begin{aligned}
\Gamma &= \frac{\left(\frac{M_6}{15} - 1\right) - 3\left(\frac{M_4}{3} - 1\right)}{\left(\frac{M_4}{3} - 1\right)^2} - 1, & (11)\\
\sigma_0 &= \sqrt{\frac{1}{1 + \Gamma^{-1}}}, \text{ and} & (12)\\
A &= \frac{\sigma_0^4\Gamma^{-2}}{\frac{M_4}{3} - 1} & (13)
\end{aligned}$$

As in the Bernoulli-Gaussian case, the Middleton class A model has also a feasible region for their moments (which is given by $M_4 > 3$ and $\frac{M_6}{15} > \frac{M_4^2}{9} + \frac{M_4}{3} - 1$). Therefore, if the estimated moments fall outside this reagion, no Middleton process with such values do exist. In this case, the model can also be used using new values close to those estimated.

IV. Choosing a model

The bulk of results on "transmission over non-Gaussian channels" focus on discussing "proposed models", "estimating the parameters of proposed models", and "calculating the capacity of the correspondent channels" but do not address the problem of choosing a good model (check the recent survey by T. Bai et alli [8] and references therein). Finding the parameters of a stochastic process which model the noise perturbing the transmission medium does not guarantee that such a model is the best choice. Many written papers addressing the subject of reaching channel capacity neglects the fact that that channel capacity limits the optimum transmission rate only if one is using the proper model. This is crucial point emphasized in our manuscript—it sounds like a trivial statement but it is, in many cases, an often misunderstood fact.

An analysis addressing this issue is presented next. In our experiments we examine the performance of systems which transmit LPDC encoded information over a binary input channel impaired by additive noise. This analysis, which follows a practical approach, can indicate (as shown by our results) which model is subject to the worst mismatch between the theoretical and simulated performance.

V. Experimental results (simulations)

To clearly understand the benefits of choosing the right model the performance of three types of noise were examined. Three sequences of noise samples (16M samples) synthetically generated were examined. The first sequence (NoiseSeq01-BGsamples) bears the characteristics of a BG-process. The pseudo-random behavior of the second (NoiseSeq02-MDDsamples) sequence is ruled by the characteristics of a MDD-process. The nature of the randomness of the third sequence (NoiseSeq03-gBGsamples) bears the characteristics of a process with five states each of them adding an independent impulsive gaussian component with arbitrary power. This third sequence is called a generalized Bernoulli-Gaussian noise (gBG-noise, in short).

Figure 1 exhibits the performance of the communication system when *NoiseSeq01-BGsamples* is the sequence of noise samples used (which corresponds to samples of noise synthetically generated by using the probability density function of a BG stochastic process).

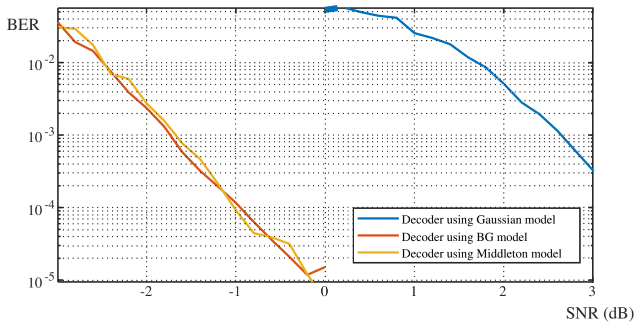

Fig. 1. Noise sequence (*NoiseSeq01-BGsamples*) generated with parameters $\left(\mathbb{E}[|Z|], \mathbb{E}[|Z|^2], \mathbb{E}[|Z|^3]\right) = (0.60, 1.00, 3.00)$ or yet $(\sigma_0, \sigma_1, p_1) = (0.4421, 2.1082, 0.1811)$. The estimated parameters under the assumption that these are noise samples taken from a BG-process are $\left(\hat{\sigma}_0, \hat{\sigma}_1, \hat{p}_1\right) = (0.4417, 2.1071, 0.1813)$. The estimated parameters under the assumption that these are samples taken from a noise random behavior is modeled as a MDD-process are $\left(\hat{\sigma}_0, \hat{A}, \hat{\Gamma}\right) = (0.1807, 0, 3199, 0.0338)$.

As it can be observed the system performance of both BG and Middleton models are better than the performance when the receiver is built under the assumption that the noise is purely Gaussian. It can also be observed that both BG and MDD exhibit quite similar performance. In fact, the LDPC decoders designed with the Log-Likelihood-Ratio (LLR) based on these models can lead to improvements over 4 dB when compared to the LDPC decoder opperating with the Gaussian LLR.

