AN INDEX ASSIGNMENT ALGORITHM FOR IMPROVING THE TRANSMISSION OF VECTOR-QUANTIZED IMAGES OVER A RAYLEIGH FADING CHANNEL

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ABSTRACT

A serious problem related to the transmission of vector-quantized images through noisy channels is that whenever errors occur, the nearest neighbor rule is broken. As a consequence, very annoying blocking effects may appear in the reconstructed images. In the present work, a simple and fast method for organizing the vector quantization (VQ) codebooks is presented. The key idea behind the proposed method is to ensure that similar (dissimilar) binary representations of the indexes of the codevectors correspond to similar (dissimilar) codevectors themselves. It is shown that the organized codebooks improve the performance of the transmission system in the sense that they lead to reconstructed images with better quality when compared to the ones obtained by using non-organized codebooks.

1. INTRODUCTION

Vector quantization (VQ) [1], [2] is a well-known coding technique that plays an important role in many applications such as speech/image compression and speech/speaker recognition.

A vector quantizer (block quantizer) is a mapping Q of a k-dimensional Euclidean space R^k into a finite subset W of R^k given by

$$Q : R^k \to W, \tag{1}$$

where the codebook $W = \{\vec{w}_i;\ i = 0,\ 1,\ \ldots,\ N-1\}$ is the set of codevectors (reproduction vectors), k is the dimension of the vector quantizer and N is the codebook size, that is, the number of codevectors in W. The corresponding code rate, which measures the number of bits per vector component, is $R = (\log_2 N)/k$.

Fig. 1 shows that a vector quantizer can be considered as a combination of two functions: a VQ encoder and a VQ decoder. The former maps an input vector $\vec{x}$ to a codevector $\vec{w}_I$ if $d(\vec{x},\vec{w}_I) < d(\vec{x},\vec{w}_i),\ \forall i \neq I$, where $d(\cdot)$ is some distortion function (the Euclidean distance is widely used). In other words, it follows the nearest neighbor rule to find the codevector that presents the greatest similarity to vector $\vec{x}$. Then, the $\lceil \log_2 N \rceil$ binary representation of the index I, denoted by b_I, is transmitted to the VQ decoder [3], [4]. Upon receiving the binary word b_I, the VQ decoder simply looks up the I-th codevector, $\vec{w}_I$, from a copy of the codebook W, and outputs $\vec{w}_I$ as the reproduction of $\vec{x}$. Therefore, it follows the decoding rule $\text{dec}(b_I) = \vec{w}_I$ [5]. As shown in Fig. 1, the mapping of $\vec{x}$ into $\vec{w}_I$ is captured by the expression $Q(\vec{x}) = \vec{w}_I$.

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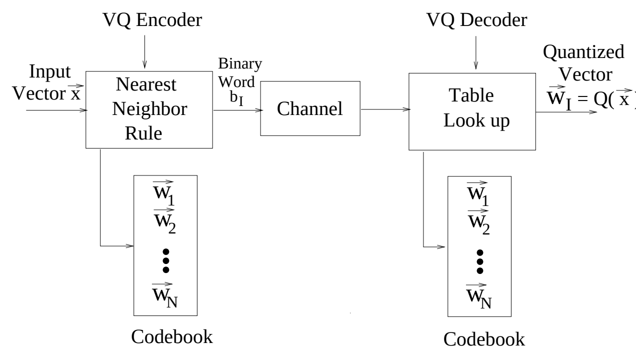


Fig. 1 - Block diagram of the coding/decoding procedure involved in vector quantization.

This paper considers the transmission of vector-quantized images through a Rayleigh fading channel. An algorithm for codebook organization (that is, index assignment) is presented for the purpose of minimizing the effects of channel errors.

The remaining of the paper is organized as follows. In Section 2, a brief description of vector quantization for noisy channels is presented. An overview of the transmission system model under consideration is given in Section 3. The index assignment algorithm is described in Section 4. Simulation results are provided in Section 5 and the concluding remarks are given in Section 6.

