ANALYTICAL SYSTEM ADJUSTMENT FOR THE DISCRETE-TO-CONTINUOUS DISTORTION

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ABSTRACT

This paper proposes an extension of the analytical compensation method for the sample-and-hold distortion presented in [1]. That distortion can introduce a degradation of up to 4 dB in the magnitude of the system frequency response. The proposed compensation method adjusts the system parameters without changing the system order. Using an adequate set of adjustment frequencies, we obtain new parameters by solving a linear system of equations. Following the adjustment, we apply frequency scaling to the transfer function. The method is applicable to monotonic and equiripple (Butterworth, Legendre, Chebyshev, Cauer, etc.) approximations. Examples are given to illustrate the compensation accuracy. Results indicate a better compensation than existing iterative and non-iterative techniques.

1. INTRODUCTION

Continuous-time signals are often processed by digital or sampled-data systems. In those systems, the interpolation process distorts the frequency spectrum of the continuous-time output signal. The most important distortion is the well-known $(\sin x)/x$, which introduces a significant distortion (up to 4 dB) for low sampling-to-signal frequency ratios [2-5]. Thus, many practical processing systems require $(\sin x)/x$ equalization. To achieve this equalization, we look for a new transfer function to accomplish both the required signal processing and equalization [2-6]. Moreover, we would like to achieve the necessary equalization with minimum design effort and little dependence on the designer's expertise.

By using numerical optimization techniques [2,7], one can achieve accurate distortion compensation. Applications that require optimization include the design of nonsymmetrical filters and the frequency response approximation with simultaneous specification of magnitude and phase. The accuracy and convergence are strongly dependent on both the system order and the initial set of adjustment frequencies. In general, to achieve an adequate compensation one requires expertise on optimization techniques. Moreover, numerical optimizations may increase the system order and the associated hardware [6].

These issues justify the search for simpler and efficient compensation methodology. Analytical equalization methods have been proposed for specific distortion problems [2,4-6] when enough information is available. Typical examples are compensation for the $(\sin x)/x$ distortion and rolloff in telephone channels. Analytical methods usually produce minimum order systems, permit convergence control and provide a better insight into the problem.

Analytical approaches for the compensation of the $(\sin x)/x$ distortion include the system design with compensation [8] or only the adjustment of an existing system transfer function [9]. The former approach may increase the order of the system. The latter adjusts the coefficients of the existing system. Usually system adjustment is a more cost-effective solution whenever a transfer function is already available. Among the existing analytical adjustment techniques, some are iterative [5,9] and other are non-iterative [4,9] methods. Non-iterative methods determine the compensated system function in one step. They usually require smaller computational effort as compared with the iterative methods. On the other hand, iterative methods equalize the distortion sequentially at a set of adjustment frequencies over a number of iterations, which may provide better compensation results than the non-iterative approaches.

This paper presents an extension of the analytical approach to compensate for the $(\sin x)/x$ distortion introduced in [1]. Now, the method is applicable to monotonic and equiripple (Butterworth, Legendre, Chebyshev, Cauer, etc.) approximations. It compensates for the distortion at every adjustment frequency in a single step. However, successive application of the method further reduces the distortion. By using an adequate set of frequencies and assuming small pole displacement, we find a linear system of equations. The solution of this set of equations produces the parameters of the compensated system. Following the adjustment, we apply frequency scaling to the transfer function. The method leads to compensation results comparable to those obtained by the best techniques. The achieved accuracy is compatible with the usual precision level limitations imposed by component precision and signal quantization. Examples using monotonic and equiripple approximations illustrate the accuracy of the method.

2. COMPENSATION METHOD

Let us consider an *s*-domain transfer function $H(s)$. This function may represent an analog reconstruction filter or a discrete-time system. In the latter, we can determine $H(s)$ from the *z*-domain transfer function through a *z*-to-*s* transformation. For this purpose, the inverse bilinear transformation is much used [5,9]. Since the processing system and the distortion are both in the signal path, it is possible to reduce the overall output spectral distortion by compensating $|H(j\omega)|$ [9].