For this first sequence, it is also important to mention that the estimated moments fell outside the feasible region of the Middleton model. Figure 2 illustrates the position of the pair (M_4, M_6) and compares it with the validity region. Therefore, the performance of the decoder based on the Middleton model, shown in Figure 1, was obtained with values close to those estimated. It is interesting to observe that even with a small adjustment in the estimated values, the results of the MDD model are close to the correcto model (BG-noise). In fact, in the simulations, small deviations in the estimated moments did not lead to significant changes in performance. Therefore, chossing a good model seems to be more important than using an optimal estimator.

Figure 3 exhibits the performance of another system: a system in which the transmission is perturbed by MDD-noise (*NoiseSeq02-MDDsamples* corresponds to samples of noise synthetically generated by using a 10 states probability density

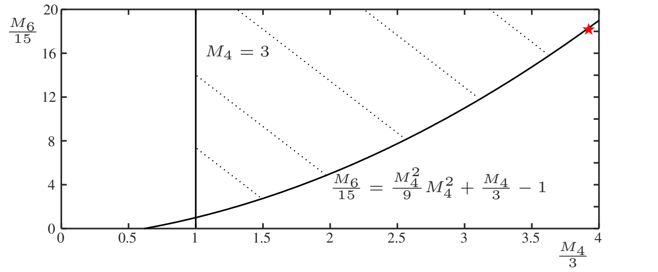

Fig. 2. The red star shows the position of the pair (M_4, M_6), falling outside the feasible region

function of a MDD stochasticv process). For this second sequence, the estimated moments fell within the feasible regions of the two models (BG and MDD).

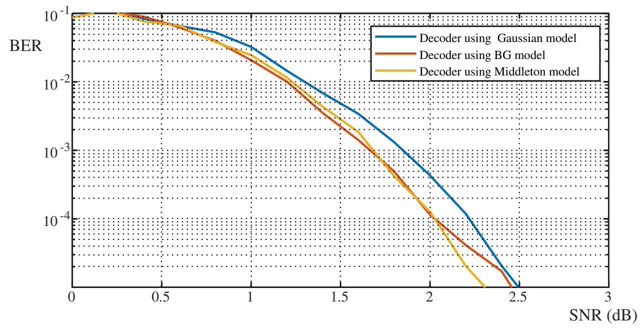

Fig. 3. Noise sequence (*NoiseSeq02-MDDsamples*) was generated by taking $p_1 = 0.20$ and $\Gamma = 0.19$ (and, of course $\sigma_Z = 1.00$) which rendered the parameter $A = 1.6094$. The estimated parameters under the assumption that these are samples of a BG-process are $\left(\hat{\sigma}_0, \hat{\sigma}_1, \hat{p}_1\right) = (0.5837, 1.1369, 0.5101)$. The estimated moment under the assumption that these are samples of a MDD-process are $\left(\hat{\sigma}_0, \hat{A}, \hat{\Gamma}\right) = (0.3786, 1.6772, 0.1673)$.

Again it can be observed that the system performance of both BG and MDD models are better than the performance when the receiver is built under the assumption that the noise is purely Gaussian. It can be also noticed that both BG and MDD models have quite similar performances, even though, in this case, they are close to the results achieved by the decoder based on the Gaussian model. It is important to mention that this similarity with the result of the Gaussian model depends on the parametes of the model used. Actually, for the BG model, it would also be possible to achieve similar results for the three models, by adjusting the parameters. Analogously, we believe that the choice of distinct parameters for the MDD model can also deviate the curve of the Gaussian model.

Finally, Figure 4 exhibits the performance of a system in which the transmission is perturbed by a gBG-noise (*NoiseSeq03-gBGsamples* corresponding to samples of noise synthetically generated by using the probability density function of gBG stochastic process, which corresponds to the mixture of 5 Gaussians random variables with the background variance equal to $\sigma_0 = 1.0$ (state 0) and states variances σ_i are equal to 2.0, $\sqrt{2.0}$, $\sqrt{3.0}$ and 4.0 for i respectively equal to 1, 2, 3, and 4). Also in this third case, the estimated moments fell within the feasible regions of the models.

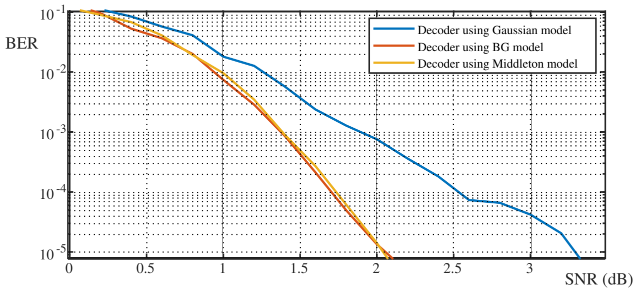

Fig. 4. Noise sequence (*NoiseSeq03-gBGsamples*) was generated by mixing five Gaussian random variables. During 88 percent of the time the noise is purely Gaussian (is in the background state). As for the other states we have used $(p_1, p_2, p_3, p_4) = (0.032, 0.064, 0.016, 0.008)$. The selected variances are, respectively, $(\sigma_0, \sigma_1, \sigma_2, \sigma_3, \sigma_4) = (1, 2, \sqrt{2}, \sqrt{3}, 4)$. The estimated parameters under the assumption that these are samples from a BG-process are $(\hat{\sigma}_0, \hat{\sigma}_1, \hat{p}_1) = (0.9055, 2.5651, 0.0174)$. The estimated moment under the assumption that these are samples of a MDD-process are $(\hat{\sigma}_0, \hat{A}, \hat{\Gamma}) = (0.9321, 0.0139, 6.6242)$.

