SPACE-TIME CODING PERFORMANCE IMPROVEMENT USING A ROTATED CONSTELLATION

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ABSTRACT

This paper presents a modification on the original space-time code scheme with the objective of improving its performance. Interleaving and rotation of the signal constellation, which do not affect the performance of the system for the additive white Gaussian noise channel, are introduced. The spectrum efficiency of the scheme is not altered and the modification of the modulator is very simple. Simulations are carried out to show that the error probability is improved, for the proposed scheme, in a fading environment.

1. INTRODUCTION

THE rapid growth of wireless communications demands an improving in the capacity and performance of the transmission systems. This can be achieved by the use of spatial diversity and optimum combining techniques [1], [2]. The space-time codes [3], [4], [5] combine spatial and temporal diversity which produce a good performance for fading channels.

Another way to increase the system diversity is to introduce redundancy by combining rotation and interleaving of the constellation symbols before the modulation operation. Here, this recent technique [6], [7], [8], [9], [10] is called "modulation diversity".

The key point to increase the modulation diversity is to apply a certain amount of rotation to a classical signal constellation in such way that any two points achieve the maximum number of distinct components [8]. Fig. 1 illustrates this idea for a QPSK scheme. In fact if it is supposed that a deep fade hits only one of the components of the transmitted signal vector, then one can see that the "compressed" constellation in Figure 1(b) (empty circles) offers more protection against the effects of noise, since no two points collapse together as would happen in Fig. 1(a). A component interleaver/deinterleaver is required to assume that the in-phase and quadrature components of the received symbol are affected by independent fading.

An interesting feature of the rotation operation is that the rotated signal set has exactly the same performance of the non-rotated one when used over a pure additive white Gaussian noise (AWGN). This occurs due to the fact that the Euclidean distance between the symbols, for rotated and non-rotated QPSK constellations, is the same in the two cases.

This paper presents a modification on the original space-time code scheme that is based on the interleaving and the rotation of the signal constellation used by the space-time codes, which considerably improves the system performance. The performance improvement is obtained with a little increase of the system complexity.

The remaining of this paper is organized as follow: In Section 2, the rotation of QPSK signal constellation for performance improvement is introduced. Section 3.1 presents the basic assumptions and the transmission system model. Section 3.2 provides the construction criteria for space-time codes. Section 4 includes the numerical results and discussions, while Section 5 is devoted to conclusions.

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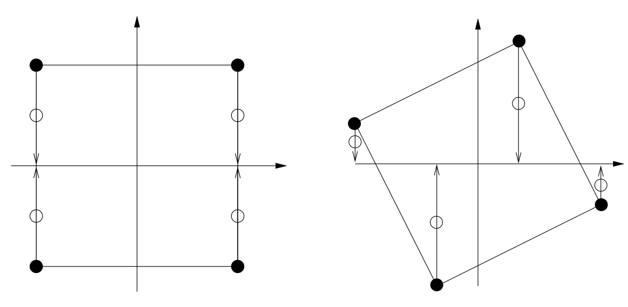
2. ROTATING THE QPSK CONSTELLATION

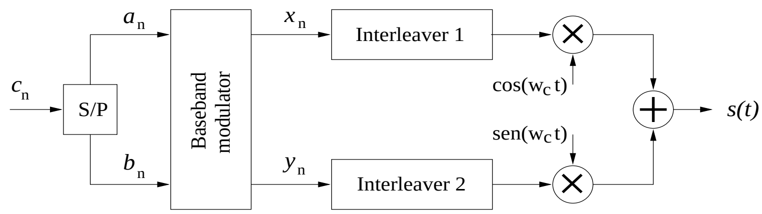
Fading causes significant degradation in the performance of digital communications systems. For a frequency-nonselective slowly fading channel, coded modulation combined with interleaving showed good performance improvement. Interleaving destroys fading correlation, adding diversity to the coded scheme [9], [10]. In [9] the author used a new form of interleaving, where each coordinate is interleaved independently to enhance the system performance. In this paper, the key idea is to analyze the influence of rotating the QPSK constellation, by a constant phase θ, in the system performance. The block diagram of the modified QPSK scheme is shown in Fig. 2.

