FRACTAL IMAGE COMPRESSION VERSUS WAVELET TRANSFORM COMPRESSION APPLIED TO COMPUTERIZED TOMOGRAPHY IMAGES

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ABSTRACT

This paper studies the advantages and disadvantages of Fractal Image Compression (FIC) compared to Wavelet Transform Compression (WTC). The main point here is the quality x compression relation. As this paper deals with medical images, the image quality is vital, therefore a quantitative analysis was done using the PSNR (Peak Signal-Noise Ratio) criteria, and a qualitative analysis was performed by medical doctors, experts in this field. Our results indicate that there is an advantage in FIC for high compression ratios when we take the image quality into account. Considering the compression speed we have a significant difference, which indicates that WTC is faster than FIC.

1. INTRODUCTION

Digital images require a large amount of data to be represented. To make image storage and transmission practical and economical, image compression has became a major issue. Image compression takes advantage on the one hand of human eye limits, and on the other hand of the natural redundancy of images. Based on the fact that human eye can tolerate small errors in images, several compression methods have been developed during the last two decades.

In the last few years, several image compression methods using fractals and wavelets theory have been developed. This methods promise better compression performances with better image quality [7, 10].

The focus on this paper is the comparison of FIC to WTC applied to computerized tomography (CT) images in the sense of image quality. The importance of this is that medical images can not lose the diagnosis properties.

The reason for use CT images is that they are high resolution gray levels images. We could use SPECT[1] or PET[2] images but those kind of images are poor in spatial resolution. Another point is that CT images are analyzed in grey level, and SPECT images, for example, are analyzed with pseudocolor techniques.

This work was supported by Fundação Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES), Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), and by Pontifícia Universidade Católica do Rio Grande do Sul (PUCRS).

[1]Single Photon Emission Computed Tomography

[2]Positron Emission Tomography

In section 2 we introduce the concepts behind fractal image compression, with a quick view to some partition schemes. Section 3 describes the functioning of wavelet transforms applied to image compression. Experimental results, implementation details, and some methods discussion are given in section 4.

2. FRACTAL IMAGE COMPRESSION

Fractal techniques have been applied to several areas of digital image processing, such as image segmentation [15], image analysis [16], and texture coding [12]. FIC was first proposed by Barnsley: objects or images could be modeled by deterministic fractal objects (attractors of sets of two dimensional affine transformations [3, 10]).

The method consist of finding a construction rule that produces a fractal image which approximates the original image. Redundancy reduction is achieved by describing the original image through contracted parts of the same image [7].

2.1. Fractals and Iterated Function Systems

The construction of an iterated function system that approximates the original image is basis for FIC. An IFS is a union of contractive transformations that maps to itself. For a transformation W to be contractive, equation (1) must be satisfied. This equation states that the distance $d(P_1, P_2)$ between any two points, $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, is diminished by applying the transformation W. A metric used to measure distance, standard Euclidean metric, in a two dimensional space is given in (2).

$$d(W(P_1), W(P_2)) < d(P_1, P_2) \tag{1}$$

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{2}$$

With fractal image compression, an image is encoded as the fixed point of a contractive mapping. The image is partitioned into blocks of square shape, named *range* blocks [23]. The goal for the encoding is to determine, for each range block, the best matching block, named *domain* block. The domain block should be larger in size than the range block to which it maps in order to fulfill contractivity requirements [7].

The range-domain set forms an IFS that has the decoded image as the attractor. Besides, the attractor should exhibit less complexity than the original image. Complexity should here be understood as the amount of storage needed to describe the object or the transformations. To store the compressed image, only the coefficients of each IFS must be stored. Given an image S, the algorithm must produce a union of transformations $w_1, w_2, \ldots, w_n$ that satisfies (3).

$$S = W(S) = w_1(S) \cup w_2(S) \cup \ldots \cup w_n(S) \qquad (3)$$

Equation (3) shows how the construction of an attractor can be achieved, i.e. by partitioning S into n pieces. Finding W so that $S = W(S)$ exactly with less complexity than the original image is not likely to be obtained [10]. Since, fractal image compression is a lossy compression method, the equality $S = W(S)$ does not have to be exactly fulfilled. Instead, S can be approximated by $W(S)$. Assuming a distortion measure $d(S, S')$ between the images S and S' exists, the problem is to minimize $d(S, W(S))$ and the complexity of W. This can also be stated as minimizing the loss and maximizing the compression ratio. For all practical purposes, the choice is more a trade-off between compression ratio and distortion.