Consider an *n*-th order transfer function $H(s)$ given by

$$H(s) = \frac{N(s)}{D_0(s)\prod_{i=1}^{k}(s^2 + a_i s + b_i)} . \qquad (1)$$

For n even, $k = n/2$ and $D_0(s) = 1$. For n odd, $k = (n-1)/2$ and $D_0(s) = (s + a_0)$.

$$\Delta\left|H(j\omega)\right| \cong \sum_{i=1}^{k} \frac{\partial\left|H(j\omega)\right|}{\partial a_i}\cdot\Delta a_i + \frac{\partial\left|H(j\omega)\right|}{\partial b_i}\cdot\Delta b_i \qquad (5)$$

Application of (5) at $2k$ adjustment frequencies leads to a linear system of $2k$ equations in Δa_i and Δb_i. Thus, for a set $A = \{\omega_1, \omega_2, \ldots, \omega_{2k}\}$ of adjustment frequencies, (5) yields

$$\begin{bmatrix} \Delta a_1 \\ \vdots \\ \Delta a_k \\ \Delta b_1 \\ \vdots \\ \Delta b_k \end{bmatrix} = \begin{bmatrix} \Psi(\omega_1, a_1) & \cdots & \Psi(\omega_1, a_k) & \Phi(\omega_1, b_1) & \cdots & \Phi(\omega_1, b_k) \\ \vdots & & \vdots & \vdots & & \vdots \\ \Psi(\omega_k, a_1) & \cdots & \Psi(\omega_k, a_k) & \Phi(\omega_k, b_1) & \cdots & \Phi(\omega_k, b_k) \\ \Psi(\omega_{k+1}, a_1) & \cdots & \Psi(\omega_{k+1}, a_k) & \Phi(\omega_{k+1}, b_1) & \cdots & \Phi(\omega_{k+1}, b_k) \\ \vdots & & \vdots & \vdots & & \vdots \\ \Psi(\omega_{2k}, a_1) & \cdots & \Psi(\omega_{2k}, a_k) & \Phi(\omega_{2k}, b_1) & \cdots & \Phi(\omega_{2k}, b_k) \end{bmatrix}^{-1} \cdot \begin{bmatrix} \Delta\left|H(j\omega_1)\right| \\ \vdots \\ \Delta\left|H(j\omega_k)\right| \\ \Delta\left|H(j\omega_{k+1})\right| \\ \vdots \\ \Delta\left|H(j\omega_{2k})\right| \end{bmatrix} \qquad (6)$$

The spectral distortion caused by a D/A conversion can be modeled by the function [9]:

$$S(\omega) = S_a(\omega)e^{-j\omega\tau/2} \qquad (2)$$

where

$S_a(\omega) = \frac{\sin(\omega\tau/2)}{(\omega\tau/2)}$,
$\alpha = \tau/T_s$,
τ is the width of the sampling pulse,
$T_s = 1/f_s = 2\pi/\omega_s$ is the sampling period,
ω is the frequency in rad/s.

Thus, we model the system frequency response $H_d(j\omega)$ including the distortion by:

$$\left|H_d(j\omega)\right| = \left|H(j\omega)\right|\left|S(\omega)\right| , \qquad (3)$$

for $H(j\omega)$ (the desired frequency response) given by (1) with $s = j\omega$.

The compensation can be achieved by adjusting the coefficients a_i and b_i $(i = 1, \ldots, k)$ of $H(s)$. Ideally, that compensation generates a modified function $H_m(j\omega)$ such that $\left|H_m(j\omega)\right|\left|S(\omega)\right| = \left|H(j\omega)\right|$ at the adjustment frequencies. At those frequencies, the compensation causes a correction of $\Delta\left|H(j\omega)\right|$ in the system frequency response, given by

$$\Delta\left|H(j\omega)\right| = \left|H_m(j\omega)\right| - \left|H(j\omega)\right| = \frac{\left|H(j\omega)\right|}{\left|S(\omega)\right|} - \left|H(j\omega)\right| . \qquad (4)$$

Assuming independent and small parameter changes Δa_i and Δb_i, we can approximate the correction $\Delta\left|H(j\omega)\right|$ by:

where

$$\Psi(\omega_x, a_i) = \left.\frac{\partial\left|H(j\omega)\right|}{\partial a_i}\right|_{\omega = \omega_x} \qquad (7a)$$

and

$$\Phi(\omega_x, b_i) = \left.\frac{\partial\left|H(j\omega)\right|}{\partial b_i}\right|_{\omega = \omega_x} . \qquad (7b)$$

The new parameters $\hat{a}_i$ and $\hat{b}_i$ of the modified transfer function $H_m(s)$ are then

$$\hat{a}_i = a_i + \Delta a_i \text{ and } \hat{b}_i = b_i + \Delta b_i , \qquad (8)$$

for $i = 1, 2, \ldots, k$. The choice of a set A of adjustment frequencies affects the compensation results. We discuss this issue in the following section.

