Bromwich Method for Solving Maxwell's Equations Applied to Cylindrical Resonators

Abelardo Podcameni
CETUC/PUC-Rio
abelardo@cetuc.puc-rio.br
Rua Marques de São Vicente 225
Rio de Janeiro, RJ – 22453-900, Brasil

***Abstract:** The Bromwich method for solving Maxwell's equations is firstly introduced. This allows for establishing the grounds for reaching solutions in a number of different coordinate systems. Secondly, the cylindrical system is put, leading to a pertinent solution for cylindrical resonators. Next, practical procedures are described for identifying desired resonant mode and for rejecting the undesired ones. Finally, a TE_{O12} resonator is accomplished and the appropriate results are described.*

***Key terms:** Bromwich, Maxwell equations, coordinate systems, cylindrical resonators, mode rejection.*

I. INTRODUCTION

Stable microwave oscillators are required in Communications Systems for providing carrier generation. Microwave resonator with an unloaded quality factor [1], Q_O, comparable with that one of a RF crystal [2], is still an open issue. The usual practical solution for generating a stable microwave frequency range carrier is to lock a microwave oscillator (slave) to a stable RF crystal oscillator (master), by means of a PLL [3,4].

However, it is mandatory that the slave microwave oscillator should be relatively stable, otherwise the lock operation will not be feasible. The more stable the microwave oscillator is, the less stringent the locking operation requirements shall be [5].

Although the dielectric resonator [1] is well accepted, when miniaturization is a paramount, the metallic-wall hollow cylindrical cavity resonator is able to reach a higher quality factor [6].

Here, a cylindrical cavity will be designed to operate in 8.3 GHz. In Section II, the Bromwich method for solving Maxwell's equations will be presented and the reason why this specific method is used will be discussed. In Section III, the theoretical solution for the cylindrical cavity, together with its prospective physical dimensions, will be obtained. In Section IV, practical considerations concerning undesirable mode rejection will be presented. In Section V, measurements are carried out and the results obtained are discussed.

II. BROMWICH METHOD FOR MAXWELL'S EQUATIONS

Consider a three-dimension generic curvilinear coordinate system, where the length element, ds, is related to the three generic coordinates u, v and w by a relation such as:

$$ds^2 = e_1^2\, du^2 + e_2^2\, dv^2 + e_3^2\, dw^2 \qquad (1)$$

where e_1, e_2 and e_3 are the local unitary length elements, which are related to the conventional rectangular system coordinates (x, y and z) by:

$$e_1^2 = \left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial u}\right)^2 \qquad (2a)$$

$$e_2^2 = \left(\frac{\partial x}{v}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2 \qquad (2b)$$

$$e_3^2 = \left(\frac{\partial x}{\partial w}\right)^2 + \left(\frac{\partial y}{\partial w}\right)^2 + \left(\frac{\partial z}{\partial w}\right)^2 \qquad (2c)$$

Now, let the Maxwell's equations be written in an homogeneous medium, free of charges, with a dielectric constant ε and a magnetic permittivity μ:

$$\text{rot}\,\vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad ; \quad \text{rot}\,\vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} \qquad (3a, b)$$

$$\text{div}\,\vec{E} = 0 \quad ; \quad \text{div}\,\vec{H} = 0 \qquad (3c, d)$$

A. Maxwell's Equations in Curvilinear System:

It is then possible to rewrite the above equations in a generic curvilinear coordinate system, as given by (1)

$$\frac{\partial}{\partial v}(e_3 E_3) - \frac{\partial}{\partial w}(e_2 E_2) = -\mu e_2 e_3 \frac{\partial H_1}{\partial t} \qquad (4a)$$

$$\frac{\partial}{\partial w}(e_1 E_1) - \frac{\partial}{\partial u}(e_3 E_3) = -\mu e_3 e_1 \frac{\partial H_2}{\partial t} \qquad (4b)$$

$$\frac{\partial}{\partial u}(e_2 E_2) - \frac{\partial}{\partial v}(e_1 E_1) = -\mu e_1 e_2 \frac{\partial H_3}{\partial t} \qquad (4c)$$

$$\frac{\partial}{\partial v}(e_3 H_3) - \frac{\partial}{\partial w}(e_2 H_2) = \varepsilon e_2 e_3 \frac{\partial E_1}{\partial t} \qquad (5a)$$

$$\frac{\partial}{\partial w}(e_1 H_1) - \frac{\partial}{\partial u}(e_3 H_3) = \varepsilon e_3 e_1 \frac{\partial E_2}{\partial t} \qquad (5b)$$

$$\frac{\partial}{\partial u}(e_2 H_2) - \frac{\partial}{\partial v}(e_1 H_1) = \varepsilon e_1 e_2 \frac{\partial E_3}{\partial t} \qquad (5c)$$