2. VECTOR QUANTIZATION FOR NOISY CHANNELS

An important problem regarding a communications system involving simple vector quantization is the transmission of the binary word b_I through a noisy channel. Since b_I was output by the VQ encoder, the nearest neighbor rule ensures that $d(\vec{x},\vec{w}_I) < d(\vec{x},\vec{w}_i),\ \forall i \neq I$. If b_I is corrupted by channel noise, the VQ decoder receives a binary word $b_{\hat{I}}$ that differs from that generated by the VQ encoder. As a consequence, the decoder outputs the codevector $\vec{w}_{\hat{I}}$. In other words, due to the channel error, the nearest neighbor rule is broken since the input (source) vector $\vec{x} \in R^k$ is not decoded as its corresponding closest codevector $\vec{w}_I$. Accordingly, the channel error leads to a distortion increase, since $d(\vec{x}, \vec{w}_{\hat{I}}) > d(\vec{x}, \vec{w}_I)$. It is worth mentioning that the distortion increase depends on the Euclidean distance between the codevectors $\vec{w}_I$ and $\vec{w}_{\hat{I}}$. It is high when the Euclidean distance is high. On the other hand, it may be negligible if the Euclidean distance is very small. Thus, if it is ensured that a small Hamming distance between the binary words b_I and $b_{\hat{I}}$ corresponds to a small Euclidean distance between the corresponding codevectors $\vec{w}_I$ and $\vec{w}_{\hat{I}}$, the distortion increase due the the channel error may be reduced. The algorithm proposed in the present paper attempts to organize the VQ codebook in order to ensure that similar binary words (small Hamming distance) correspond to similar corresponding codevectors (small Euclidean distance).

Hence, considering the problem of vector quantization for noisy channels, the indexing of the codevectors in a codebook influences considerably the average distortion caused by channel errors. By suitably arranging (organizing) the codebook such that the errors introduced in the binary representation of the indexes of the codevectors cause incorrectly decoded codevectors to be close on average to the transmitted codevectors, the average distortion due to channel noise can be reduced. The problem of finding the best codebook arrangement (organization) involves searching every possible index assignment for the one that yields the best possible performance. This task requires enormous computational complexity due to the combinatoric nature of the problem, since there are $N!$ possible assignments of indexes to N codevectors. As an example, a codebook with $N = 32$ reconstruction vectors has approximately 10^{35} different codebook arrangements to be investigated. In this context, the proposed algorithm may be viewed as a suboptimal solution to the index assignment problem, since it does not perform a complete search in the entire set of $N!$ possible codebook configurations.

It is important to point out that, concerning the transmission of vector-quantized images through noisy channels, when a binary word corresponding to a codevector index is corrupted by noise, an entire block of $k = k_1 \times k_2$ pixels is damaged, where k is the dimension of the vector quantizer. Correspondingly, typical annoying blocking artifacts of $k_1 \times k_2$ pixels may appear in the reconstructed image.

Several approaches have focused on the problem of vector quantization for noisy channels [6], such as the channel-optimized quantization [7], [8] and the use of index assignment methods [9], [10]. Zeger and Gersho [9] proposed a Binary Switching Algorithm that performs Pseudo-Gray coding on a given VQ codebook. Farvardin [7] developed an algorithm based on *simulated annealing* for assigning binary codewords to the vector quantizer codevectors. However, in systems with dynamically changing codebooks, those algorithms can become very costly in terms of computational complexity due to the required number of operations involved. Thus, when time is an important factor, the use of those algorithms is not recommended.

In the present work, a new, simple, and fast index assignment algorithm for improving the performance of the transmission of vector-quantized images through noisy channels is introduced. The next section considers the transmission system model used to test the proposed algorithm.

3. THE TRANSMISSION SYSTEM MODEL

Consider a communications system where the transmitter is equipped with n antennas and the receiver is equipped with m antennas. After the channel encoding, the data goes through a serial-to-parallel (S/P) converter and it is divided into n streams of bits. Each stream is used as the input to a pulse shaper. The output of each pulse shaper is then modulated. At each time slot t, the output of modulator i is a signal c_t^i, that is transmitted using antenna i for $1 \leq i \leq n$ (see Fig. 2 for the case of two antennas). The signal at each receiving antenna is a noisy superposition of the n transmitted signals corrupted by Rayleigh fading. This kind of fading is usual when one considers the absence of a direct path between the transmitting antenna and the receiving antenna. It is assumed that the elements of the signal constellation are contracted by a factor of $\sqrt{E_S}$, chosen so that the average energy of the constellation is 1.