Consider a conventional QPSK scheme. The transmitted

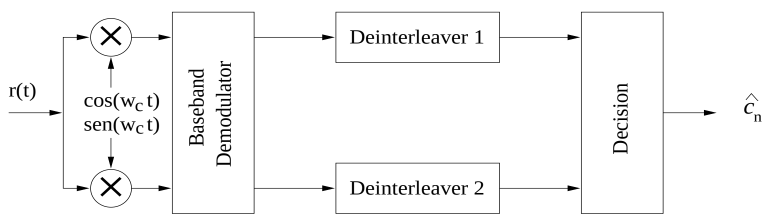


(a)   (b)

Fig. 1 - The basic idea of performance improvement using a rotated constellation.



(a) Modulator.



(b) Demodulator.

Fig. 2 - Block diagram of the modified QPSK scheme.

signal is given by

$$\begin{aligned} s(t) = A \sum_{n=-\infty}^{+\infty} a_n p(t - nT_S) \cos(2\pi f_c t) \\ +A \sum_{n=-\infty}^{+\infty} b_n p(t - nT_S) \sin(2\pi f_c t) \end{aligned} \quad (1)$$

where

$$a_n, b_n = \pm 1 \qquad \text{with equal probability,}$$

$$p(t) = \begin{cases} 1, & 0 \le t \le T_S \\ 0, & \text{elsewhere,} \end{cases}$$

f_c is the carrier frequency and A is the carrier amplitude.

When the signals in phase (I channel) and quadrature (Q channel) are interleaved independently, a diversity gain can be obtained because the fading in one channel is independent from the other channel. From Equation 1 the sequence $\{a_n\}$ is independent of the sequence $\{b_n\}$. In this case the system cannot take advantage of the previous described diversity unless some kind of redundancy between the two quadrature channels is introduced. Introducing redundancy in the QPSK scheme can be achieved by rotating its signal constellation by a constant phase θ, as show in Fig. 3.

After the rotation and the interleaving, the transmitted signal becomes

$$\begin{aligned} s(t) = A \sum_{n=-\infty}^{+\infty} x_n p(t - nT_S) \cos(2\pi f_c t) \\ +A \sum_{n=-\infty}^{+\infty} y_{n-k} p(t - nT_S) \sin(2\pi f_c t), \end{aligned} \quad (2)$$

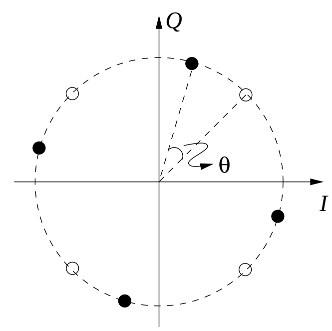


Fig. 3 - The rotated QPSK constellation.

where k is an integer representing the time delay, in number of symbols, introduced by the interleaving between the I and Q components and

$$\begin{aligned} x_n &= a_n \cos\theta - b_n \sin\theta \\ y_n &= a_n \sin\theta + b_n \cos\theta \end{aligned} \quad (3)$$

are the rotated symbol components. It is important to note that the rotation does not affect the system spectral efficiency for the rotated system also transmits two bits in one symbol interval.

The digital communication channel is assumed to be frequency-nonselective slowly fading with a multiplicative factor representing the effect of fading and an additive term representing the AWGN channel. Perfect phase recovery and channel state information (CSI) are assumed at the receiver. It is also assumed that the system is unaffected by intersymbol interference. The received signal is then written as

$$r(t) = \alpha(t)s(t) + \eta(t) \tag{4}$$

where $\alpha(t)$ is modeled as zero-mean complex Gaussian process.