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$$\frac{\partial}{\partial u}(e_2 e_3 E_1)+\frac{\partial}{\partial v}(e_1 e_3 E_2)+\frac{\partial}{\partial w}(e_1 e_2 E_3)=0 \quad (6)$$

$$\frac{\partial}{\partial u}(e_2 e_3 H_1)+\frac{\partial}{\partial v}(e_1 e_3 H_2)+\frac{\partial}{\partial w}(e_1 e_2 H_3)=0 \quad (7)$$

It is well known that the most general solution associated with the above (4), (5), (6) and (7) equation set is a superposition of two particular solutions. The first one is obtained with H=0, called as TM. The second, with E=0 is the TE

B. Solution by the Bromwich's Method

The Bromwich method does not apply to all curvilinear systems. It is a necessity that the local length unitary element e_1 must be a function - only - of the coordinate that is associated to it: u. Additionally, the two others should be independent of u. All other permutations must also be fulfilled. Consequently, it is possible to do:

$e_1 = 1$; and (e_2 / e_3) is independent of u (8a,b)

B.1. Transversal Magnetic Solution

By setting $H_1 = 0$, equation (4a) yields:

$$\frac{\partial}{\partial v}(e_3 E_3)=\frac{\partial}{\partial w}(e_2 E_2) \quad (9)$$

in the above equation, let P be an arbitrary auxiliary function such that its derivatives – with respect to v and w – are respectively:

$$\frac{\partial P}{\partial v}=e_2 E_2 \quad ; \quad \frac{\partial P}{\partial w}=e_3 E_3 \quad (10a, b)$$

By substituting E_2 and E_3 as from (10a, b) in (4b) and (4c), and also taking into account (8a,b), it will be obtained:

$$\varepsilon\frac{\partial^2 P}{\partial v\partial t}=-\frac{e_2}{e_3}\frac{\partial}{\partial u}(e_3 H_3)=-\frac{\partial}{\partial u}(e_2 H_3) \quad (11a)$$

$$\varepsilon\frac{\partial^2 P}{\partial w\partial t}=-\frac{e_3}{e2}\frac{\partial}{\partial u}(e_2 H_2)=-\frac{\partial}{\partial u}(e_3 H_2) \quad (11b)$$

The components of E and H will be now obtained. Let be put: $P = (\partial U/\partial u)$, with U to be defined ahead. Since $H_1=0$, E_2 and E_3 are obtained as follows:

$$E_2=\frac{1}{e_2}\frac{\partial^2 U}{\partial u\partial v} \quad ; \quad E_3=\frac{1}{e_3}\frac{\partial^2 U}{\partial u\partial w} \quad (12a, b)$$

For obtaining H_2 and H_3, it must be first considered:

$$\frac{\partial}{\partial u}\left(\varepsilon\frac{\partial^2 U}{\partial v\partial t}\right)=-\frac{\partial}{\partial u}\left(e_2 H_3\right) \ ; \ \frac{\partial}{\partial u}\left(\varepsilon\frac{\partial^2 U}{\partial w\partial t}\right)=\frac{\partial}{\partial u}\left(e_3 H_2\right)$$

Next, from (11), H_2 and H_3 are readily obtained:

$$H_2=\varepsilon\frac{1}{e_3}\frac{\partial^2 U}{\partial w\partial t} \quad ; \quad H_3=-\varepsilon\frac{1}{e_2}\frac{\partial^2 U}{\partial v\partial t} \quad (12c, d)$$

By using (12b) and (12c) in (4b), the last component, E_1, will result:

$$E_1=\frac{\partial^2 U}{\partial u^2}-\mu\varepsilon\frac{\partial^2 U}{\partial t^2} \quad (12e)$$

