A SIMPLE PDF FITTING APPROACH FOR BLIND EQUALIZATION

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ABSTRACT

A new blind equalization algorithm for digital communication systems is presented. This algorithm is based on the adjusting of a linear equalizer in such a way that the probability density function (PDF) of its output matches a parametric target function. A link between the proposed cost function and that used by the Constant Modulus Algorithm is also pointed out. Some simulation results are presented and compared to that provided by the Godard's equalizer.

1. INTRODUCTION

This work deals with blind equalization of linear channels in digital communication systems. A new blind equalizer is presented, based on the adaptation of the equalizer taps until that of the output equalizer PDF matches a target multimodal function. In this moment we can guarantee that equalization is performed [1]. In Section 2, the system model is presented along with the new algorithm. A link between this new algorithm and the CMA is presented in Section 3. Simulations and conclusions are presented in the last two sections.

2. A NEW BLIND ALGORITHM

Fig. 1 shows the single-input-single-output system model we are concerned with. The equalizer input is obtained after demodulation and sampling of the received signal.

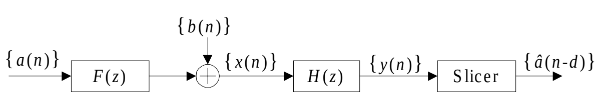

Figure 1: Baseband System Model.

The stream $a(n)$ carries the information and each element in this stream is modeled as a random variable which takes values from an S-sized symbolic alphabet. We further assume that this alphabet is complex-valued and symmetric w. r. t. the origin. The noise sequence $b(n)$ is i.i.d., zero-mean Gaussian and statistically independent of $a(n)$. Then, and assuming that the channel transfer function, $F(z)$, has no spectral nulls, a possible equalization strategy is to make the global response $G(z) = F(z) \cdot H(z)$ as close as possible to the ideal one: $G(z) = z^{-d}$, where d is a suitable decision delay. This is, in fact, the so-called zero forcing (ZF) equalization [2].

Assuming that perfect ZF is obtained, it is easy to show that the PDF of $y(n)$ is:

$$p_{Y,ideal}\left(y; \mathbf{h}_{ZF}, \sigma_b^2\right) = \frac{1}{\sqrt{2\pi\sigma_b^2}} \sum_{s=1}^{S} \exp\left(\frac{-\left|y(n) - a_s\right|^2}{2\sigma_b^2}\right) \tag{1}$$

where $\mathbf{h}_{ZF} = [\ h_0^{ZF}\ h_1^{ZF}\ \ldots\ h_{M-1}^{ZF}\ ]^T$ is a normalized vector of equalizer coefficients and σ_b^2 is the noise variance. In fact, we assume $M \gg 1$ to perform equalization.

Then, in order to match this desired PDF of a perfectly equalized system, we construct a parametric target function given by $f(y, \mathbf{h}, \hat{\sigma}_b^2) = p_{Y,ideal}(y; \mathbf{h}, \hat{\sigma}_b^2)$, and we compare it to the actual PDF, $p_Y(y)$, by means of the following measurement of the extend to which the model density and the true density agree [3, p. 59]:

$$J = -\int p_Y(y; \mathbf{h}_{ZF}, \sigma_b^2) \ln\left(f(y, \mathbf{h}, \hat{\sigma}_b^2)\right) dy \tag{2}$$

where the minimum of J, obtained when the two functions are equal, is the *entropy* of Y.

Clearly, the estimated noise variance $\hat{\sigma}_b^2$ depends on the equalizer coefficients. However, in order to be simple, we can replace it by a constant parameter Υ which, in fact, can play a crucial role on the final algorithm (see Section 3). Then, expanding Equation (2) and eliminating the constant terms, our effective cost function becomes

$$J = -E\left\{\ln\left[\sum_{s=1}^{S} \exp\left(\frac{-\left|y(n) - a_s\right|^2}{2\Upsilon}\right)\right]\right\} \tag{3}$$

Now, taking the stochastic gradient of Equation (3) w.r.t. the equalizer parameters, our blind equalization algorithm can be summarized as follows:

$$\mathbf{h}(n+1) = \mathbf{h}(n) - \alpha_h \nabla J$$

where

$$\nabla J = \frac{\sum_{s=1}^{S} \exp\left(\frac{-|y(n)-a_s|^2}{2\Upsilon}\right)(y(n) - a_s)}{\Upsilon \sum_{s=1}^{S} \exp\left(\frac{-|y(n)-a_s|^2}{2\Upsilon}\right)} \mathbf{x}^* \neq \mathbf{0}$$

3. A LINK TO THE CM CRITERION

After the milestone paper of Sato [4], in 1975, blind equalization to compensate for intersymbol interference (ISI) in digital communication has been intensively studied. Among the most known blind equalization criteria, those based on Constant Modulus are probably the most popular. For instance, we can point out the so-called Godard equalizer [5]. Moreover, since a great number of publications on CMA performance limits and convergence issues is available, it is useful to find similarities between new approaches and CMA. Indeed, some relevant works like [6],[7] and [8] point out links between other approaches and the CM criterion.