If coherent demodulation is used the fading coefficients can be modeled, after phase elimination, as real random variables with Rayleigh distribution and unit second moment ($E[\alpha^2] = 1$). The independence of the fading samples represents the situation where the components of the transmitted points are perfectly interleaved. An undesirable effect of the component interleaving is the fact that it produces nonconstant envelope transmitted signals [8]. In the next section, the space-time code are presented.

3. THE SPACE-TIME CODES

3.1 The Transmission System Model

Consider a communication system where the transmitter is equipped with n antennas and the receiver is equipped with m antennas. The data is encoded by the channel encoder, the encoded data goes through a serial-to-parallel converter and is divided into n streams of data. Each stream of data is used as the input to a pulse shaper. The output of each pulse shaper is then modulated. At each time slot t, the modulator output i is a signal c_t^i that is transmitted using transmitting antenna i for $1 \leq i \leq n$ (see Fig. 4 for the case of two antennas). The signal at each receiving antenna is a noisy superposition of n transmitted signals corrupted by Rayleigh fading. Assume that the elements of the signal constellation are contracted by a factor of $\sqrt{E_S}$, chosen so that the average energy of the constellation is 1.

At the receiver, the demodulator computes a decision statistic based on the received signals arriving at each receiving antenna j, $1 \leq j \leq m$. The signal d_t^j received by antenna j at time t is given by

$$d_t^j = \sum_{i=1}^{n} \alpha_{i,j} c_t^i \sqrt{E_S} + n_t^j, \tag{5}$$

where the noise at time t is modeled as independent samples of a zero-mean complex Gaussian random variable with variance $N_0/2$ per dimension. The coefficient $\alpha_{i,j}$ is the path gain from transmitting antenna i to receiving antenna j. It is assumed that these path gains are constant during a frame and vary from one frame to another.

3.2 Code Construction

Reference [3] presented a criterion for the design of space-time codes based on certain matrices constructed from pairs of distinct code sequences. The minimum rank among those matrices quantifies the *diversity gain*, while the minimum determinant of those matrices quantifies the *coding gain*. This criterion is summarized in the following.

Consider the probability that a maximum-likelihood receiver decides erroneously in favor of a signal

$$\boldsymbol{e} = e_1^1 e_1^2 \cdots e_1^n e_2^1 e_2^2 \cdots e_2^n \cdots e_l^1 e_l^2 \cdots e_l^n, \tag{6}$$

if

$$\boldsymbol{c} = c_1^1 c_1^2 \cdots c_1^n c_2^1 c_2^2 \cdots c_2^n \cdots c_l^1 c_l^2 \cdots c_l^n \tag{7}$$

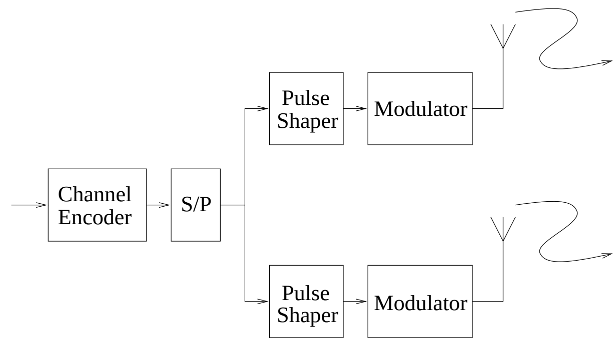


Fig. 4 - Block diagram of the transmitter.

was transmitted.