In this way all the six components have been obtained. However, all of them are a function of U, which by its turn has been defined from P, with this last being an arbitrary function. Therefore, still is necessary to properly define the function U. For doing so, a second expression for E_1 must be obtained. By using (12c) and (12d) in (5a), another expression for E_1 becomes apparent:

$$E_1=-\frac{1}{e_2 e_3}\left[\frac{\partial}{\partial v}\left(\frac{e_3}{e_2}\frac{\partial U}{\partial v}\right)+\frac{\partial}{\partial w}\left(\frac{e_2}{e_3}\frac{\partial U}{\partial w}\right)\right] \quad (13)$$

Comparison of (12e) and (13) defines U:

$$\mu\varepsilon\frac{\partial^2 U}{\partial t^2}=\frac{\partial^2 U}{\partial u^2}+\frac{1}{e_2 e_3}\left[\frac{\partial}{\partial v}\left(\frac{e_3}{e_2}\frac{\partial U}{\partial v}\right)+\frac{\partial}{\partial w}\left(\frac{e_2}{e_3}\frac{\partial U}{\partial w}\right)\right] \quad (14)$$

Therefore, U is the function that solves the differential equation (14).

B.2. Transversal Electric Solution

By setting $E_1 = 0$, and using a procedure similar to the one used for TM, the E and H components are easily obtained:

$$E_1=0 \ ; \ E_2=-\mu\frac{1}{e_3}\frac{\partial^2 U}{\partial w\partial t} \ ; \ E_3=\mu\frac{1}{e_2}\frac{\partial^2 U}{\partial v\partial t} \quad (15a,b,c)$$

$$H_1=\frac{\partial^2 U}{\partial u^2}-\mu\varepsilon\frac{\partial^2 U}{\partial t^2} \ ; \ H_2=\frac{1}{e_2}\frac{\partial^2 U}{\partial u\partial v}; \quad (15d, e)$$

$$H_3=\frac{1}{e_3}\frac{\partial^2 U}{\partial u\partial w} \quad (15f)$$

Again, the U function is defined exactly as in (14).

B.3. Simplification for Sinusoidal Field

Frequently, the electromagnetic field is a sinusoidal time function. In this case, $(\partial /\partial t)$ may be substituted by jkv_p, where k is a propagation constant – which will be discussed ahead – and v_p is the phase velocity. Additionally, f is the electromagnetic field frequency and the following relations apply:

$$v_p=\frac{1}{\sqrt{\mu\varepsilon}} \quad ; \quad k=\frac{2\pi f}{v_p}$$

If the previous paragraph derivations were done again considering the field as a sinusoidal function of time, it would be equivalent to write the U(u,v,w,t) function as follows: $U=e^{jk v_p t}\,U(u,v,w)$.

The field components, according to the TM and TE solutions, and the U function would be as follows considering the factor $e^{jk v_p t}$ as an implicit one:

B.3.1. - TM Solution

$$E_1 = \frac{\partial^2 U}{\partial u^2} + k^2 U \quad ; \quad E_2 = \frac{1}{e_2}\frac{\partial^2 U}{\partial u \partial v} \qquad (16a, b)$$

$$E_3 = \frac{1}{e_3}\frac{\partial^2 U}{\partial u \partial w} \quad ; \quad H_1 = 0 \qquad (16c, d)$$

$$H_2 = \frac{jk}{e_3}\sqrt{\frac{\varepsilon}{\mu}}\frac{\partial U}{\partial w} \quad ; \quad H_3 = -\frac{jk}{e_2}\sqrt{\frac{\varepsilon}{\mu}}\frac{\partial U}{\partial v} \qquad (16e, f)$$

B.3.2. - TE Solution

$$E_1 = 0 \quad ; \quad E_2 = \frac{jk}{e_3}\sqrt{\frac{\varepsilon}{\mu}}\frac{\partial U}{\partial w} \qquad (17a, b)$$

$$E_3 = \frac{jk}{e_2}\sqrt{\frac{\varepsilon}{\mu}}\frac{\partial U}{\partial v} \quad ; \quad H_1 = \frac{\partial^2 U}{\partial u^2} + k^2 U \qquad (17c, d)$$

$$H_2 = \frac{1}{e_2}\frac{\partial^2 U}{\partial u \partial v} \quad ; \quad H_3 = \frac{1}{e_3}\frac{\partial^2 U}{\partial u \partial w} \qquad (17e, f)$$

B.3.3. – U Function

$$\frac{\partial^2 U}{\partial u^2} + \frac{1}{e_2 e_3}\left[\frac{\partial}{\partial v}\left(\frac{e_3}{e_2}\frac{\partial U}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{e_2}{e_3}\frac{\partial U}{\partial w}\right)\right] + k^2 U = 0 \quad (18)$$

The equation (18) is called as the Bromwich equation.