3. CHOICE OF THE ADJUSTMENT FREQUENCIES

This method requires a choice of the frequencies where the distortion is to be minimized. However, we are restricted to a set of $2k$ frequencies, since $2k$ is the number of parameters (a_i and b_i) to adjust.

Most practical filtering applications uses two major classes of approximation functions: those with equiripple magnitude in the passband (such as Cauer and Chebyshev), and those with monotonic passband (such as Butterworth, Legendre and Inverse Chebyshev). We propose different strategies for choosing the adjustment frequencies for each of these classes.

For equiripple functions, the magnitudes at the peaks and valleys of the passband frequency response are very important. Minimization of the magnitude distortion at those points also restrains it to the specified passband ripple. Thus, we choose the frequencies corresponding to the maxima and minima of the desired passband frequency response as adjustment frequencies.

For functions with monotonic passband behavior, such as Butterworth and Inverse Chebyshev, the choice of adjustment frequencies is based on a logarithmic distribution within the passband. For Legendre monotonic approximation, one uses the adjustment frequencies, ω_x, such that $\left.\frac{d^2|H(\omega)|}{d\omega^2}\right|_{\omega=\omega_x} = 0$.

4. REPEATED APPLICATIONS OF THE METHOD

We use a first order approximation for $\Delta|H(j\omega)|$ (Eq. (5)). The approximation is more accurate for smaller Δa_i and Δb_i. The required Δa_i and Δb_i depend on the distortion amplitude and the characteristics of the desired frequency response. In most cases, the set of adjustment frequencies proposed results in sufficiently small Δa_i and Δb_i and it reduces the passband distortion to acceptable levels. In all cases tested, the distortion level was reduced.

For more demanding applications, the results obtained from a single application of the method may not satisfy the design requirements. We process the remaining distortion as another compensation problem. However, in this case, the passband distortion has been reduced by the previous application of the method. Hence, the required $\Delta|H(j\omega)|$ is smaller and approximation (5) will be more accurate.

To reapply the compensation method we replace (4) by

$$\Delta|H(j\omega)| = \frac{|H(j\omega)|}{|S(\omega)|} - |H_m(j\omega)| \qquad (9)$$

since now $|H_m(j\omega)|$ is the new starting frequency response. Moreover, $|H(j\omega)|$ is replaced by $|H_m(j\omega)|$ in Eq. (7).

We can repeat this process more times to improve the level of compensation. Three applications are sufficient to produce an accurate compensation in a first stage of the process.

5. FREQUENCY SCALING

Although the errors at adjustment frequencies are very small, there is a perceptible difference among the peaks of the original function and of the function compensated by the method. This shifting among peaks is caused by alterations made in the pole magnitudes. An approach to reduce this effect is presented in [9]. With this technique, the peaks of both functions, original and compensated, get very close and there is a small increase in distortion at the adjustment points. Finally, this shifted function can be re-submitted to the iterative process. Consequently, the shifting of peaks is reduced and the errors at adjustment frequencies are kept very small. For some cases, we observed no significant improvement by reapplying our method.

6. EXAMPLES AND COMPARISONS

This section presents several examples to demonstrate the applicability of the proposed compensation method. The selection of adjustment frequencies is based on the criteria explained in Section 3.

Figures 1 to 3 illustrate the method application to a transfer function based on an 8-th order Butterworth function. Figures 4 to 6 depict the results for a 4-th order Chebyshev function. Figures 7 to 9 show the results for a 6-th order Legendre function.

We notice a very small difference between the desired and compensated responses in the passband. Outside the passband frequencies, the attenuation requirements are met. Moreover, we observe a very small change in the phase response due to the adjustment.

All examples use three applications of the method followed by frequency scaling. For some cases, one last adjustment can still be done.