Likewise, a first step toward the performance characterization of our proposal has been to find a link between our approach and CMA. This link is clearer when a BPSK (± 1) modulation scheme is applied. Then, a parametric target function can be defined as follows:

$$f_{CM}(y, \mathbf{h}, p) = \frac{\exp\left(|\ |y|^p - 1\ |^2\right)}{\int\limits_{-\infty}^{+\infty} \exp\left(|\ |\xi|^p - 1\ |^2\right) d\xi}$$

where p is a strictly positive integer parameter.

It is straightforward to show that, applying it in Equation (2), the resulting cost function, apart from some constants, is indeed the well-known constant modulus criterion:

$$J_{CM} = -E\left\{\ln\left[f_{CM}(y, \mathbf{h}, p)\right]\right\} = E\left\{||y|^p - 1|^2\right\}$$

As a result, from this point of view we can regard the CM criterion as being a measure of divergence between the actual equalizer output PDF and the target function $f_{CM}(\cdot)$. Moreover, we can show that our proposed target function $f(y, \mathbf{h}, \Upsilon)$ can be set close to either $f_{CM}(y, \mathbf{h}, 1)$ (i.e., the Sato's criterion) or $f_{CM}(y, \mathbf{h}, 2)$ (i.e., the Godard's criterion), by choosing a suitable value for Υ.

Fig. 2 shows some such target functions. Similarities between the target functions, along with some simulation results, have shown that an appropriate choice of Υ can afford performance characteristics close to those of either the Godard or Sato equalizer, when the modulation scheme is the BPSK.

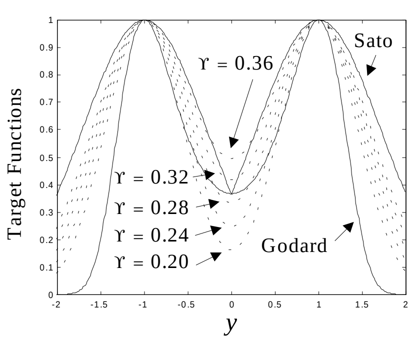

Figure 2: Comparison between Target Functions with some Υ values.

Moreover, for more complex modulation schemes, the new family of target functions can better fits the idealized equalizer output PDF with S Gaussian kernels. Thus, since Equation (1) can take into account even complex symbol parts and multilevel alphabets, we can predict an improved performance w.r.t. the CMA.

4. SIMULATION RESULTS

All simulations presented in this section were done in order to verify the relationship between our proposal and CMA for $p = 2$ (i.e., the Godard's equalizer). Moreover, only two modulation schemes were considered, namely, BPSK and 4-QAM.

The signal-to-noise ratio is defined as $\mathrm{SNR} = 10\log_{10}\left(\frac{\sigma_a^2 \sum_{i=0}^{N-1} f_i^2 + \sigma_b^2}{\sigma_b^2}\right)$ where σ_a^2 and σ_b^2 are the symbol and noise variance respectively. Performance measures (i.e., Decision Squared Error (DSE) $(\varepsilon(n) = y - \mathrm{Dec}(y))^2$ and Symbol Error Rate (SER)) are averaged from 200 Monte-Carlo trials.

For clarity's safe, our approach is called NBE (New Blind Equalizer) in the figures.

Simulations with BPSK modulation were done with two channels. The discrete impulse response of such channels can be represented in vectorial form as follows:

$$\mathbf{f}_1 = [\,0.1 \quad 0.5 \quad 1 \quad -0.6 \quad -0.2\,]$$

and

$$\mathbf{f}_2 = [\,1 \quad -0.2 \quad 0.71 \quad 0.282 \quad 0.8658\,]$$

Channel $\mathbf{f}_1$ has no spectral near-nulls as we can see in Fig. 3 (The representation of channel zeros is also presented in Fig. 4). Therefore this channel can be "easily equalized" by means of a linear transversal equalizer (LTE).

Fig. 5 shows the performance of both NBE and CM equalizers measured in terms of DSE. It can be easily seen that our approach is equivalent to the CM criterion after convergence. However, the convergence of the NBE is faster than that of the CMA because its adaptation step size is the greater. We highlight that a step size greater than 10^{-3} provokes divergence of the CM Algorithm. Table 1 specifies the simulation parameters. That fact indicates higher robustness to the noise by the NBE.