C. Applicability of the Method

Due to the restriction described in (8), the Bromwich method can not be used in all curvilinear coordinate systems. Only systems whose coordinate relationships are such that comply with (8) are to be used. A few examples are given now:

(a) – rectangular; (b) – cylindrical; (c) – spherical; (d) - elliptic cylinder, (e) - parabolic cylinder; and (f) – bi-axial cylinder.

It is then observed that the Bromwich method leads to Maxwell's equation solutions that may be used in a vast majority of problems normally occurring in the Classical Electromagnetism.

III. CYLINDRICAL CAVITY APPLICATION

For solving a problem with cylindrical symmetry, it is advised to use a cylindrical coordinate system. Let be a hollow cylindrical cavity with perfect conductor walls. The height is h and the radius is r. The z-axis is coaxial with the cylinder as shown in Fig.1.

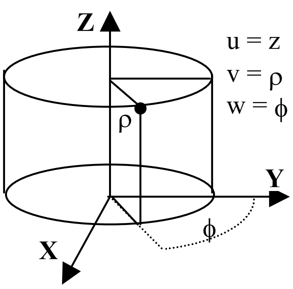

Fig. 1 – Cylindrical coordinates are used to represent a cylindrical cavity with metallic walls.

The bottom and top covers are located in z = 0 and in z = h, respectively. The entire lateral surface is located at $\rho = r$. By Fig. 1, $e_1 = e_2 = 1$ and $e_3 = \rho$, with $u = z$, $v = \rho$ and $w = \phi$. The Bromwich function is then:

$$\frac{\partial^2 U}{\partial z^2} + \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}\left(\rho\frac{\partial U}{\partial \rho}\right)\right] + \frac{1}{\rho^2}\frac{\partial^2 U}{\partial \phi^2} + k^2 U = 0 \qquad (19)$$

Observe the simplicity of the Bromwich method. Since an expression for U, *in general curvilinear coordinates* is already available, as (18), it is just a matter of rewriting it in the cylindrical system, for obtaining (19). Next, U is written in terms of Laplace products [7], yielding:

$$U = {}^{\cos}_{\text{sen}}\, n\phi \bullet {}^{\cos}_{\text{sen}}\, qz \bullet {}^{J_n}_{Y_n}(a\rho) \bullet e^{jkv_p t} \qquad (20)$$

where $a^2 = k^2 - q^2$ (q definition will be presented ahead) and $v_p = \frac{1}{\sqrt{\mu\varepsilon}}$.

As the cylinder is empty and the U function assumes finite values for the axis, $\rho = 0$, then the Bessel function of second kind, Y, should be rejected. As the field pattern is such that presents a revolving symmetry and repeats itself at each 2π radial rotation, then n must be an integer. Therefore (20) simplifies to:

$$U = {}^{\cos}_{\text{sen}}(n\phi) \bullet {}^{\cos}_{\text{sen}}(qz) \bullet {}^{J_n}_{Y_n}(a\rho) \bullet e^{jkv_p t} \qquad (21)$$

The previously exposed equation sets (16) and (17), while using U as (21) will give the TM and TE solutions. For doing so, the boundary conditions are first introduced:

(a) - for $z = 0 \rightarrow (E_\rho = E_\phi = 0)$
(b) - for $\rho = r \rightarrow (E_z = E_\phi = 0)$
(c) - for $z = h \rightarrow (E_\rho = E_\phi = 0)$

For TM: $U = J_n(a\rho) \bullet {}^{\cos}_{\text{sen}}(n\phi) \bullet \cos(qz)$

For TE: $U = J'_n(a\rho) \bullet {}^{\cos}_{\text{sen}}(n\phi) \bullet \text{sen}(qz)$

with:

$[J_n(a\rho)]_{\rho=r} = [J'_n(a\rho)]_{\rho=r} = 0$; and $q = \frac{m\pi}{h}$ with m integer

Observe that q has been defined as related to number of oscillations occurring along the cylinder axis. Additionally, let p_{in} and p'_{in} be the i-th non-zero roots of $J_n(a\rho)$ and of $J'_n(a\rho)$, respectively.