2.2. Image Partitioning Schemes

An obvious partition scheme of S is to divide the image into a number of non-overlapping quadratic square range blocks as implemented in the first algorithms developed [7, 10]. An image of size 256×256 is partitioned into squares of 8×8 pixels. As required by equation (3), the union of the set range blocks covers the entire image. A corresponding set of domain blocks is similarly constructed with the exceptions that the domain blocks are chosen to be twice the size of the range blocks (to insure contractivity for the domain-to-range block transformation) and the domain blocks are allowed to overlap. Overlapping increases the size of the domain block pool thus increase the probability of finding a good range-domain match. The partition of an image of size 256×256 into domain blocks of size 16×16 will result in a large number of domain blocks (58081). This large search space is the reason for the high computational costs associated with fractal image compression. To obtain the best match, the entire domain pool must be searched, with the best match chosen for each range block. In practice, the first domain block matching the range block within an acceptable distance measure is used.

This simple partition scheme, with a fixed block size, has limitations. For large range blocks, good matchings with domain blocks become unlikely. To overcome this limitation, a quadtree partitioning scheme can be employed. A quadtree decomposition initially partitions the image into large range blocks, typically 16×16 pixels. Then, the best possible transformation for each block is found. This best transformation is compared with the original block using a distance metric. An acceptable threshold is set before the transformation. The transformation is accepted if the distance between the blocks is lower than the threshold. If the transformation is discarded, the range block is divided into four sub-blocks and the search for a best transformation for each sub-block is initiated.

This partition scheme can be recursively continued for several levels (typically $2-4$) until either all blocks are covered with an acceptable transformation or until a certain minimum range block size is reached for which the best matching transformation is used. The range block sizes for the quadtree partitioning employed here are 16, 8×8 and 4×4 pixels. The corresponding domain block sizes are 32×32, 16×16 and 8×8 pixels, respectively.

Other schemes, using not only different partition sizes but also different partition geometries, have been implemented [7]. These schemes typically attempt to decompose the image in some content dependent manner. Different partition geometries which have been employed include horizontal-vertical (HV), triangular and polygonal decompositions.

3. WAVELET TRANSFORM

The first wavelet was found by Haar early in this century [22]. But the construction of more general wavelets to form bases for square-integrable functions was investigated in the 1980s, along with efficient algorithms to compute the expansion. At the same time, applications of these techniques in signal processing have blossomed.

The Wavelet theory [20, 22] provides a powerful tool to solve many signal processing problems. For example, multiresolution signal processing [13], computer vision [22], subband coding [14, 20] (dealing with image and speech compression), and Wavelet series expansion (applied mathematics). In fact, the Wavelet theory covers a wide area that treats time continuous and discrete cases.

The principle behind the wavelet transform, as elaborated in a number of recent papers [2, 13], is to hierarchically decompose an input signal into a series of successively lower-resolution reference signals and their associated detail signals. At each level, the reference signal and detail signal contain the information needed to reconstruct the reference signal at the higher resolution level.

3.1. Continuous Wavelet Transform

The one dimension continuous wavelet transform (CWT) is defined [18] by:

$$\begin{aligned} CWT(a,b) &= \langle f, \Psi_{a,b} \rangle \\ &= \frac{1}{\sqrt{|a|}} \int_{-\infty}^{+\infty} f(t)\, \Psi^* \left(\frac{t-b}{a} \right) dt \quad (4) \end{aligned}$$

where $\langle\ \rangle$ represents a two functions inner product, a and b are real numbers and $\Psi_{a,b}(t)$ is a set of orthogonal functions, the so called *wavelets*.

3.2. Discrete Wavelet Transform

In accordance to [19], a discrete wavelet transform (DWT) can be defined as:

$$DWT(m,n) = d_{m,n} = \langle f, \Psi_{m,n} \rangle \tag{5}$$

where $\Psi_{m,n}$ is a discrete *wavelets* version given by:

$$\begin{aligned} \Psi_{m,n} &= \frac{1}{\sqrt{a_0^m}} \Psi\left(\frac{t}{a_0^m} - nb_0\right) \\ &= \frac{1}{\sqrt{a_0^m}} \Psi\left(\frac{t - na_0^m b_0}{a_0^m}\right) \end{aligned} \tag{6}$$

where a_0, b_0 are constants, and m, n are integers.

