Burst-Error Statistics Obtained by ML Estimation of a New Structured Markov Channel Model

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***Abstract*— A new Markovian model (MM) for burst-error generation has been recently presented by the authors. This model is conveniently structured, with the double purpose of reproducing the generation of some typical patterns of burst error sequences and facilitating the maximum-likelihood (ML) estimation of the its parameters. In the present work we show that this model also turns out to be useful for obtaining simple expressions for several statistics of great interest in the context of modeling and performance evaluation of communications systems degraded by burst-errors. So we propose to use these analytical expressions fulfilled with ML estimates of the model parameters in order to obtain good models for the above mentioned statistics. Some numerical results are presented that illustrate the effectiveness of this approach.**

Keywords*— *Markov model; ML estimation; burst; simulation.

I. Introduction

The investigation of tools for evaluating the impact of burst errors on the performance of networks, by means of analysis and simulation, has been a field of great interest over the last decades [1].

The search for simple models with the ability to reproduce important statistics of the error processes is an issue of utmost importance in this context. Markov Models (MM) and Hidden Markov Models (HMM) have been frequently adopted for this aim.

Hidden Markov Models (HMM) have also been successfully applied to satellite and wireless communications [1]–[4]. Besides, simple HMM such as the Gilbert-Elliot model have been shown to be useful for performance analysis, not only to evaluate the impact of errors generated in the physical layer, but also in regard of the errors produced in higher layers [1].

In spite of the widespread interest in using Markov models for burst-error processes, several works have pointed out some limitations of these approaches, specially for frequently requiring the increase in the number of states in order to fit well some desired burst-error statistics [5]. This increase may lead to an unfeasible number of parameters to be adjusted, eliminating this way the main advantage of those models.

Some recent works have been devoted to improvements in Markovian channel models, by proposing moderate increases in the model complexity, such as the use of hierarchical models [6], [7] and the so-called bipartite models [8].

A new way to circumvent some usual limitations of the Markovian approach for burst channel modeling without any sacrifice of simplicity was proposed by the authors in [9]. Our basic idea consists of imposing a convenient structure to the Markov channel model, inspired in some usual mechanisms of burst error generation. This structure is conceived in such a way that the number of parameters to be estimated is decoupled from the order of the MM. Besides, the derivation of analytical expressions for ML estimation of those parameters is facilitated.

In the present paper we show that this model is also appropriate for analytical modeling of several statistics of burst error processes. We present the analytical expressions of some statistics of great interest and show that these expressions may be used jointly with ML estimates of the model parameters in order to easily provide descriptive models of burst errors. Numerical results here presented show the effectiveness of this approach.

This paper has 6 sections. The proposed model is summarized in Section II. The maximum likelihood estimation of its parameters is dealt with in section III. The analysis of some useful distributions in the context of burst error investigation is the subject of section IV. Numerical results are given is section V. Finally, our findings are summarized in section VI.

II. Structured Markov Model

An error sequence is initially represented as a finite sequence of ones and zeros associated with correct and error events, respectively. A gap (cluster) of length n is defined as a group of n consecutive zeros (ones) between two ones (zeros).

The proposed model is illustrated in Fig. 1. It is composed of $L+3$ states and split in two sub-models named *length-limited gaps* and *length-unlimited gaps*.