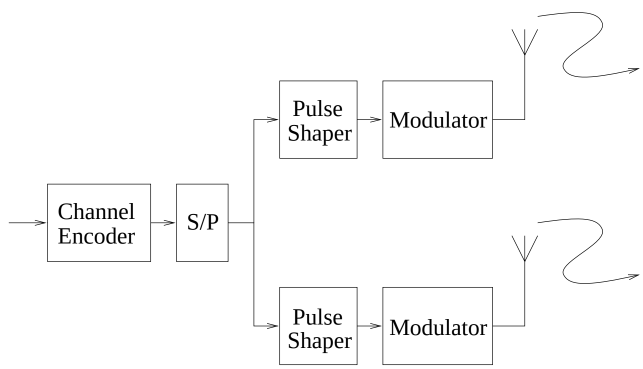


Fig. 2 - The block diagram of the transmitter.

At the receiver, the demodulator computes a decision statistics based on the received signals arriving at each receiving antenna j, $1 \leq j \leq m$. The signal d_t^j received by antenna j at time t is given by

$$d_t^j = \sum_{i=1}^{n} \alpha_{i,j} c_t^i \sqrt{E_S} + n_t^j, \tag{2}$$

where the noise n_t^j is modeled as independent samples of a zero-mean complex Gaussian random variable with variance $\mathcal{N}_0/2$ per dimension. Coefficient $\alpha_{i,j}$ is the path gain from transmitting antenna i to receiving antenna j. It is assumed that these path gains are constant during a frame and vary from one frame to another.

In the present work two different transmission schemes, for evaluating the performance of the proposed index assignment method, are considered. The first scheme uses a space-time code [11] with 8 states. The second one uses a repetition code with rate equals 1/2. This rate is chosen in order to assure that the application of the repetition code leads to the same channel transmission rate obtained by using the space-time code. These two transmission systems under consideration are suitable to analyze the performance of the index assignment algorithm when the bit error rate (BER) is low, using space-time code, and high, for the system using the repetition code. It is worth mentioning that the space-time code is suitable for transmission over Rayleigh fading channels, due to its potential regarding trellis complexity, diversity advantages, data rate and bit error rate. Criteria for designing space-time codes may be found in [11].

4. THE INDEX ASSIGNMENT ALGORITHM

In this section, the new index assignment algorithm is described in details. The proposed method provides an improvement on the average distortion due to the channel errors, over an arbitrary index assignment, as confirmed by the simulation results presented in Section 5.

Consider the original VQ codebook $W = \{\vec{w}_i;\ i = 0,\ 1,\ \ldots,\ N-1\}$ and let $N = 2^\beta$, that is, β represents the number of bits per vector. Let $Y = \{\vec{y}_i;\ i = 0,\ 1,\ \ldots,\ N-1\}$ denote the organized (ordered, rearranged) codebook, that is, the output of the index assignment algorithm.

Let b_i denote the binary word (binary representation) of the i-th codevector, $\vec{w}_i$. Let $d(\vec{w}_i, \vec{w}_j)$ be the Euclidean distance (measured in the input space R^k) between codevectors $\vec{w}_i$ and $\vec{w}_j$ and let $h(b_i, b_j)$ denote the Hamming distance between the corresponding binary words b_i and b_j. The fundamental purpose of the proposed algorithm is to organize the codebook in a such way that similar codevectors (in R^k) are represented by similar binary words (in $\{0,1\}^\beta$), in order to minimize the effect of an error introduced by a noisy channel. In other words, the proposed codebook organization method attempts to ensure that small Euclidean distances $d(\vec{w}_i, \vec{w}_j)$ correspond to small Hamming distances $h(b_i, b_j)$. With such approach, if a channel error occurs, the received distorted binary representation of the index of a codevector will be decoded to a codevector that is not very "dissimilar" to the one that would be decoded upon an error-free transmission.