The six field-components will now be obtained, using as parameters the integers – n, i and m – which designate the different oscillatory modes.

◊ TM CYLINDRICAL CAVITY FIELD COMPONENTS

$$E_z = \frac{p_{in}^2}{r^2} J_n\left(\frac{p_{in}}{r}\rho\right) \bullet {}^{\cos}_{\sin}(n\phi) \bullet \cos\frac{m\pi z}{h} \qquad (22a)$$

$$E_\rho = -\frac{m\pi p_{in}}{hr} J_n^{|}\left(\frac{p_{in}}{r}\rho\right) \bullet {}^{\cos}_{\sin}(n\phi) \bullet \sin\frac{m\pi z}{h} \quad (22b)$$

$$E_\phi = -\frac{nm\pi}{h\rho} J_n\left(\frac{p_{in}}{r}\rho\right) \bullet {}^{-\sin}_{\cos}(n\phi) \bullet \sin\frac{m\pi z}{h} \quad (22c)$$

$$H_z = 0 \quad (22d)$$

$$H_\rho = -j\frac{nk}{\rho}\sqrt{\frac{\varepsilon}{\mu}} J_n\left(\frac{p_{in}}{r}\rho\right) \bullet {}^{\sin}_{-\cos}(n\phi) \bullet \cos\frac{m\pi z}{h} \quad (22e)$$

$$H_\phi = -j\frac{p_{in}k}{r}\sqrt{\frac{\varepsilon}{\mu}} J_n^{|}\left(\frac{p_{in}}{r}\rho\right) \bullet {}^{\cos}_{\sin}(n\phi) \bullet \cos\frac{m\pi z}{h} \quad (22f)$$

For the vacuum the above field frequency is:

$$f_{n,i,m} = \frac{kc}{2\pi} = \frac{c}{2}\sqrt{\frac{p_{in}^2}{\pi^2 r^2} + \frac{m^2}{h^2}} \quad (23)$$

The lowest frequency is the one for m = 0 and for the first non-zero root of J_{in}. This last root occurs for i = 1 and n = 0. In this case the oscillation frequency is not dependent upon the height of the cylinder, as below:

$$f(n = 0, i = 1, m = 0) = \frac{72{,}145}{2\pi r} 10^9 \text{ (Hz) ; r in cm.}$$

◊ TE CYLINDRICAL CAVITY FIELD COMPONENTS

$$E_z = 0 \quad (24a)$$

$$E_\rho = j\frac{nk}{\rho}\sqrt{\frac{\varepsilon}{\mu}} J_n\left(\frac{p_{in}^{|}}{r}\rho\right) \bullet {}^{\sin}_{-\cos}(n\phi) \bullet \sin\frac{m\pi z}{h} \quad (24b)$$

$$E_\phi = j\frac{p_{in}^{|}k}{r}\sqrt{\frac{\varepsilon}{\mu}} J_n^{|}\left(\frac{p_{in}^{|}}{r}\rho\right) \bullet {}^{\cos}_{\sin}(n\phi) \bullet \sin\frac{m\pi z}{h} \quad (24c)$$

$$H_z = \frac{p_{in}^{|2}}{r^2} J_n\left(\frac{p_{in}^{|}}{r}\rho\right) \bullet {}^{\cos}_{\sin}(n\phi) \bullet \sin\frac{m\pi z}{h} \quad (24d)$$

$$H_\rho = \frac{m\pi p_{in}^{|}}{hr} J_n^{|}\left(\frac{p_{in}^{|}}{r}\rho\right) \bullet {}^{\cos}_{\sin}(n\phi) \bullet \cos\frac{m\pi z}{h} \quad (24e)$$

$$H_\phi = -\frac{nm\pi}{h\rho} J_n\left(\frac{p_{in}^{|}}{r}\rho\right) \bullet {}^{\text{sen}}_{-\cos}(n\phi) \bullet \cos\frac{m\pi z}{h} \quad (24f)$$

In the vacuum the above field frequency is:

$$f_{n,i,m} = \frac{kc}{2\pi} = \frac{c}{2}\sqrt{\frac{p_{in}^{|2}}{\pi^2 r^2} + \frac{m^2}{h^2}} \quad (25)$$

Now, the lowest frequency does not occur for m=0, as for m=0, the six field components are zero. Rather, the lowest frequency occurs for m=1. Another interesting point is that the first non-zero root of J'_O is 3,8317, which is greater than the first non-zero root of J'_1, which is 1.8412. Therefore, the lowest frequency occurs for n =1, i = 1, m = 1.