Assuming the ideal channel state information (CSI), the probability of transmitting $\boldsymbol{c}$ and deciding in favor of $\boldsymbol{e}$ is well approximated by

$$P(\boldsymbol{c} \rightarrow \boldsymbol{e} | \alpha_{i,j}) \leq \exp(-d^2(\boldsymbol{c}, \boldsymbol{e}) E_S / 4N_0), \tag{8}$$

where $N_0/2$ is the noise variance per dimension and

$$d^2(\boldsymbol{c}, \boldsymbol{e}) = \sum_{j=1}^{m} \sum_{t=1}^{l} \left| \sum_{i=1}^{n} \alpha_{i,j} (c_t^i - e_t^i) \right|^2. \tag{9}$$

Design criteria were defined for construction of space-time codes [3]. These criteria are the rank criterion and the determinant criterion that are based on matrix $B(\boldsymbol{c}, \boldsymbol{e})$, defined for each pair of possible codewords $(\boldsymbol{c}, \boldsymbol{e})$ as

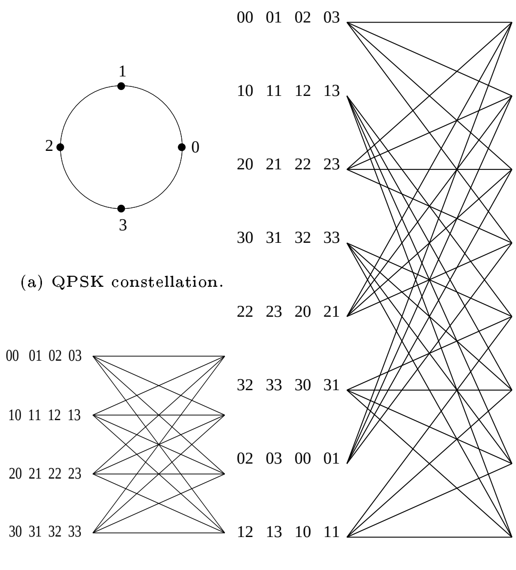
$$B(\boldsymbol{c}, \boldsymbol{e}) = \begin{pmatrix} e_1^1 - c_1^2 & e_2^1 - c_2^1 & \cdots & e_l^1 - c_l^1 \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & \cdots & e_l^2 - c_l^2 \\ e_1^3 - c_1^3 & e_2^3 - c_2^3 & \cdots & e_l^3 - c_l^3 \\ \vdots & \vdots & \ddots & \vdots \\ e_1^n - c_1^n & e_2^n - c_2^n & \cdots & e_l^n - c_l^n \end{pmatrix}. \tag{10}$$

The rank criterion establishes that to achieve maximum diversity mn, the matrix $B(\boldsymbol{c}, \boldsymbol{e})$ must be full rank for any codewords $\boldsymbol{c}$ and $\boldsymbol{e}$. If $B(\boldsymbol{c}, \boldsymbol{e})$ has rank r, where $r < n$, the maximum diversity is rm.

Defining matrix $A(\boldsymbol{c}, \boldsymbol{e}) = B(\boldsymbol{c}, \boldsymbol{e}) \cdot B^*(\boldsymbol{c}, \boldsymbol{e})$, where $B^*(\boldsymbol{c}, \boldsymbol{e})$ denotes the Hermitian (transpose conjugate) of $B(\boldsymbol{c}, \boldsymbol{e})$ taken over all pairs of distinct codewords and assuming that its rank is r, the code advantage can be defined as the minimum of the r-th roots of the sum of determinants of all $r \times r$ principal cofactors of $A(\boldsymbol{c}, \boldsymbol{e})$. That is the determinant criterion.

The space-time code with 4 and 8 states (Fig. 5(b) and Fig. 5(c), respectively) for a QPSK constellation in Fig. 5(a) was constructed based on the above performance criteria. It specifies two transmitting antennas and a spectral efficiency of 2 bits/s/Hz.

At each time t, a branch transition is chosen based on the state of the encoder and the input bits. Each branch is labeled $q_t^1 q_t^2$ when the constellation symbol q_t^1 is transmitted by the first antenna and symbol q_t^2 is transmitted by the second antenna. The constellation symbols can be seen in Fig. 5(a).



(a) QPSK constellation.

(b) 4 states.

(c) 8 states.

Fig. 5 - Space-time code.