7. CONCLUSIONS

We achieve a significant improvement in accuracy provided by the analytical compensation method. The proposed method can also be applied to compensate for the typical rolloff distortion. This effect is found in analogue telephone channels [9]. For this purpose, it is necessary to exchange $S(\omega)$ by the rolloff magnitude (or an estimation of it) in all expressions. Other similar distortions may also be compensated by the proposed method.

8. REFERENCES

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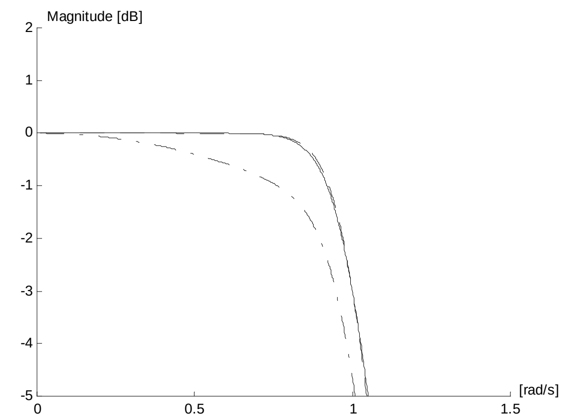

Fig. 1 - Butterworth 8-th order function. (—) Desired response; (---) compensated response; (-.-.-) without compensation.

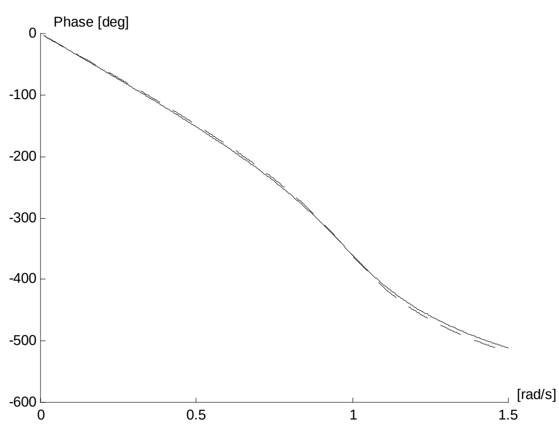

Fig. 2 - Illustrate phase of desired and compensated response for Butterworth 8-th order. (—) Desired response; (---) compensated response.

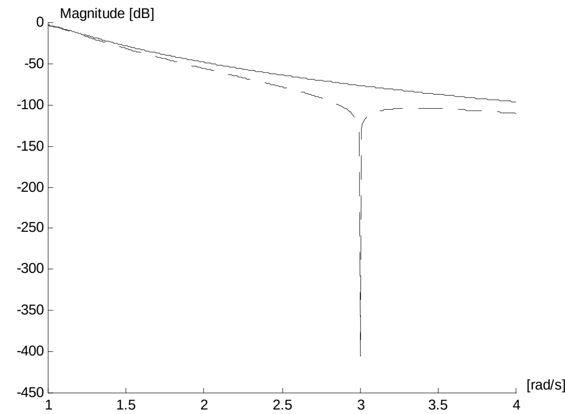

Fig. 3 - Illustrate response in the rejection band for the 8-th order Butterworth transfer function. (—) Desired response; (---) compensated response.

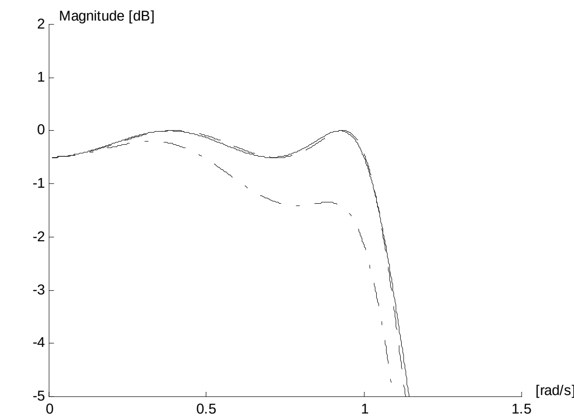

Fig. 4 - Chebyshev 4-th order. (—) Desired response; (---) compensated response; (-.-.-) without compensation.