IV. DESIGNING A TE_{012} CAVITY

If a cavity is used for an oscillator application, then its unloaded quality factor, Q_O, should be as high as possible. For a given metallic cavity material, different quality factors are obtained, according to the different chosen mode [6]. The highest quality factors are obtained with the TE_{0im} modes. Usually, the TE_{011} mode is the one chosen, although, the TE_{012} presents a higher Q_O. There are two reason for preferring TE_{011} with respect to TE_{012}: (a) - the cavity results smaller, roughly with reduction of a third in the volume; (b) – the operational range of the TE_{011} occurs in a region far less populated by extraneous modes than the TE_{012}.

However, in this work, the TE_{012} mode will be used, in order to demonstrate an expertise in rejecting undesired modes. The onus will be a larger size cavity, while the bonus will be a higher quality factor, by a ratio of approximately 25%, with respect to TE_{011}.

Additionally, Collin [6] has also determined the optimum ratio diameter/height for maximizing the quality factor, according to the desired mode. For both TE_{011} and TE_{012} this ratio is 1.1. By using (25) the TE_{012} cavity dimensions for 8.3 GHz are:

Radius r = 29.7 mm ; Height h = 54.0 mm

By using (23) and (25), it is seen that within a suitable operational frequency range, in the vicinity of 8.3 GHz, between 7.7 and 8.9 GHz, the following modes may generated and deserve a careful analysis:

(i) - TM_{210} at 8256 MHz

(ii) - TM_{112} at 8295 MHz

o TE $_{012}$ at 8295 MHz (desired)

(iii) - TM_{211} at 8711 MHz

(iv) - TE_{312} at 8745 MHz

(v) - TE_{113} at 8843 MHz

(vi) - TM_{020} at 8874 MHz

There are six modes to be eliminated. Although the frequency of the TM_{112} mode occurs (theoretically) in the same as TE_{012}, both have different Q´s. In practice, the loading effect will displace one from the other. An oscillator may then keep jumping between these two frequencies. Observe that closing a hollow metallic cylindrical pipe body with two circular covers usually makes a cylindrical resonant cavity. The precious point is that a bad electrical contact may be deliberately done while closing the cavity. For instance: the touching surfaces may be covered with an insulating varnish. By doing so, the modes TE_{113} and TM_{112} will not be supported, as they have strong currents at the junction of the cover with the lateral cylinder surface. However, for the TE_{012} this current is zero ($H_\rho = 0$ and $E_\phi = 0$, for $\rho = r$).

Additionally, the cavity will operate in a transmission mode, by using two connectors placed on the same cover. The connectors are located in two radii with an angle of ϕ=135^{O} between them. The excitation of the TE_{012} will not be affected, as with n=0, there is no dependence upon ϕ. However, all modes TX_{2im} will be rejected. This is easily seen from the equation (22f) and from (24f), which describe the H_{ϕ} field component for TM and TE modes, respectively.

Concerning the cavity excitation through one of the covers, it must be observed that on the covers one has $E_{\phi} = 0$ and $H_z = 0$. Therefore, it was decided to use a magnetic excitation over H_{ρ}, which incidentally, reaches its maximum value on the covers, producing circumferential currents. These are such that the maximum intensity current circumferences – on the covers - are located at the coordinate ρ = 0.48 r. This fact provides for the information for the best location for placing the connectors: yielding maximum excitation and maximum reception.

Additionally, by placing input and output connectors at ρ = 0.48 r some other modes that possess H_{ϕ} nodal circles at (or near) this coordinate are eliminated (or substantially attenuated).

In this way, simple methods – based on field configurations – will enhance the desired mode and eliminate (or at least attenuate) undesired ones.

V. Practical Results

Two SMA connectors were next introduced in the top cavity cover, at the coordinate ρ = 0.48 r, and, their radii were doing an angle of 135^{O}. A short circular loop was responsible for magnetic excitation. Empirically, the loop size was adjusted until the insertion loss at the TE_{012} peak reached – 3 dB.