Assuming that the decoder knows the fading coefficients $\alpha_{i,j}$ and that r_t^j is the signal that arrives at the decoder at time t, the branch metric for a transition $q_t^1 q_t^2$, given by

$$d^2 = \sum_{i=1}^{m} \left| r_t^j - \sum_{i=1}^{2} \alpha_{i,j} q_t^i \right|^2, \qquad (11)$$

can be used to decode the information.

The Viterbi decoding algorithm [11] was used in [3] to decode the space-time codes. It has been showed in [12] that the use of the stack algorithm in the decoding stage reduces considerably the processing decoding time as compared to the original scheme, maintaining an acceptable performance in terms of the bit error probability.

4. SIMULATIONS RESULTS

This section presents the simulations used to find out the rotation phase which produces the best performance, in terms of bit error rate. Next, this rotation is applied to the space-time scheme and its performance is assessed by simulations.

In order to determine the phase rotation θ that achieves the best performance, the transmission system in Fig. 2 was simulated for θ varying from zero to $\pi/2$. Fig. 6 shows the system performance, measured in terms of bit error probability, for $E_b/N_0 = 10$, 15 and 20 dB. It can be seen that the optimum performance is obtained for θ approximately equal to $\pi/7$ for the three curves presented. Considering this optimum phase rotation, Fig. 7 compares the performance of the original QPSK scheme ($\theta = 0$) and it rotated version for E_b/N_0 varying from zero to 30 dB. When $\theta = 0$, the performance of the system reduces to that of a conventional QPSK scheme, measured in terms of the bit error rate P_b, given by [13]

$$P_b = \frac{1}{2}\left[1 - \sqrt{\frac{E_b/N_0}{1 + E_b/N_0}}\right], \qquad (12)$$

which is also plotted in Fig. 7. It can be noted that a considerable performance improvement is obtained compared to the conventional QPSK scheme, which can reach 8 dB at a bit error probability of 10^{-3}.

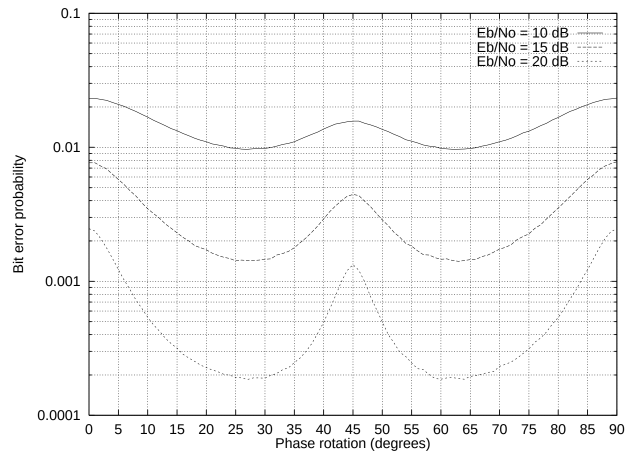


Fig. 6 - The bit error probability for the modified QPSK scheme as a function of the phase rotation θ.

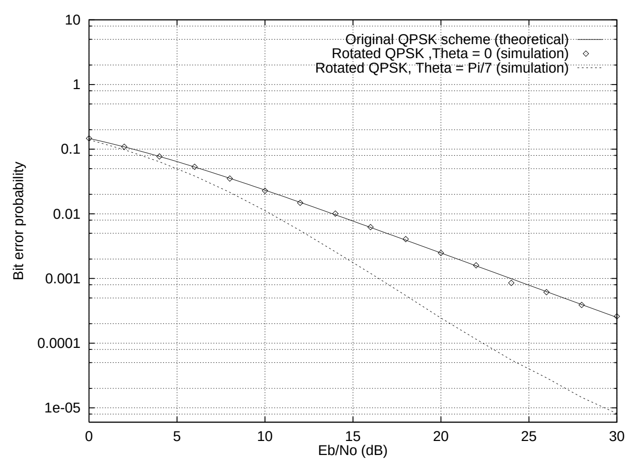


Fig. 7 - The bit error probability for the modified QPSK scheme and the original QPSK scheme as a function of E_b/N_0.