4. RESULTS AND CONCLUSIONS

A 512×512 computerized tomography image was used to evaluate FIC and WTC applied to CT images. This image is shown in figure 1. To better evaluate the results obtained in this work, we have performed a qualitative and a quantitative analysis of the compressed images. For the quantitative analysis the Peak Signal Noise-Ratio (PSNR[3]) was used to compare the image quality. For the qualitative analysis we created a set of images and submitted those images to the analysis of four radiologist (see table 1 for details).

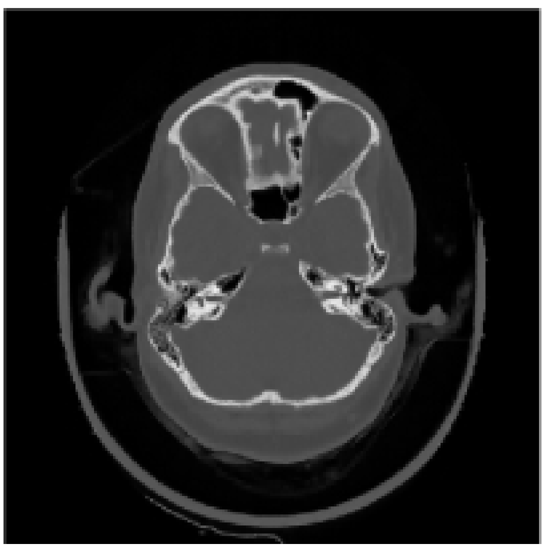

Figure 1: Original 512×512 CT image

Before present the results we would like to show the implementation of both techniques involved in this paper.

4.1. Implementation Description

Here we describe the implementation of each technique used. The wavelet program we used was developed by Simoncelli [1], and its called EPIC[4]. The FIC program was completely based on the LIMBO software which was developed by Frigaard [8], and adapted by the authors of this paper.

Basically, the EPIC is an image coder based on a biorthogonal critically-sampled dyadic wavelet decomposition and a combined run-length/Huffman entropy coder. EPIC is available via anonymous ftp at `ftp.cis.upenn.edu` in the file `pub/eero/epic.tar.Z`.

The skeleton of LIMBO has not been changed. Its use a Quadtree partitioning system. We modified the final structure of the program inserting an adaptive Huffman coder. The original version of LIMBO can be downloaded from `ftp.vision.auc.dk` in the directory `pub/Limbo/`.

4.2. Quantitative Analysis

In this section we present a qualitative analysis comparing fractal image compression to wavelet transform compression.

The graphic in figure 2 represents the compression ratio by the PSNR considering the softwares discussed in section 4.1, and a FIC version of LIMBO with a simple Huffman coder.

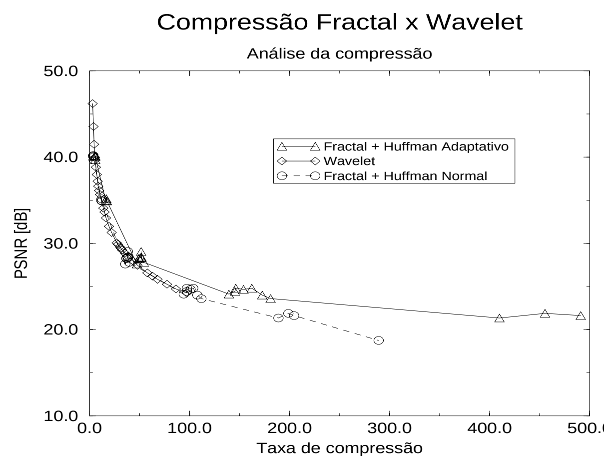

Figure 2: Fractal $\times$ Wavelet

From the graphic we can see that an adaptive Huffman coder is more interesting than a simple Huffman for the FIC method. Besides, the FIC method using an adaptive Huffman becomes even more interesting than the WTC method when we compare the compression ration $\times$ image quality. We were not worried on computational time.

4.3. Qualitative Analysis

In this section we present the qualitative analysis. Here we had to create a set of images to show to the medical doctors that were collaborating in this work.

The images were presented randomly to each doctor. They were asked to separate the images by quality. This way we were trying to get the filling of a radiology expert. Of course they had not received any information about the images (they were not looking for some disease but just comparing image quality by their experience).