This – 3 dB value was chosen as it is was satisfactory practical value for exacting counter-balancing the gain of a typical active element (for instance, an AsGa FET) at 8.3 GHz, operating at large signal condition. Fig. 2 depicts a possible prospective arrangement of the present described cavity working together with an active device for an oscillator implementation.

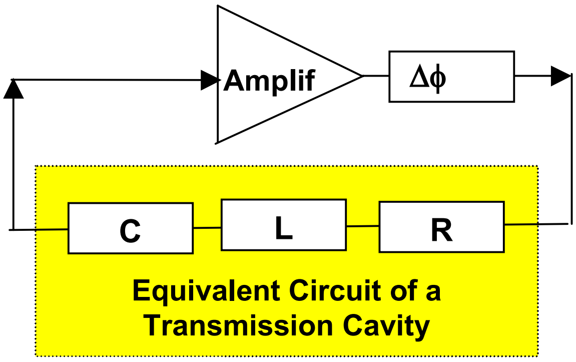

Fig. 2 – Practical arrangement for implementing an oscillator, using a transmission mode cavity.

In Fig. 2, it is seen a loop containing an amplifier, a phase-shift adjustment and an equivalent RLC circuit

The amplifier provides for gain, in order to compensate the losses of the cavity and some small losses of the phase-shifter.

The phase-shifter provides for a phase adjustment in order to have a positive feedback at the desired frequency.

The transmission mode cavity is equivalent to a series RLC circuit. As the input and output impedances of the other components are 50 ohm and the cavity has a 3 dB insertion loss, then the R value is 100 ohm.

Finally, the cavity was assembled, as described, with a coat of insulating varnish between the covers and the lateral body. The cavity has its insertion loss measured, from 7.7 to 8.9 GHz. The obtained results are shown in Fig. 3.

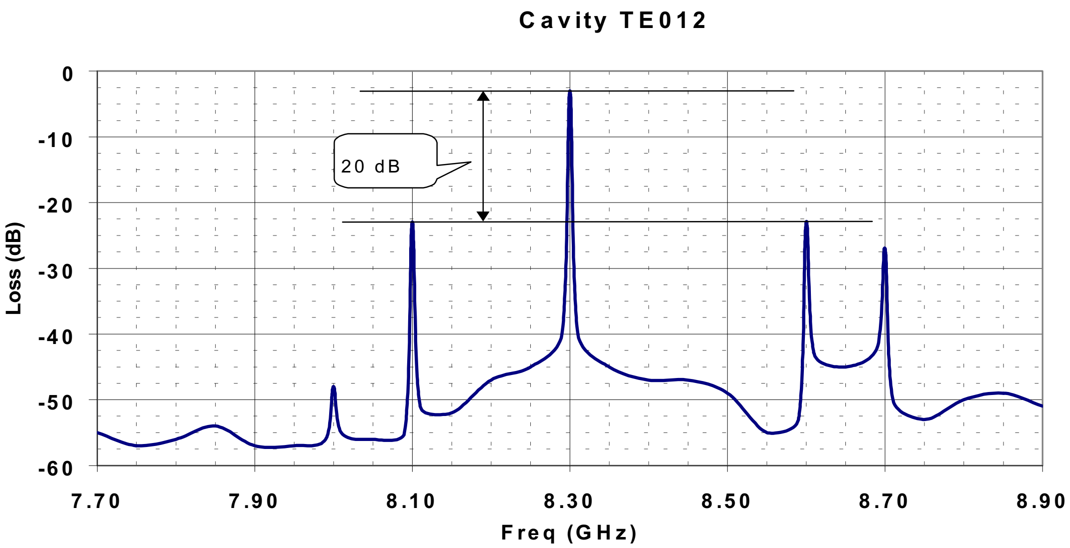

Fig. 3 – Obtained measured results for the insertion loss of a TE_{012} transmission mode cavity

The first observation, in Fig. 3, is a pronounced peak at 8.3 GHz, resulting from the transmission of the TE_{012} mode.

Next, it is seen that there several other peaks, however, they are – at least – 20 dB below the TE_{012} one. Therefore, the simple countermeasure methods that have been used to combat extraneous modes have been successful, as a 20-dB rejection is a figure well below the gain of Fig. 2 amplifier.