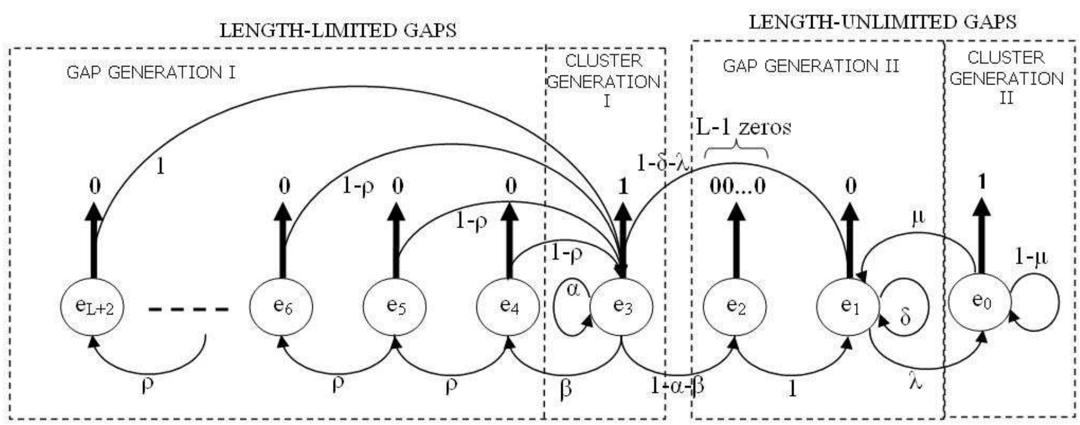

Fig. 1. Proposed model

Despite this nomenclature, the above mentioned sub-models may be involved in the generation of both gaps and clusters, with the basic difference that the gaps generated within the first sub-model are limited in length to a maximum of $L-1$. The states directly involved in the generation of such gaps (e_4, e_5, . . ., e_{L+2}) comprise the group that is called *gap generation I* in Fig. 1.

On the other hand, unrestricted-in-length gaps may be generated by the so called *length-unlimited gaps* sub-model. In particular, gaps of any length may be originated from a number of self-transitions of state e_1. Besides, gaps of lengths equal to or greater then L may also be generated within the group named *gap generation II* in Fig. 1, which is composed by states e_1 and e_2.

In respect of the generation of clusters in the *length-unlimited gaps* sub-model, it is worth noticing that any cluster's length may be obtained with transitions to e_0 and its self-transitions. Clusters of unrestricted length may also be generated within the other sub-model, with transitions to state e_3 followed by self-transitions.

There are significant differences in the roles played by states e_0 and e_3 in the generation of error sequences. In particular, it should be noted that state e_3 is more closely connected with both the mechanisms for producing gaps (*gap generation I* and *gap generation II* in Fig. 1) and in this sense it connects the sub-models of length-limited and length-unlimited gaps. On the other hand, e_0 is a cluster-generating state with a direct connection to the group *gap generation II* only. In order to highlight the differences in the generation of clusters by states e_3 and e_0, they are labeled *cluster generation I* and *cluster generation II*, respectively, in Fig. 1.

III. ML ESTIMATION

We collect the 7 parameters of the model in the vector $\theta \triangleq (\alpha,\beta,\delta,\lambda,\mu,\rho,L)$ and assume that the observation starts with an error. This observation is regarded as a sequence of $K+1$ pairs of clusters and gaps. It is represented by the vector $\mathbf{n} \triangleq (n_0, n_1, \cdots, n_{2K}, n_{2K+1})$, where n_{2k} and n_{2k+1} denote the lengths of the cluster and the gap in the $(k+1)$-th pair, respectively.

We also define for convenience the auxiliary function $\gamma_t(i) \triangleq P(n_0, n_1, \cdots n_{2t}, n_{2t+1})$, for $t \in \{0,1,\cdots K\}$, which is the probability of occurrence of the first $t+1$ cluster-gap pairs. It can be recursively calculated as follows:

$$\gamma_t(i) = \sum_{j\in\{0,3\}} \gamma_{t-1}(j) P[n_{2k}, n_{2k+1}, e_i | e_j], \tag{1}$$

where $P[n_{2k}, n_{2k+1}, e_i|e_j]$ denotes the conditional probability of emitting a gap of length n_{2k} followed by a cluster of length n_{2k+1} and being after that at state e_i, given that the model was in state e_j before these emissions.

By analyzing the model in Fig. 1 we obtained the following expressions, after some manipulations:

$$\begin{aligned}
P[n_g, n_c, e_0|e_0] &= \mu\delta^{n_g-1}\lambda(1-\mu)^{n_c-1}\\
P[n_g, n_c, e_3|e_0] &= \mu\delta^{n_g-1}(1-\delta-\lambda)\alpha^{n_c-1}\\
P[n_g, n_c, e_0|e_3] &= \begin{cases} 0, \ n_g < L;\\ \eta\delta^{n_g-L}\lambda(1-\mu)^{n_c-1}, \ n_g \geq L.\end{cases}\\
P[n_g, n_c, e_3|e_3] &= \begin{cases} \beta\rho^{n_g-1}(1-\rho)\alpha^{n_c-1}, \ n_g \leq L-2;\\ \beta\rho^{L-2}\alpha^{n_c-1}, \ n_g = L-1;\\ \eta\delta^{n_g-L}(1-\alpha-\lambda)\alpha^{n_c-1}, \ n_g \geq L,\end{cases}
\end{aligned} \tag{2}$$

being $\eta = 1-\alpha-\beta$.