	CMA	NBE
TAPs	30	30
Step size	10^{-3}	25.10^{-3}
Υ	—	0.3
Initial TAPs	$[\,0\cdots\ 0\ 1\ 0\cdots\ 0\,]$	$[\,0\cdots\ 0\ 1\ 0\cdots\ 0\,]$

Table 1: Simulation Parameters for $\mathbf{f}_1$

Channel $\mathbf{f}_2$ has two spectral near-nulls, as we can see in Fig. 6 that shows the frequency response of the channel and Fig. 7 shows the channel zeros. It is worth noting that those spectral near-nulls rend the equalization with a LTE more difficult.

Fig. 8 shows the performance of both the NBE and CMA in an environment with SNR = 30 dB. Table 2 specifies simulation parameters.

	CMA	NBE
TAPs	30	30
Step size	10^{-3}	25.10^{-3}
Υ	—	0.29
Initial TAPs	$[\,0\cdots\ 0\ 1\ 0\cdots\ 0\,]$	$[\,0\cdots\ 0\ 1\ 0\cdots\ 0\,]$

Table 2: Simulation Parameters for $\mathbf{f}_2$

Simulations with 4-QAM modulation scheme were done with the channel proposed in [9] that has the discrete impulse response given by:

$$\mathbf{f}_3 = [\, 2 - 0.4j \quad 1.5 + 1.8j \quad 1 \quad 1.2 - 1.3j \quad 0.8 + 1.6j]$$

This channel has been used in some recent works to illustrate the robustness of algorithms facing a very distorsive channel. The frequency response of this channel is depicted in Fig. 9 whereas Figure 10 shows the channel zeros.

The performances of both equalizers are depicted in Fig. 11. As we can see, they have an almost identical performance. However, we emphasize that in this case the CMA requires a phase recovering device to work whereas the NBE compensates for phase distortion by itself. Table 3 specifies simulation parameters.

	CMA	NBE
TAPs	30	30
Step size	10^{-3}	2.10^{-3}
Υ	—	0.35
Step Size Phase	10^{-3}	—
Initial Phase	0	—
Initial TAPs	$[\,0 \cdots 0\,1\,0 \cdots 0\,]$	$[\,0 \cdots 0\,1\,0 \cdots 0\,]$

Table 3: Simulation Parameters for $\mathbf{f}_3$

5. CONCLUSIONS AND PERSPECTIVES

Simulation results had confirmed that when $\Upsilon \cong 0.3$ our proposal has a final performance practically equivalent to that of the Godard's equalizer for both BPSK and 4-QAM modulation schemes. On the other hand, we also conclude that, thanks to its capacity of convergence with step-sizes greater than Godard's, the NBE can provide faster convergence in some cases.

Furthermore, as illustrated in the last simulation result, the NBE has no need for a phase recovering device.

Actually, this work has just started and the continuity of it includes tests involving multi-amplitude complex modulation, like 16-QAM schemes, and the study of an adaptation strategy for the parameter Υ. The use of a feedback structure cascaded with the filter is also under study.

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6. REFERENCES

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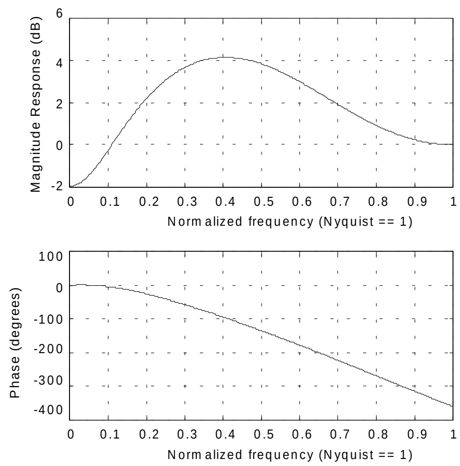

Figure 3: Frequency Response of Channel $\mathbf{f}_1$.

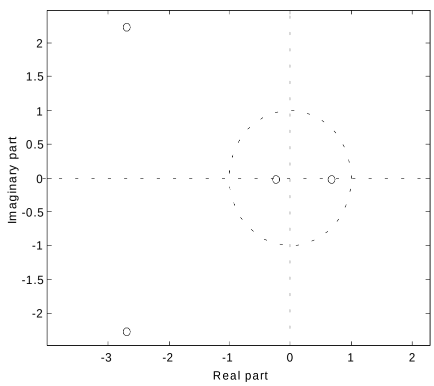

Figure 4: Root Locus of Channel $\mathbf{f}_1$.