The index assignment algorithm consists of the following steps:

Step 1: Let $\vec{y}_0$ be equal to the vector $\vec{w}_i$ with maximum Euclidean norm among all (code)vectors in W and exclude this vector $\vec{w}_i$ from the search space;

Step 2: Among the remaining vectors $\vec{w}_i$ choose the vector with the largest Euclidean distance to $\vec{y}_0$ and let $\vec{y}_{N/2}$ be this vector. Exclude vector $\vec{w}_i$ from the search space.

Step 3: For $i = 1$ to $N/2 - 1$, do:

- (A) Among the remaining vectors $\vec{w}_i$ choose the vector with the lowest Euclidean distance to $\vec{y}_{i-1}$ and let $\vec{y}_i$ be this vector. Exclude vector $\vec{w}_i$ from the search space.
- (B) Among the remaining vectors $\vec{w}_i$ choose the vector with the lowest Euclidean distance to $\vec{y}_{(N-i+1) \bmod N}$ and let $\vec{y}_{N-i}$ be this vector. Exclude vector $\vec{w}_i$ from the search space.

Step 4: To this ordered vector set $Y = \{\vec{y}_i;\ i = 0,\ 1,\ \ldots,\ N-1\}$ apply a Gray code with $\log_2 N$ bits.

A Gray code is constructed based on the constraint that neighbor codewords differ in a single bit. Fig. 3 shows Gray codes with 2, 3 and 4 bits. The conversion between a binary positional code and a Gray code is a very simple process [12].

00
01
11
10

(a) 2 bits.

000
001
011
010
110
111
101
100

(b) 3 bits.

0000
0001
0011
0010
0110
0111
0101
0100
1100
1101
1111
1110
1010
1011
1001
1000

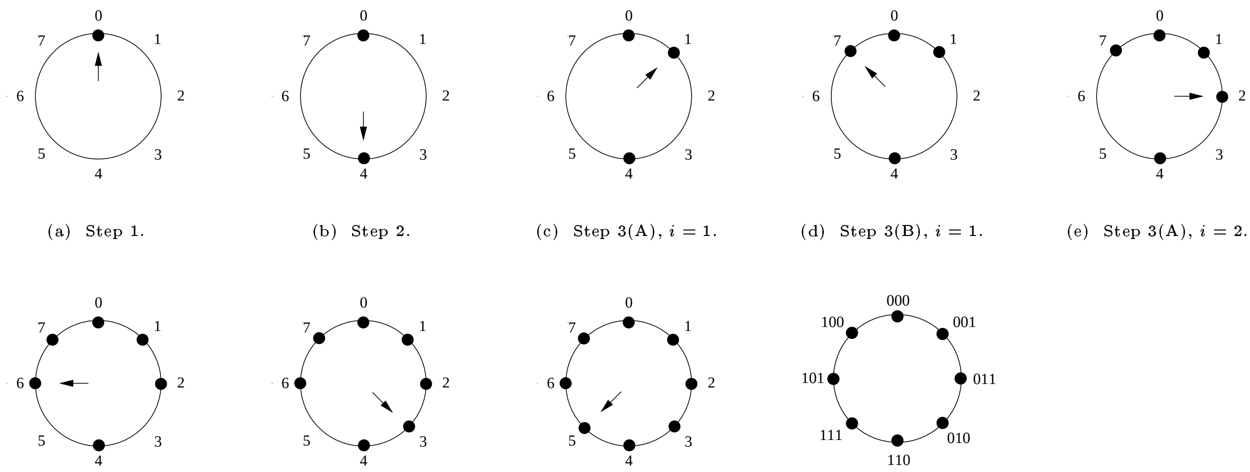
(c) 4 bits.

Fig. 3 - Gray codes.