The likelihood function may be expressed as

$$V(\theta) \triangleq \sum_{i\in\{0,3\}} \gamma_K(i;\theta), \tag{3}$$

where the dependence of $\gamma_K(.)$ with θ has been made explicit for the sake of clearness.

This form of calculating the ML metrics is one important advantage provided by the proposed model. As the vector θ contains a discrete parameter (L), the ML estimation is performed in two main steps. Initially $V(\theta)$ is maximized in the continuous parameters for fixed values of L taken in a large pre-established set. The best estimates of the continuous elements of θ, as well as the corresponding values of the objective function are retained in this phase. The second step consists on searching for the best value of the objective function among those previously retained, in order to arrive to the ML estimate of θ.

IV. Analytical Expressions for the Statistics of Interest

Besides its advantages for facilitating ML estimation, the model in Fig.1 also provides easy mathematical analysis of statistics of great interest in the context of burst channel investigation.

We adopt the CCITT definition of burst [10], which states that an error burst is a group of bits that starts and ends with a “1” in which the number of contiguous “0” is less than a maximum number that is here called *interburst threshold* and is denoted by X. An error-free burst is defined as a sequence of zeros with length great or equal to X.

We focus on the four statistics defined below. For the sake of clearness in the presentation of our numerical results in the following section, the definitions of these statistics were slightly changed but we maintained the usual nomenclature of [5], as follows:

- gap distribution (GD) - the complementary cumulative distribution of gap lengths G, $P(G \geq m)$. A gap (G) of an error sequence is defined as a run of the consecutive zeros between two ones and has a length equals to the number of consecutive zeros;
- error cluster distribution (ECD) - the complementary cumulative distribution function of cluster lengths C, $P(C \geq m)$. A cluster (C) of an error sequence is defined as a run of the consecutive ones between two zeros and has a length equals to the number of consecutive ones;
- error-free run distribution (EFRD) - defined as the conditional probability of the occurrence of m consecutive error-free bits, given a previous occurrence of an error, which we denote by $P[0^m|1]$;
- error-free burst distribution (EFBD) - the complementary cumulative distribution of error-free burst lengths EFB, $P(EFB \geq m)$. An error-free burst is a gap of length greater than or equal to a predefined parameter L;

For obtaining the gap distribution, it is enough to have an expression for the probability of the occurrence of an amount m of correct decisions in between two errors, which we denote by $P[10^m1]$. By analyzing the model in Fig. 1 it may be verified that

$$P[10^m1] = x_0\mu(1-\delta)\delta^{m-1} + x_3D, \tag{4}$$

where

$$D = \begin{cases} \beta(1-\rho)\rho^{m-1},\ m \in \{1,2,\ldots,L-2\} \\ \beta\rho^{L-2},\ m = L-1, \\ (1-\alpha-\beta)(1-\delta)\delta^{m-L},\ m \geq L \end{cases} \tag{5}$$

The parameters x_k for $k = 0,1,2,\ldots,L+2$ represent the steady-state values of the state probabilities.

Using the above expression, the probability of a gap of length m may be expressed as

$$P[G=m] = \frac{P[10^m1]}{\mu x_0 + (1-\alpha)x_3},\ m \geq 1. \tag{6}$$

In a similar way, the probability of a cluster of length m may be shown to be given by

$$P[C=m] = \frac{P[01^m0]}{\mu(1-\delta)x_0/\lambda + \beta x_3} \tag{7}$$

$$= \frac{x_1(1-\delta)+\beta x_3}{\mu x_0 + (1-\alpha)x_3}, m \geq 1. \tag{8}$$

The probability of an error-free burst of a given length $m \geq X$ may be expressed as $P(EFB=m) = P(G=m)/P(G \geq m)$. Using (4), it may be verified that

$$P(EFB=m) = \frac{P[10^m1]}{\mu\delta^{X-1}x_0 + (1-\alpha-\beta)\delta^{X-L}x_3}. \tag{9}$$

Finally, the conditional probability of occurring m error-free decisions, given the previous occurrence of an error, may be verified to be given by

$$P[0^m|1] = \begin{cases} \frac{F+x_3[(1-\alpha-\beta)+\beta\rho^{m-1}]}{x_0+x_3},\ m < L \\ \frac{F+x_3(1-\alpha-\beta)\rho^{m-1}}{x_0+x_3},\ m \geq L, \end{cases} \tag{10}$$

where $F = x_0\mu\delta^{m-1}$.

V. Results

We have evaluated the ability of the proposed model to capture the statistical properties of target error sequences produced in a typical scenario of burst error generation, namely the transmission over flat-fading channels.