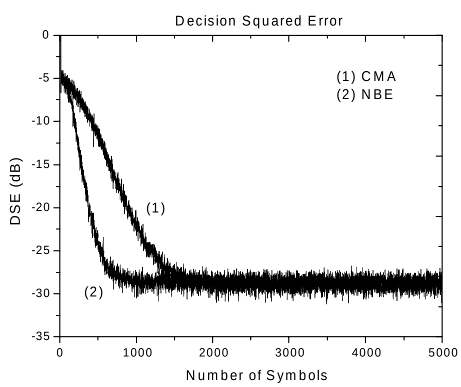

Figure 5: DSE for Channel $\mathbf{f}_1$ (SNR = 30 dB).

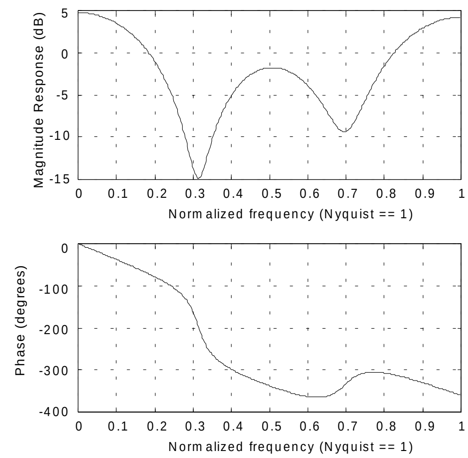

Figure 6: Frequency Response of Channel $\mathbf{f}_2$.

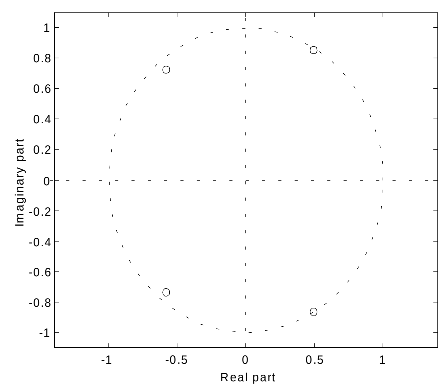

Figure 7: Root Locus of Channel $\mathbf{f}_2$.

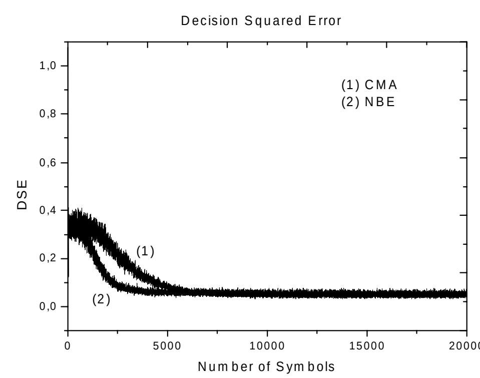

Figure 8: DSE for Channel $\mathbf{f}_2$ (SNR=30 dB).

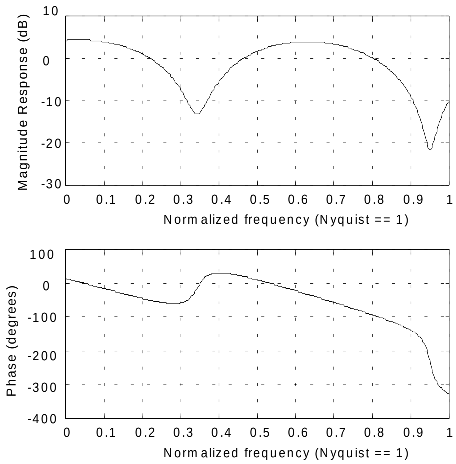

Figure 9: Frequency Response of Channel $\mathbf{f}_3$.

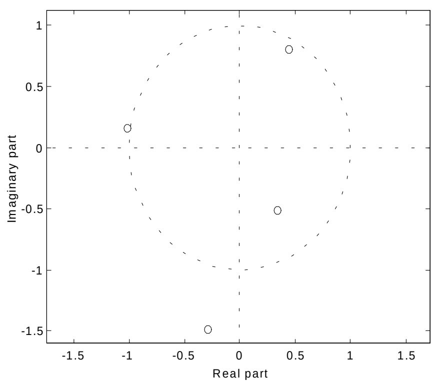

Figure 10: Root Locus of Channel $\mathbf{f}_3$.

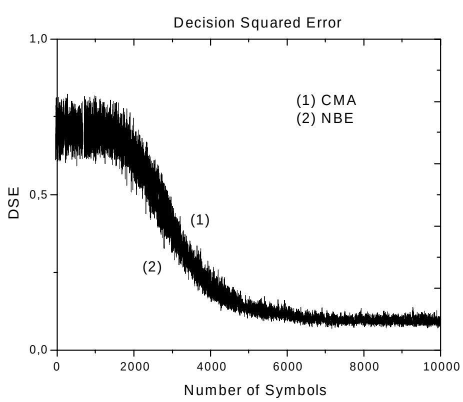

Figure 11: DSE for Channel $\mathbf{f}_3$ (SNR = 30 dB).