The second set of simulations was used to verify the influence of the rotation and interleaving on the performance of space-time codes. A space-time code with 8 states was used, as depicted in Fig. 5(c). In the simulations, the frame consisted of 256 bit of information and the interleaving depth was $k = 16$. The Viterbi algorithm was used in the decoding process. Figs. 8 and 9 present the simulation results when the system is equipped with one receiving antenna and two receiving antennas, respectively. Notice that a performance improvement is obtained (approximately 3 dB gain at a frame-error probability of 10^{-2}), compared to the original space-time code scheme, for the receiver equipped with one receiving antenna. It is worth to mention, that this performance improvement is obtained with little increase in the complexity of the system.

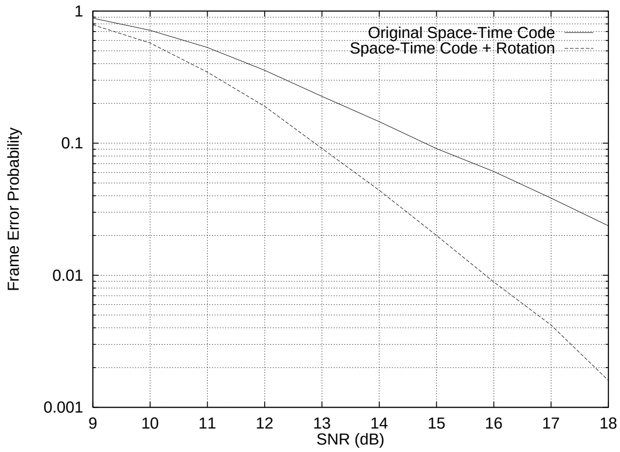


Fig. 8 - Comparing original and rotated QPSK constellation applied to the space-time code with 8 states (system with 2 transmission antennas and 1 receiving antenna).

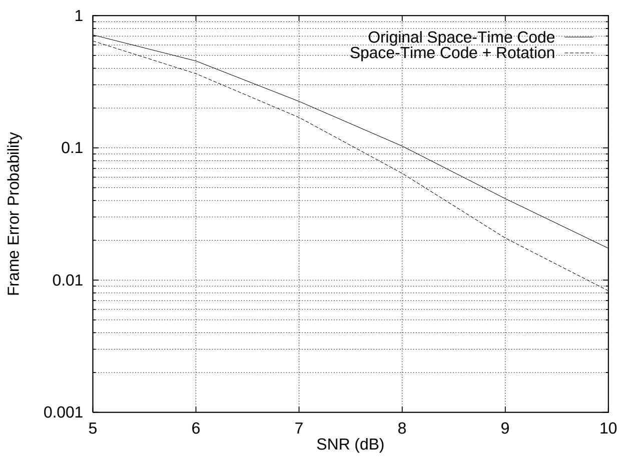


Fig. 9 - Comparing original and rotated QPSK constellation applied to the space-time code with 8 states (system with 2 transmission antennas and 2 receiving antennas).

5. CONCLUSIONS

This paper presented a modification in the original space-time code scheme with the objective of improving its bit error probability. The Space-Time Codes combine spatial and temporal diversity which produces a good performance for fading channels. The modification is based on interleaving and rotation of the signal constellation, which do not affect the performance of the system over the AWGN channel. The spectral efficiency of the QPSK scheme was not changed and the modification of the modulator is very simple. The optimum phase rotation was obtained and it has been shown that the bit error probability of the modified scheme was improved in a fading environment. The use of a rotated QPSK scheme together with the Space-Time code improves the performance of the transmission system, in terms of the bit error probability, due to the addition of a new kind of diversity, called modulation diversity.

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