In more specific terms, we have modeled error sequences obtained by simulation of the transmission of PSK-4 modulated signals over time-varying Rayleigh channel with Doppler spectrum modeled by the Jakes' model and normalized maximum Doppler shift of 10^{-1}. The E_b/N_0 ratio was fixed at 10 dB and perfect phase synchronization in the receiver was assumed.

After performing the ML estimation of the proposed-model parameters we have applied the analytical expressions of the four statistics of interest and compared the results so obtained with their counterparts empirically obtained from the original data sequence. The curves so obtained

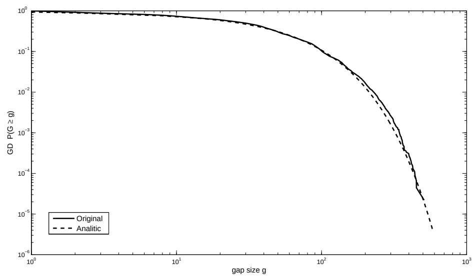

Fig. 2. Gap distribution (GD).

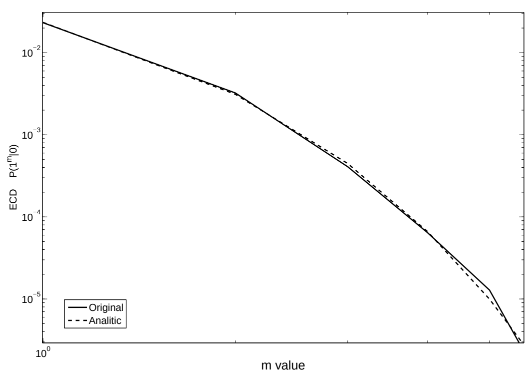

Fig. 3. Error cluster distribution (ECD).

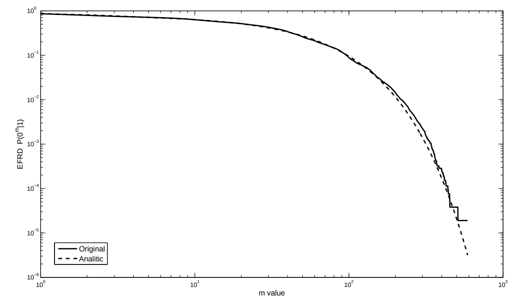

Fig. 4. Error-free run distribution (EFRD).

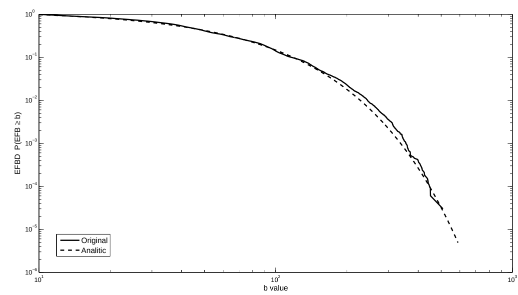

Fig. 5. Error-free burst distribution (EFBD).

are respectively named ”‘analytical”’ and ”original”’ in the following.

Fig. 2 shows the estimate of the gap distribution produced by the proposed model with the analytical expressions and the estimated values of its parameters, as well as that obtained with the original data. An almost perfect agreement between these estimates is observed.

A similar behavior may be observed in Fig. 3, that shows the results obtained in the estimation of the error cluster distribution.

Fig. 4 shows the results obtained for the error-free run distribution. A very good agreement between the analytical curve ant that obtained from the original data may be observed once again.

Finally, Fig 5 shows the results obtained for the error-free burst distribution. The interburst threshold X has been set to the ML estimate of L, which was 10 in this case. An excellent adjustment between the curve obtained by analysis and ML estimation of the model parameters and that obtained from the original data is shown once more.

Summarizing, the above shown figures indicate that the use of the ML estimates of the parameters of the model proposed in [9] within the analytical expressions here derived for the statistics under consideration produces very good fittings to the corresponding estimates calculated from the original error data.

VI. Conclusion

The modeling of several statistics of interest in the context of burst error investigation has been here performed, taking as a tool the structured Markovian model for burst-error channel proposed in [9]. This model is specially structured for reproducing the generation of typical patterns of burst-error sequences and has only 7 parameters to be adjusted, irrespective of its number of states. Simple expressions for those statistics have been derived and have been proposed to be applied jointly with ML estimates of the model parameters in order to obtain descriptive models of burst error statistics of interest for performance investigation purposes. Numerical results showed that this approach is effective.

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