The dynamics of the proposed algorithm can be well understood from Fig. 4, that shows a hypothetical circumference upon which the codevectors are to be suitably arranged for the subsequent application of a Gray code. Fig. 4 shows all the steps involved in the indexing of a codebook with $N = 8$. In Step 1 (Fig. 4(a)), the codevector with maximum Euclidean norm is placed in position 0 of the hypothetic circumference. Then the codevector which presents the largest Euclidean distance to the first chosen codevector (selected to position 0) is placed in position 4. Subsequently, the remaining codevectors are sequentially placed in positions 1, 7, 2, 6, 3 and 5. The organization of the codebook is concluded with the application of a Gray code in a clockwise direction. It is important to mention that the positioning of the codevectors is carried out to minimize the Euclidean distance between codevectors placed in adjacent positions on the circumference. As an example, the codevector in position 2 is the closest vector (in terms of Euclidean distance) to the vector placed in position 1.

5. SIMULATION RESULTS

In all simulations reported in this section, the image block size used was 4×4 pixels and thus the vector dimension was



(a) Step 1. (b) Step 2. (c) Step 3(A), $i = 1$. (d) Step 3(B), $i = 1$. (e) Step 3(A), $i = 2$.

(f) Step 3(B), $i = 2$. (g) Step 3(A), $i = 3$. (h) Step 3(B), $i = 3$. (i) Step 4.

Fig. 4 - An example of a codebook organization. The proposed algorithm is applied to 8 codevectors.

$k = 16$. The traditional LBG (Linde-Buzo-Gray) [13] algorithm was used for designing a codebook with $N = 256$ codevectors. Thus, a 0.5 bpp source coding was considered. For evaluating the transmission of the binary representation of the index of the codevectors over a Rayleigh fading channel, the 256×256 Lena image (Fig. 5) was used. Therefore, the transmission of the whole image involves a total of $\frac{256\times 256}{16} = 4096$ binary words.

The transmission of the vector-quantized Lena image was evaluated for different values (ranging from 6 dB to 10 dB) of channel signal-to-noise ratio (SNR), which is defined as the ratio of the average faded signal power to the received noise power. The transmission system was composed of two transmitting antennas and two receiving antennas. The quality of the reconstructed (decoded) Lena image was assessed by the peak signal-to-noise ratio (PSNR) [1].

Fig. 5 - Original Lena image.

Two cases were considered: the use of repetition code and the application of space-time code. Consider, first, the use of repetition code and the occurrence of many errors. Fig. 6 clearly shows the efficiency of the index assignment algorithm. For the entire range of fading channel SNR, the organized codebook leads to higher values of PSNR when compared to to ones obtained by using the original codebook. It is observed that the gain (in terms of PSNR) introduced by the codebook organization via the index assignment algorithm decreases as the fading channel SNR increases. In fact, as can be observed from Fig. 8, the amount of error decreases as the channel SNR increases. As a consequence, the advantage of using the organized codebook in substitution to the original (non-organized) codebook is reduced.

Fig. 7 depicts the PSNR as a function of the channel SNR for a space-time code with 8 states. It is observed that the organized codebook is less sensitive to channel errors, since the image quality (in terms of the PSNR) obtained by using the organized codebook is better than that obtained by using the original codebook, for all values of channel SNR under consideration. It is important to note that the performance gain of the organized codebook over the original codebook decreases as the channel SNR increases.

Simulations were also carried out to evaluate the performance of the proposed codebook organization algorithm regarding image transmission without the application of channel coding. Figs. 9(a) and 9(b) show reconstructed Lena images after transmission through a Rayleigh fading channel with SNR=6 dB. It is important to note that the use of an organized codebook proved to be suitable to improve the quality of the reconstructed image. Indeed, a direct comparison of Figs. 9(a) and 9(b) emphasizes the effectiveness of the index assignment algorithm – the blocking artifacts of the reconstructed image obtained by using an organized codebook causes a smaller influence on the subject perception when compared to the ones obtained by using a non-organized codebook.

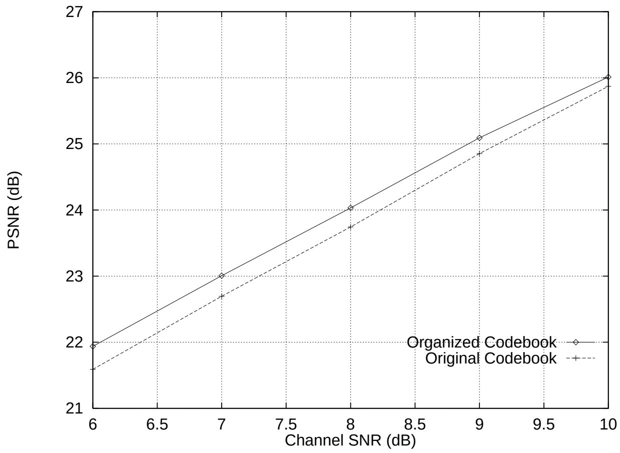


Fig. 6 - PSNR as a function of the channel SNR. A repetition code was used.