Notice that, once more, the system performance of both BG and MDD models are better than the performance of a receiver built under the assumption that the noise is purely Gaussian. It can be also noticed that both BG and MDD have a quite similar performance. Choosing to model the noise with either as a Bernoulli-Gaussian stochastic process or a Middleton would lead to a better design.

VI. Conclusion

We have examined the problem of choosing a good probabilistic model to represent the noise perturbing the communication over a transmission medium when samples of the noise are known. The performance of communication systems impaired by noise has been extensively and successively examined under the assumption that the noise impairing the system performance has a given probabilistic model. In the current paper we target the more ambitious task of choosing a probabilistic model which better match the random behavior of the noise. The optimum decoder under mismatched condition is a theoretical subject of current interest which has received recent attention [6], [7]. The ideas presented in our paper provide practical guidance to choose a good model (better matched to the true noise) based on the examination of the performance of LDPC coded transmission over BPSK systems.

A relevant task is to find the parameters of the model. Three common modeling techniques, namely, the Gaussian model (characterized by the parameter σ_0^2—Gaussian noise power), the Bernoulli-Gaussian (σ_0^2—background Gaussian noise power, σ_1^2—impulsive noise power, p_1—probability of being in the impulsive state) and Middleton Class-A model (σ_0^2 and parameters A and Γ) were examined. Many approaches to specify these models parameters have been proposed in the literature [8]. We have chosen to specify the parameters by calculating their exact values under the assumption that exact values of the expected values of the process are known. This is, of course, a theoretical assumption: in practice the expected values are never known, the "estimated expected values" thus, and only the estimates, can be obtained. In the current paper we took synthetically generated noise samples to evaluate the performance of the LDPC coded transmission over BPSK. The results have shown that the use of the Gaussian model can produce a loss above 4 dB, if the channel adds impulsive noise. The simulation results also showed that BG and MDD models render systems with similar performance for different kinds of impulsive noise. However, since the MDD model leads to a more complex implementation due to multiple states, the BG model can be considered a good choice.

Finally, we conjecture that there are many advantages from using these results. We anticipate that this would facilitate the development of adaptive receivers, assist in discovering efficient methods for transmission over a non-Gaussian channel, and pave the way for investigating the potential gains from using models that incorporate possible noise memory.

Aknowledgements

This work was supported in part by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) under grant 2022/14386-7.

References

[1] W. A. Finamore, M. da S. Pinho, M. Sharma and M. V. Ribeiro, *Modeling Noise as a Bernoulli-Gaussian Process*. JCIS, Vol. , No. , May 2023 — submitted for publication (a preliminary version is available at https://doi.org/10.36227/techrxiv.17912276.v1).

[2] D. Middleton, "Procedures for determining the parameters of the first-order canonical models of class A and class B electromagnetic interference", *IEEE Trans. Electromagn. Compat.*, vol. 14, No. 3, pp. 190-208, August 1979. (DOI: 10.1109/TEMC.1979.303731).

[3] A. Papoulis & S. U. Pillai, "Probability, Random Variables and Stochastic Processes." McGraw-Hill, 4th Ed. 2002. (ISBN 0-07-366011-6; ISBN 0-07-112256-7 (ISE)).

[4] F. Rouissi, A. J. Han Vinck, H. Gassara and, A. Ghazel, "Improved impulse noise modeling for indoor narrow-band power line communication," *International Journal of Electronics and Communications*, pp. 74-81, 2019. (DOI: 10.1109/ISPLC.2017.7897119).

[5] M. Stojanovic and J. P. Woods, "Underwater Acoustic Communication Channels: Propagation Models and Statistical Characterization," *IEEE Communications Magazine*, pp. 84-89, Jan. 2009. (DOI: 10.1109/MCOM.2009.4752682).

[6] E. A. Kangarshahi and A. Guillén i Fàbregas, *A Sphere-Packing Error Exponent for Mismatched Decoding,* IEEE Trans. Inf. Theory, Vol. 69, No. 5, May 2023

[7] N. Merhav and G. Böcherer, *Codebook Mismatch can be Fully Compensated by Mismatched Decoding,* IEEE Trans. Inf. Theory, Vol. 69, No. 4, April 2023

[8] T. Bai et alli, *Fifty Years of Noise Modeling and Mitigation in Power-Line Communications,* IEEE Communications Surveys & Tutorials., pp. 1-29, 2019. (DOI 10.1109/COMST.2020.3033748).