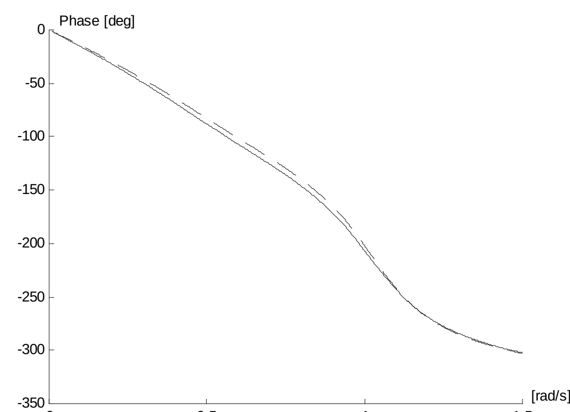

Fig. 5 - Illustrate phase of desired and compensated response for Chebyshev 4-th order. (—) Desired response; (---) compensated response.

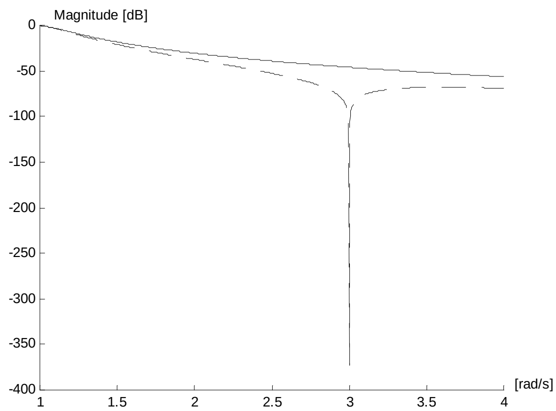

Fig. 6 - Illustrate response in the rejection band for the 4-th order Chebyshev transfer function. (—) Desired response; (---) compensated response.

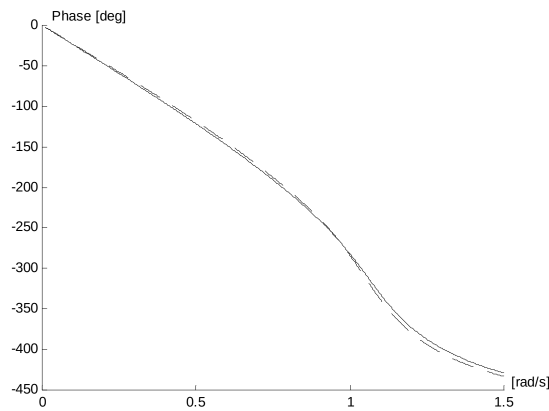

Fig. 8 - Illustrate phase of desired and compensated response for Legendre 6-th order. (—) Desired response; (---) compensated response.

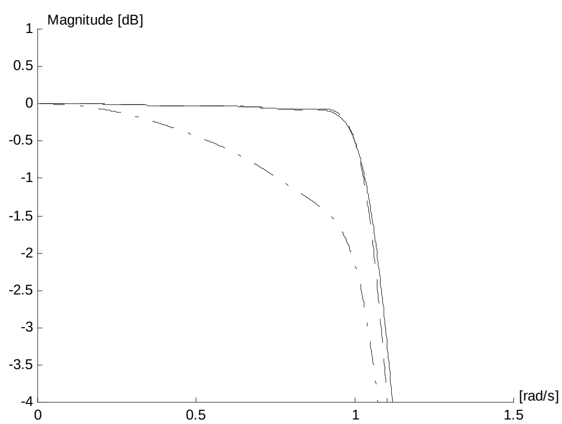

Fig. 7 - Legendre 6-th order. (—) Desired response; (---) compensated response; (-.-.-) without compensation.

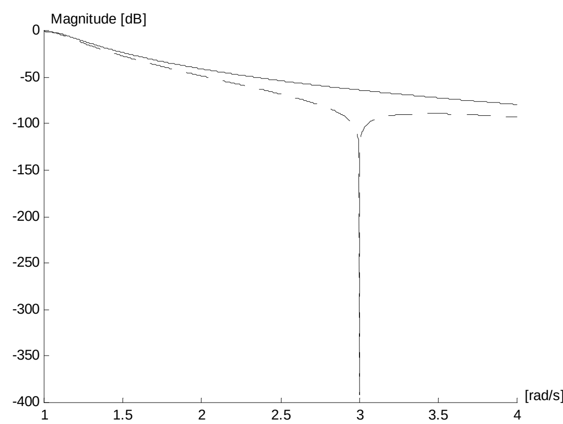

Fig. 9 - Illustrate response in the rejection band for the 6-th order Legendre transfer function. (—) Desired response; (---) compensated response.