[3]
$$PSNR = 10\, log_{10}\left(\frac{255^2}{MSE}\right)$$

[4] Efficient Pyramid Image Coder

Image	PSNR[dB]	Technique	Dr. A	Dr. B	Dr. C	Dr. D	Sum
1	**original**	**original**	5º	5º	7º	2º	19
2	34.9	Fractal	4º	7º	5º	4º	20
3	40.16	Fractal	6º	2º	2º	9º	19
4	29.06	Fractal	8º	10º	8º	10º	36
5	35.17	Fractal	3º	8º	4º	5º	20
6	40.13	Fractal	2º	3º	3º	8º	16
7	46.19	*Wavelet*	1º	1º	1º	1º	4
8	26.55	*Wavelet*	11º	11º	10º	11º	43
9	32.92	*Wavelet*	9º	6º	6º	3º	24
10	39.27	*Wavelet*	7º	4º	2º	6º	19
11	29.59	*Wavelet*	10º	9º	9º	7º	35

Table 1: Image classification in accordance to doctors' opinion.

The results are shown in table 1. This table is organized as follow: the first column represents the image number (for classification), the second column represents the image quality considering the PSNR value, the third column shows the technique used to compress that image, from the fourth to seventh columns we have each doctors' opinion, and in the eighth column we have the sum of the doctors' classification for each image.

4.4. Conclusion

The objective of this paper was to compare fractal image compression to wavelet transform compression, and its application to CT images.

From the qualitative analysis and the quantitative analysis we could verify some important points. First of all, let's consider the quantitative analysis, where the PSNR is the main point. There we could see the advantage of using an adaptative Huffman coder against the use of a normal Huffman coder. An interesting point is the wavelet curve behavior (smooth). It's because the low pass filter characteristics of wavelet transform, which contributed to the better performance of this specific method for low compression ratios.

Now, considering the qualitative analysis we have first to point out the results represented in the eighth column of table 1. Taking into account the evaluation of the doctors we could see that the PSNR has a significant value when we are comparing image compression techniques.

Following the results, some assumptions are made. For low compression ratio, WTC seems to be much better than FIC, even using an adaptive Huffman coder (AHC). For high compression ratio, FIC performed a better result compared to WTC, but just using an AHC. Another important point is that from the complete set of images showed to the doctors we could see that for a PSNR higher than $37dB$ they found very difficulty to compare images ("*they all seem to be the same*" said the doctors). Two example images can be seen in figures 3 and 4.

Acknowledgments

We would like to thank Dra. Magda Nunes, Dr. Mauro Tchasavoy Master, Dr. Matias Kronfeld and Dr. Cláudio Zaslavisky for their important collaboration regarding the analysis of the compressed CT images.

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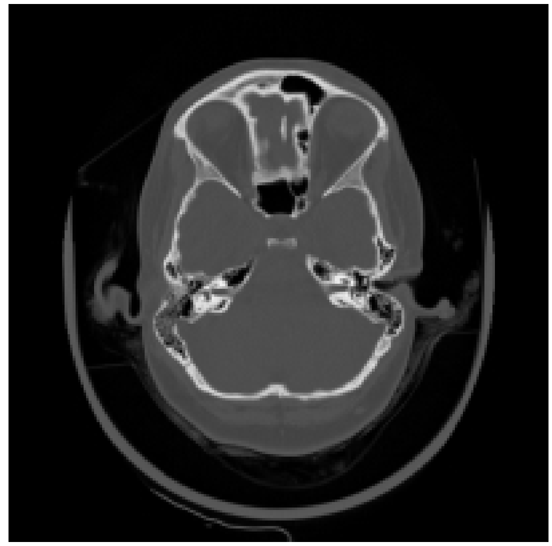

Figure 3: FIC, $CR^5 = 5.27$, $PSNR = 40.16dB$

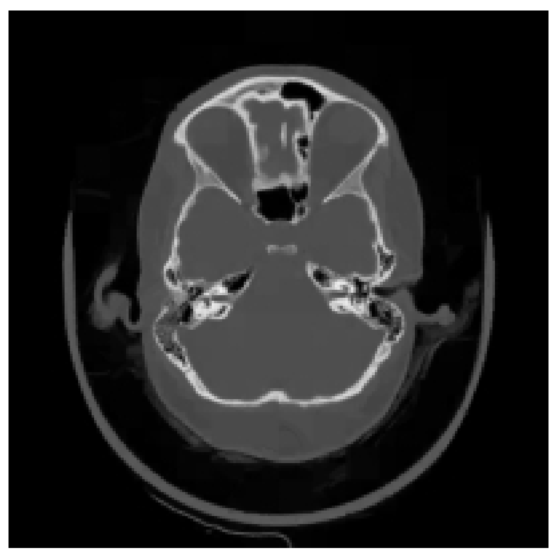

Figure 4: WTC, $CR = 2.87$, $PSNR = 46.19$