By using a stable synthesized generator together with a stable digital frequency meter, it was possible to measure the bandwidth of the TE_{012} line at the 3 dB below the peak level. Although noisy, the measured bandwidth was estimated as 600 kHz. The loaded quality factor, Q_L, is then obtained:

$$Q_L = \frac{8.3x10^9}{600x10^3} \cong 13.8x10^3$$

However, the unloaded quality factor, Q_O, as R =100 ohm (- 3dB insertion loss), is twice the last figure, i.e.:

$$\mathbf{Q_O \cong 28x10^3}$$

This value is reasonably competitive with the Q_O of a crystal, and almost an entire order of magnitude greater than the figure obtained with a dielectric resonator, at the same frequency.

Still, for comparison, the theoretical value of the same cavity, if coated with polished silver, would be $Q_O(Ag) = 38x10^3$ [6], while for polished aluminum it would be $Q_O(Al) = 31x10^3$ [8]. Here, a lower value has been obtained as a very elementary polishing has been performed.

VI. Discussion and Conclusions

In this work a metallic resonant cavity, operating at the TE_{012} mode in 8.3 GHz has been implemented, for stable microwave carrier generation.

The work has started by demonstrating that the Bromwich method to solve Maxwell's equations is a very suitable one. It may be applied in the most usual coordinate systems occurring in the Classical Electro-magnetism problems. It has also been shown that, once Maxwell's equations are written and solved in a generic curvilinear coordinate system, the trans-position to a specific coordinate system is quite straightforward. In this present work, the transposition to cylindrical coordinate system has been performed.

Once Maxwell's equations have been written in the cylindrical coordinate system, the desired cavity design was immediately obtained. Additionally, pattern field observation of several other modes has led to countermeasures in order to avoid (or at least to attenuate) extraneous modes occurring in the vicinity of the desired one.

Strictly speaking, the cavity has resonated – in the TE_{012} mode - at 8.295 GHz, only 0.06% apart from the desired value. The use of TE_{012} mode, alternatively to the usual TE_{011}, has led to a quality factor 25% higher. Although a TE_{012} cavity works in a region far more populated by extraneous modes than a TE_{011}, all undesirable modes have been attenuated by 20 dB, or more.

This cavity is a worthy piece of hardware for implementing stable microwave carrier generator, neither needing complex and costly multiplier chains nor sophisticated PLL loops. As shown in Fig. 2, the carrier generator is extremely simple. Additionally, the carrier generator will be temperature-stable if the cavity temperature has some sort of stabilization

Concluding, this work has two relevant contributions.

The first is an academic one, by bringing into focus the Bromwich method for solving Maxwell's equations and pointing out its advantages and simplicity. Unfortunately, the Bromwich method has been somewhat left aside among numerous researchers; exception done with those from French School [7].

The second is a practical one, by offering a successful design of a *risky* TE_{012} cavity (alternatively to a *safe* TE_{011}), and by reaching satisfactory and useful results for a practical utilization in stable microwave carrier generation.

References

[1] – A. Podcameni, L. F. M. Conrado and M. M. Mosso - "Unloaded Quality Factor Measurement of a Dielectric Resonator for MIC Applications, ***IEE Electronics Letters***, Vol. 18, 1982, pp. 656-658.

[2] – J. Smith - Modern Communications Circuits, ***Mc-Graw Hill***, N. Y., 1986.

[3] – H. Meyr and G. Ascheid – Synchronization in Digital Communications, Vol. 1; Phase, Frequency-Locked Loops and Amplitude Control, ***John Wiley and Sons***, N. Y., 1990.

[4] – W. Gosling (Ed.) - Radio Receivers, ***Peter Peregrinus***, London, 1986.

[5] - A. Podcameni - "Long-Loop PLL Design for Low Phase Noise AM Demodulation", in: *Proceedings of the* ***International Telecommunications Symposium - ITS 94,*** (Rio de Janeiro), 1994.

[6] – R. E. Collin – Foundations for Microwave Engineering 2nd Edtion, ***Mc-Graw Hill***, N. Y., 1992, see Section 7.4.

[7] – A. Angot – Compléments de Mathématiques, 4ème Edition, ***Edition de la Revue D'Optique***, Paris, 1961, see Sections 3.2, 3.3, 3.4, 5.3, 6.3 and 7.5.

[8] S. F. Adam – Microwave Theory and Applications, ***Prentice-Hall***, Englewood Cliffs, 1969, see Section 2.1.