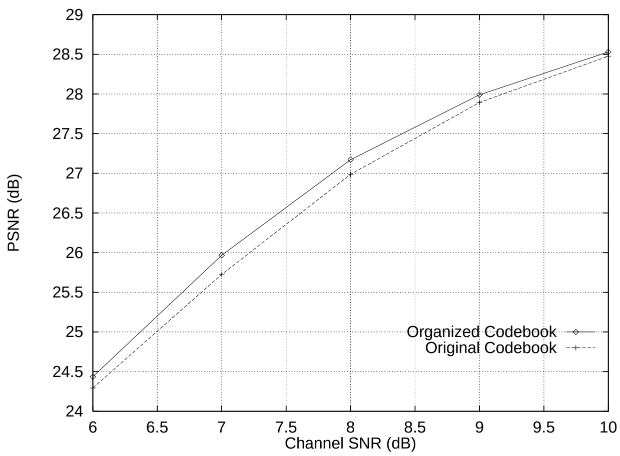


Fig. 7 - PSNR as a function of the channel SNR. A space-time code with 8 states was used.

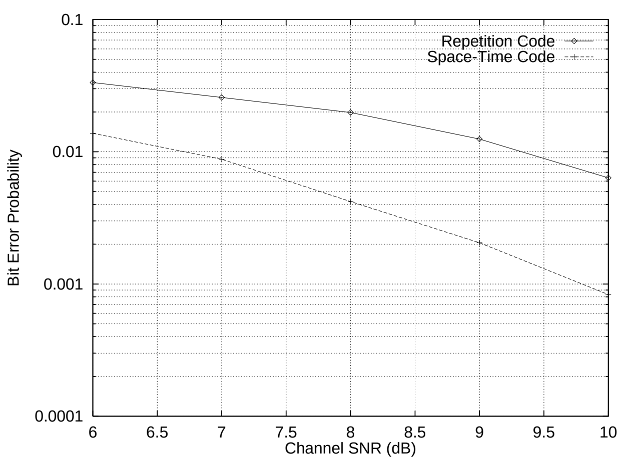


Fig. 8 - Bit error probability as a function of the channel SNR.

(a) Using a non-organized codebook.

(b) Using an organized codebook.

Fig. 9 - Reconstructed Lena image after transmission through a Rayleigh fading channel with SNR=6 dB. No channel code was used.

6. CONCLUDING REMARKS

A new, simple, and fast method for organizing a vector quantization (VQ) codebook was introduced. By suitably indexing the codevectors, the proposed algorithm orders the codebook such that codevectors with small/large Euclidean distances are represented (indexed) by corresponding binary words with small/large Hamming distances. As a consequence, considering the scenario of vector quantization for noisy channels, errors introduced in the binary words of the codevectors cause incorrectly decoded codevectors to be close on average to the transmitted codevectors. Hence, the proposed index assignment provides an improvement on the average distortion due to channel errors over an arbitrary indexing.

Simulations results concerning the transmission of vector-quantized images through a Rayleigh fading channel showed that the organized codebook leads to reconstructed images with better quality. In fact, for different values channel signal-to-noise ratio, a performance gain (in terms of the peak signal-to-noise ratio of the reconstructed image) is obtained by using an organized codebook in substitution to the original (non-organized) codebook.

It is worth mentioning that although the problem of finding the best codebook arrangement (organization) has already been addressed in many works, it is well known that most algorithms requires an enormous computational complexity. In this context, the organized codebook obtained by the proposed index assignment method may be used as the starting point to these codebook organization algorithms, leading to a reduction in terms of the computational complexity involved in their dynamics.

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